

THE NETWORK ANALYSIS OF INTERLOCKING DIRECTORS: THE CASE OF THAILAND'S LISTED COMPANIES

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1. Introduction

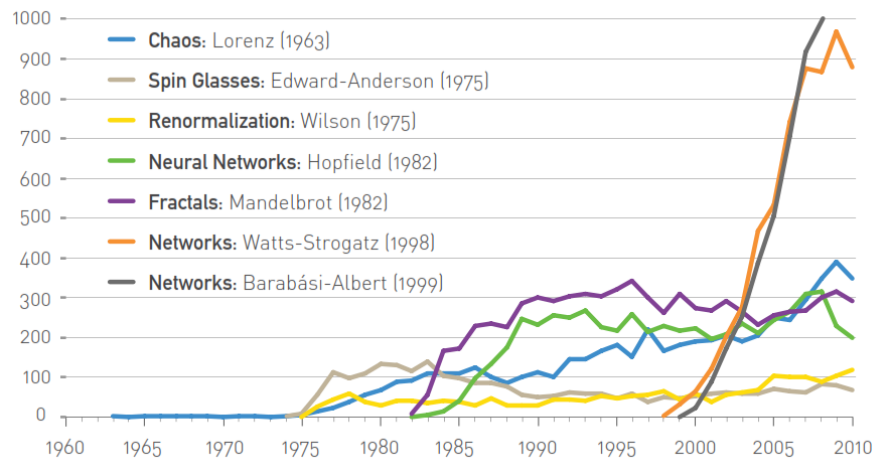
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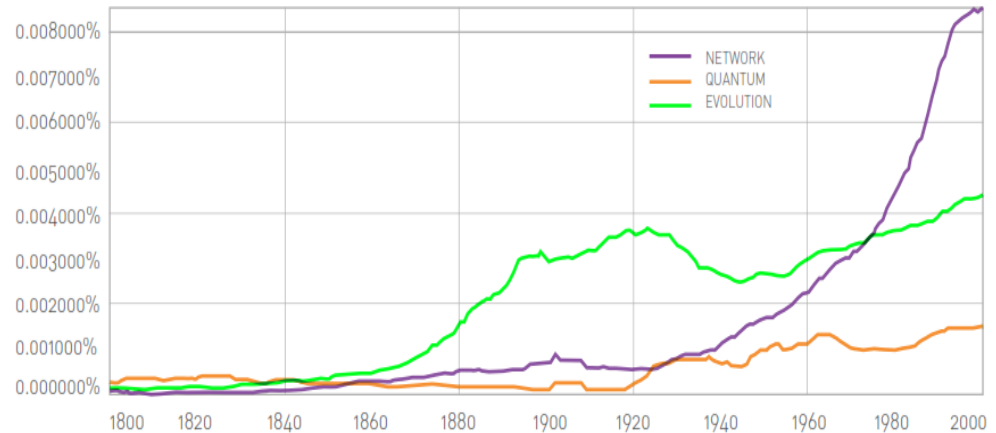
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1. Introduction



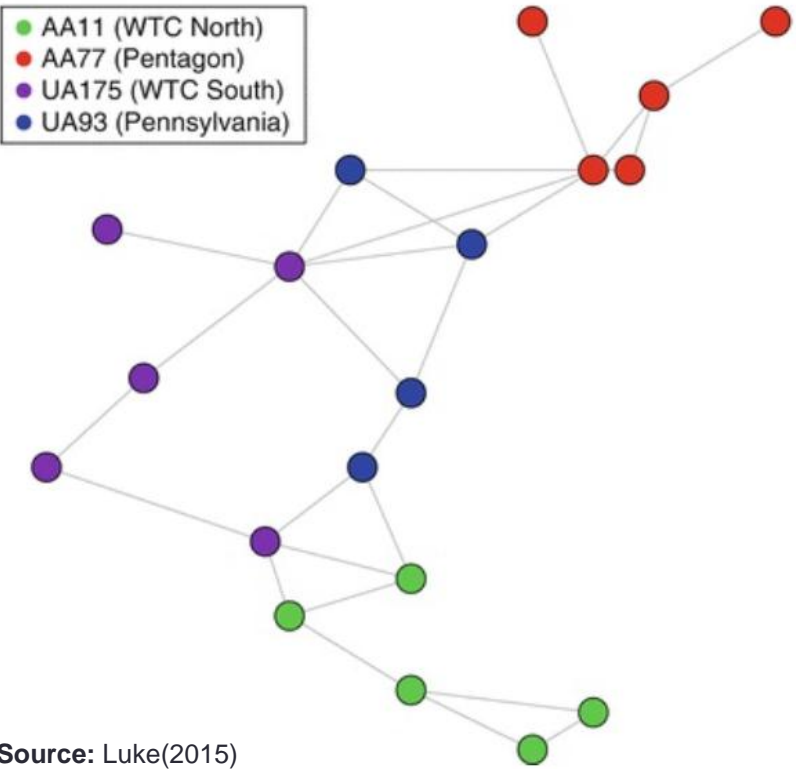
Source: Barabási (2016)



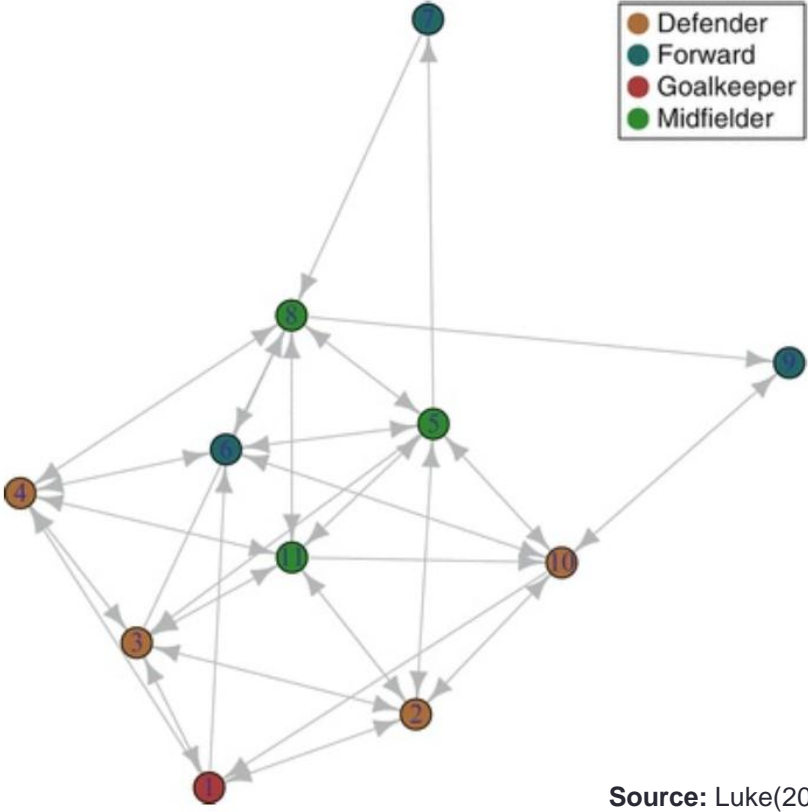
Source: Barabási (2016)

- The development of network analysis has been one of the most prominent fields of studies in the past 70 years.
- Barabási (2016) stated that when compared to other main areas of studies in science such as **quantum** and **artificial intelligence**, the **network analysis has surpassed** them in term of the **number of citations** and **the frequency of use of the words** in publications.
- Not only does the development of theories in network analysis provide the new foundation for **physic**, **chemistry**, **biology** and **engineering**, but it also broadens the knowledge in **social science**, **economics**, and even the **counterintelligence investigation** as documented by Lawton(2016).

Examples of Networks



Network of 9–11 hijackers

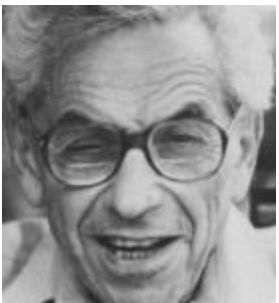


Network of Netherlands 2010 World Cup soccer team

1.1 Random Graph Model

- The **theoretical foundation** of network analysis has been initiated by **Erdős and Rényi (1959)**.
- Their work introduced the mathematical framework describing the quantitative characteristics of **network formulated by randomness** (i.e. the **random graph model**).
- Specifically, this theory enables **the prediction** of key indicators of **network's properties** such as:
 - the numbers of linkages
 - the average length of all paths
 - the diameter of the network

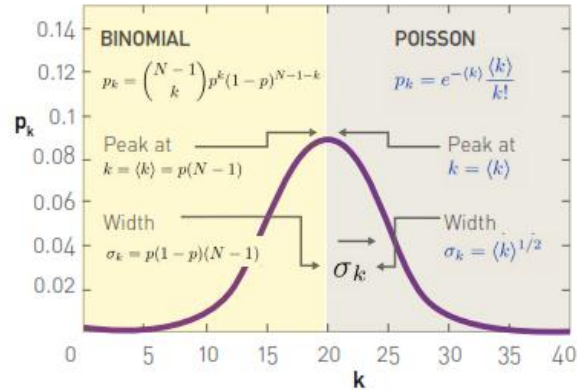
1.1 Random Graph Model



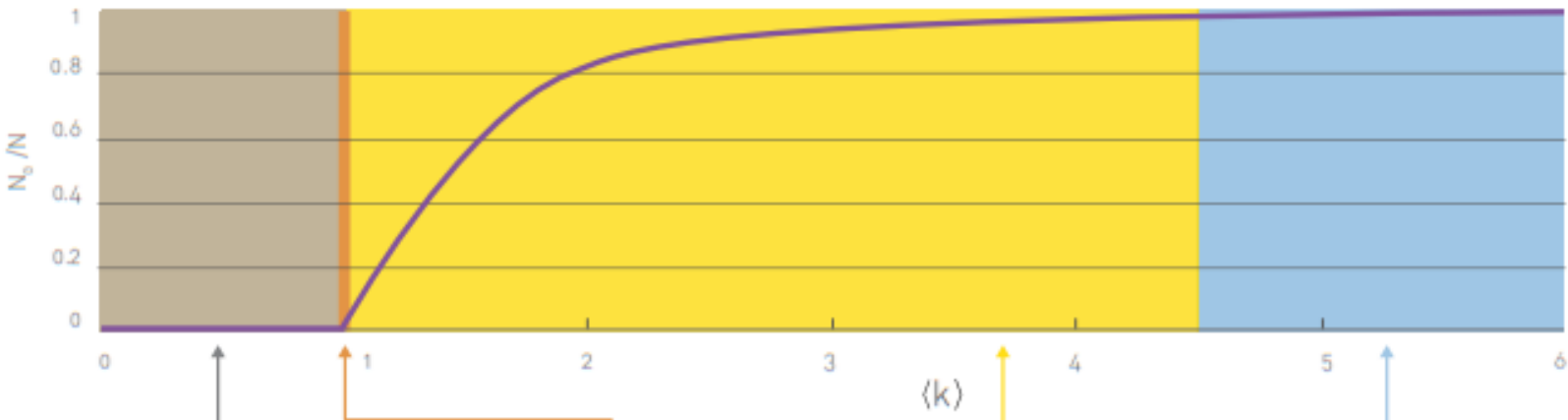
Pál Erdős
(1913-1996)



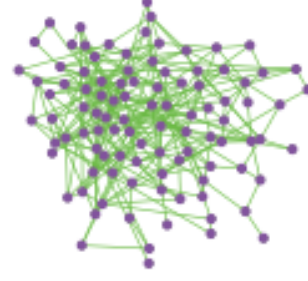
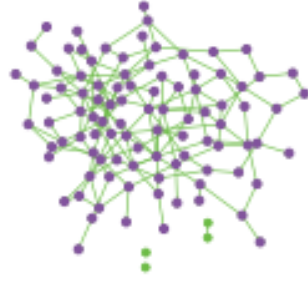
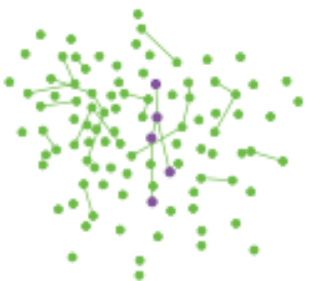
Alfréd Rényi
(1921-1970)



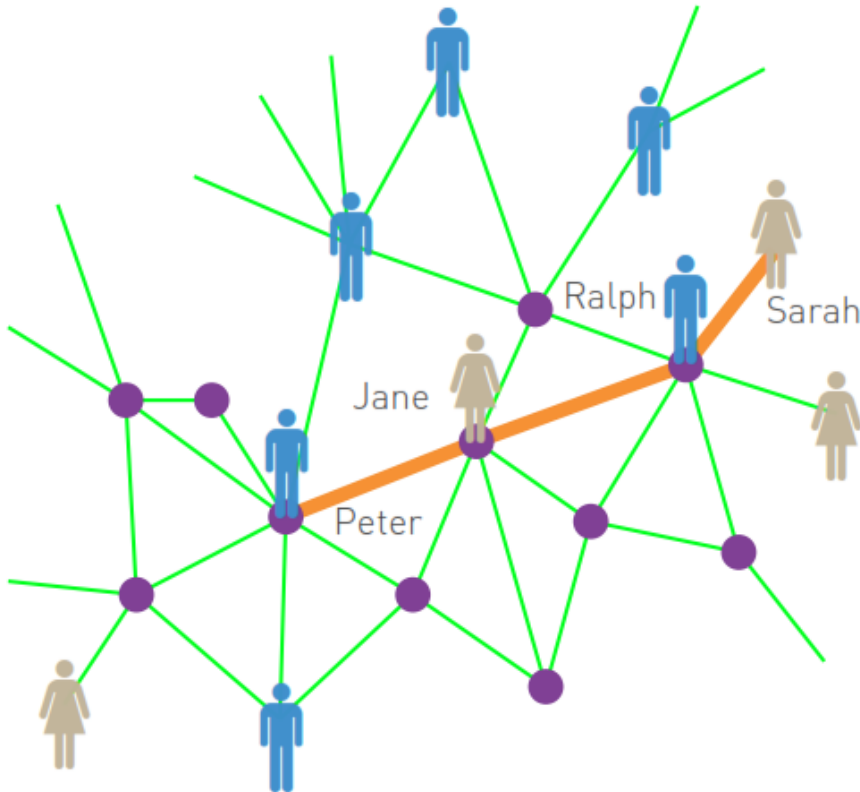
Source: Barabási (2016)



Source: Barabási (2016)



1.2 Small-World Model



Source: Barabási (2016)

- Simultaneously, Ithiel de Sola Pool and Manfred Kochen initiated the mathematical framework in 1958.
- Later, de Sola Pool and Kochen (1978) explained the **special characteristic of human's society**.
- Their work proposes the idea that there is actually **a short path linking any pair of persons in the world** (i.e. the small world effect).

1.2 Small World Model (Cont'd)



Manfred Kochen



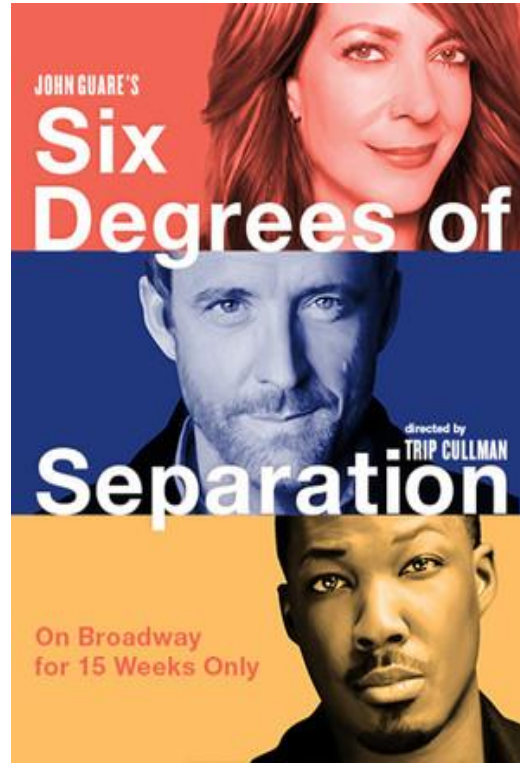
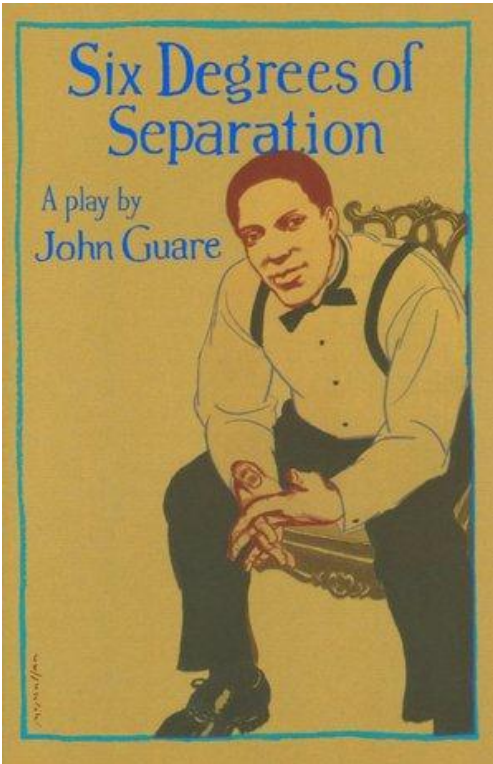
Ithiel de Sola Pool



Stanley Milgram

- Milgram(1967) documented the outcome of his **actual experiment** of small world effect using the **technique of randomly distributed mails**.
- His work, conducted in the US, has indicated that the **median of intermediates is 5.2**, implying that there are approximately the **maximum of 5-6 links connecting any pair of persons in the society**.

1.2 Small World Model (Cont'd)



- Guare(1991) applied the finding of Milgram(1967) as the fundamental of his famous Broadway play ‘**six degrees of separation**’, and this play’s title has become the popular phrase.

“Everybody on this planet is separated by only six other people. Six degrees of separation. Between us and everybody else on this planet. The president of the United States. A gondolier in Venice. ... I am bound to everyone on this planet by a trail of six people.”

Guare, 1991

1.2 Small World Model (cont'd)



Duncan J. Watts



Steven Strogatz

“The **average path length or the diameter** depends logarithmically on the system size. Hence, “small” means that the distance is proportional to **$\ln N$, rather than N or some power of N** ”

Source: Barabási (2016)

- Scientifically, Watts and Strogatz(1998) solidified the concept of small world effect through their breakthrough work.
- Their model provides the **mathematical framework** representing the **formulation and quantitative characteristics** of the small world phenomena.

1.3 Interlocking Directorate

- In finance, the pattern of **interlocking directorate** has been observed for many decades, and it triggered attentions from regulators and researchers.
- In addition, the development of methods in network analysis quantitatively **unveils two hidden networks**, which are the **affiliation among directors** and **the linkages among firms**.
- Particularly many literatures have documented the empirical evidence of **small-world pattern in stock markets** in cases of both developed and developing countries.
- These revealed outcomes shed light on the new insight of the connection among listed companies.
- Also they unveils the **new dimension of networks** that might **extend the fundamental concept of symmetric risk through interlocking directorate**.

1.4 Research Questions

With the evolving development of techniques in network analysis, this paper aims at examining the following characteristics of listed companies in Thailand using the data of 2012 - 2016.

- The **quantitative characteristics of the network** of directors and that of listed firms obtained from network analysis
- The **existence of small world pattern**
- The **quantitative features of subgroups** within the network of directors and that firms classified by network analysis

2. Literature Review

2.1 Theories of Interlocking Directorates

2.2 Network Analysis of Interlocking Directorates

2.1 Theories of Interlocking Directorates

(1) The reciprocity theory

- The interconnection of companies enabled the **mitigation of external uncertainties** through the **harmonized managerial activity** across boards.
- Pfeffer and Salancik(1978),Bazerman and Schoramn(1983), Stearns and Mizruchi (1986)

(2) The resource-based theory

- The obvious case of the resource-based formation is the **connection between the financial firm and its borrowers.**
- Likewise the similar connection is based on the **supply chain relationship**, interlocking the boards of supplier and purchaser.
- Mizruchi and Stearns (1988), Richardson(1987), Land and Lockhart(1990) and Elouaer-Mrizak(2012)

(3) The class hegemony theory

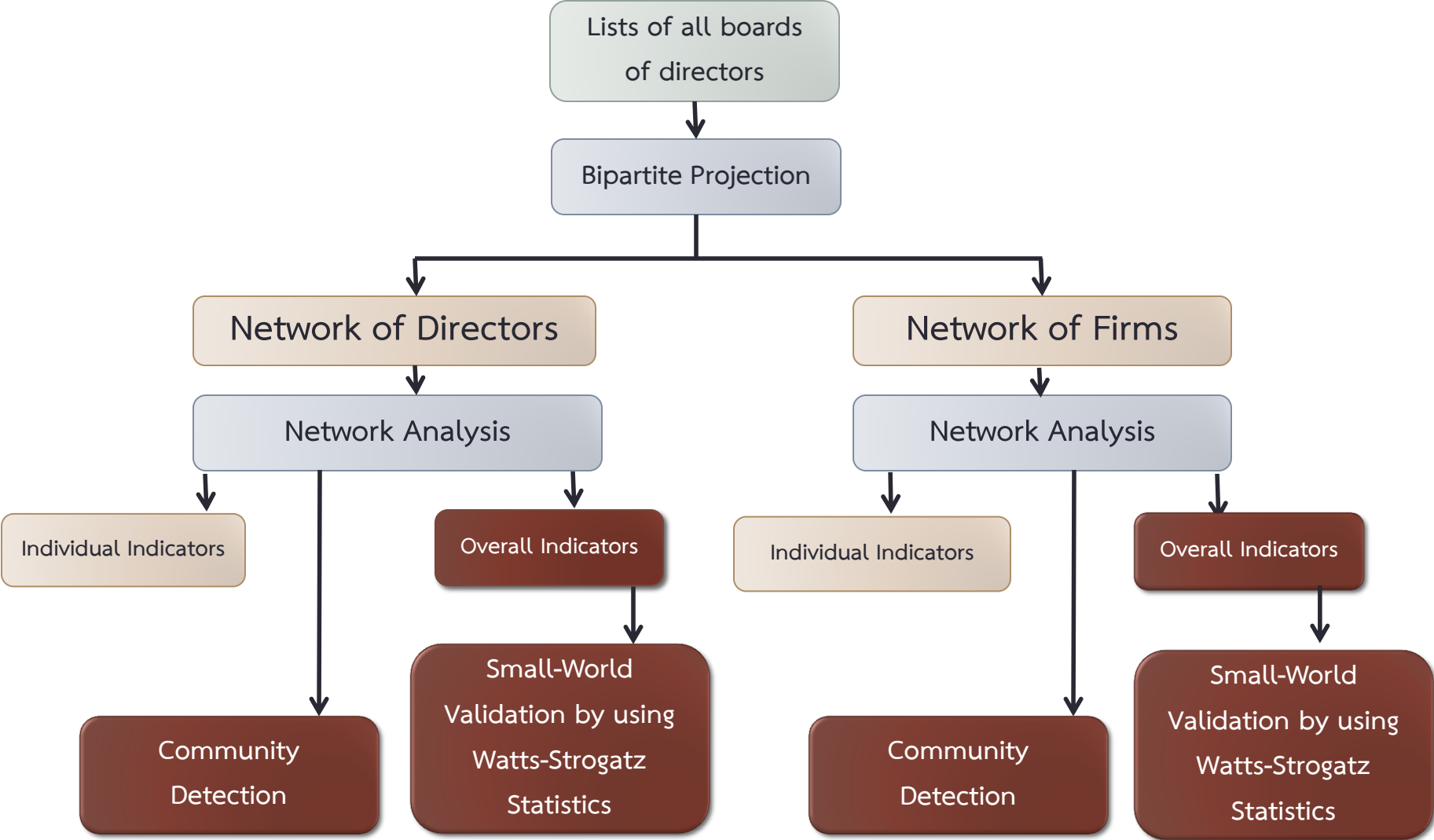
- The assignment of directors to corporates' boards is **based on the social connection**, particularly the dominant groups in the society.
- Palmer(1983), Caswell (1984) and Drago et al. (2015)

2.2 Network Analysis of Interlocking Directorates

Counties	Related literatures
US	Battiston and Catanzaro (2004)
	Caldarelli and Catanzaro (2004)
	Davis et al. (2003)
	Strogatz (2001)
	Conyon and Muldon (2006)
	Caldarelli (2007)
	Robins and Alexander (2004)
	Kogut (2012)
UK	Conyon and Muldon (2006)
	Kogut (2012)
Australia	Robins and Alexander (2004)
Canada	Kogut (2012)
Italy	Battiston and Catanzaro (2004)
	Caldarelli and Catanzaro (2004)
	Caldarelli (2007)
	Kogut (2012)
Germany	Conyon and Muldon (2006)
	Kogut (2012)
The Netherlands and Switzerland	Kogut (2012)
	Heemskerk and Schnyder (2008)
Denmark, Norway and Sweden	Kogut (2012)
	Sinani et al. (2008)
South Africa	Durbach and Parker (2009)
South Korea	Kogut (2012)
	Nam and An (2017)
Poland	Sankowska and Siudak (2016)
Greek	Dimitrios and Vasileios (2015)
Spain, France, Brazil, Chile, Israel, Mexico and Taiwan	Kogut (2012)

3. Research Methods

3. Research Methods



4. Results

4.1 Quantitative Characteristics of Networks

4.2 Validation of Small-World Networks

4.3 Community Detection

4.1 Quantitative Characteristics of Networks

Main indicators of network's properties

- **Degree:** One of main properties of each node is degree. The magnitude of degree indicates **the number of links** to other nodes. If we define k_i as the degree of the i node and N is the number of total nodes, the total number of links (L) is denoted as:

$$L = \frac{1}{2} \sum_{i=1}^N k_i$$

- **Average Degree:** One of key indicators of the network is the average degree, denoted as $\langle k \rangle$. The formula for computing $\langle k \rangle$ is defined as:

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i = \frac{2L}{N}$$

- **Average Path Length:** The **average distance between all pairs** in a particular network is defined as the average path length, mathematically denoted as $\langle d \rangle$.

$$\langle d \rangle = \frac{1}{N(N-1)} \sum_{i,j=1 \text{ and } i \neq j}^N d_{i,j}$$

4.1 Quantitative Characteristics of Networks (cont'd)

- **Shortest Path:** The shortest path is the **route between two nodes which has the minimum distance**. Alternatively, it is called Geodesic Path.
- **Diameter:** The **longest path between a pair of nodes** in the network is defined as the diameter (d_{max}). This indicator represents the width of the network.
- **Density:** The density is the **ratio of total number of links to the maximum number of potential links**. For the network with N node, the maximum possible number of links is $N(N-1)/2$. Therefore, the density(S) of the network is mathematically denoted as:

$$S = \frac{2L}{N(N - 1)}$$

4.1 Quantitative Characteristics of Networks (cont'd)

- **Betweenness Centrality:** In addition to the number of links or the distance to others, one of important properties of a particular node is **its function of transmitting physical matter or information** to other nodes. Based on this concept, the Betweenness Centrality is the metric **quantifying the significance** of each node as **the key transmitter to the network**. Mathematically, this measure is based on the computation of number of sets of shortest paths to other nodes. Therefore, the highest Betweenness Centrality node has the maximum numbers of shortest paths to other nodes, enabling it to minimize time to transmit a substance or the information to other nodes with the maximum network coverage.

4.1 Quantitative Characteristics of Networks (cont'd)

- **Eigenvector Centrality:** the Eigenvector Centrality has been developed by **summing only linkages connected to important nodes**. Mathematically speaking, the computation is based on **the eigenvalue of the adjacency matrix**, which is the tabulated representation of nodes and linkages.

4.1 Quantitative Characteristics of Networks (cont'd)

Proportion of Interlocking Directors

Year	Total Directors	Interlocking Directors	(% of total)	Single-Board Director	(% of total)	Avg. Directors per Board
2012	6,516	1,030	15.81%	5,486	84.19%	10.13
2013	6,725	1,072	15.94%	5,653	84.06%	10.27
2014	6,804	1,047	15.39%	5,757	84.61%	10.39
2015	6,896	1,089	15.79%	5,807	84.21%	10.28
2016	6,991	1,123	16.06%	5,868	83.94%	10.49

4.1 Quantitative Characteristics of Networks (cont'd)

Summary of key indicators of network's properties

(the network of directors)

Year	Type of Network	Density	Standardized average degree	Closeness index	Eigen index	Diameter	Transitivity index	Average shortest path length
2012	Director Network	0.0082	0.0335	0.0017	0.9683	10.0000	0.6413	4.1056
2013	Director Network	0.0121	0.0395	0.0058	0.9364	9.0000	0.6001	3.8134
2014	Director Network	0.0119	0.0428	0.0061	0.9464	9.0000	0.5907	3.8469
2015	Director Network	0.0101	0.0404	0.0043	0.9368	11.0000	0.6385	3.9284
2016	Director Network	0.0117	0.0423	0.0049	0.9343	10.0000	0.6130	4.0310

4.1 Quantitative Characteristics of Networks (cont'd)

Summary of key indicators of network's properties

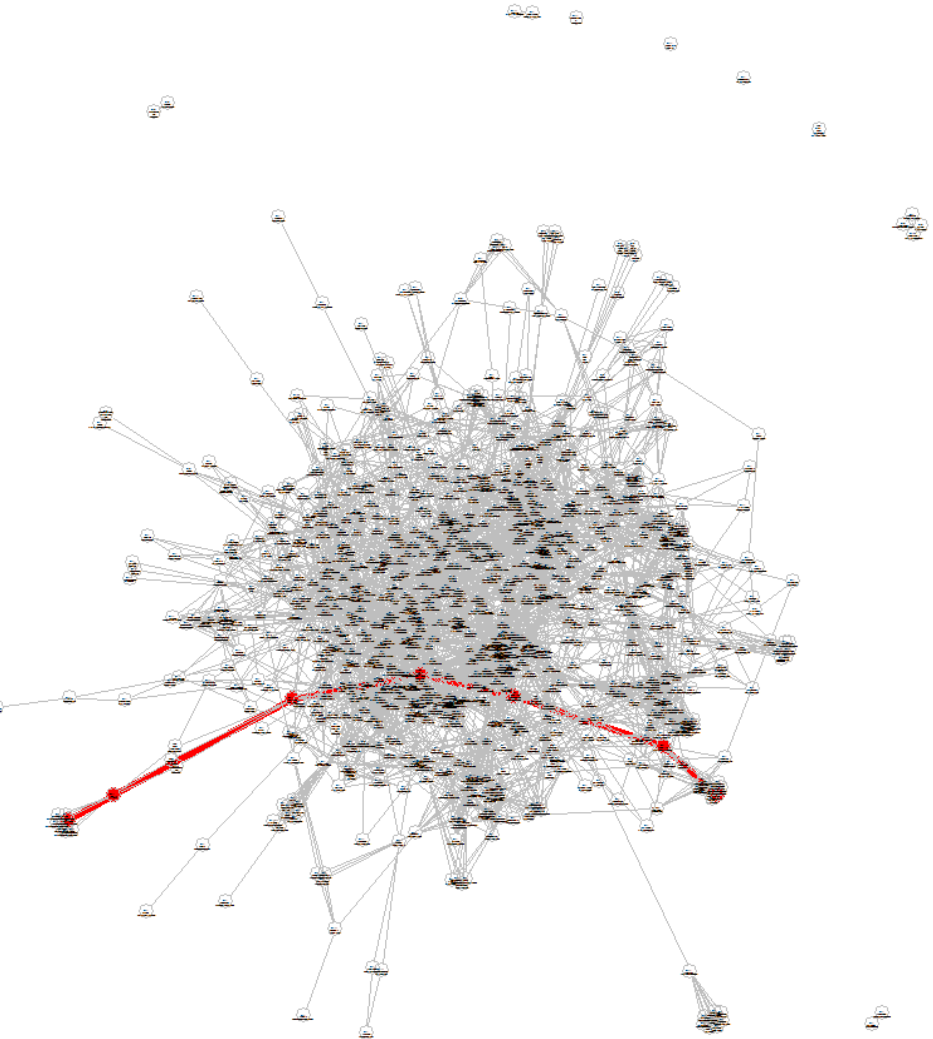
(the network of listed companies)

Year	Type of Network	Density	Standardized average degree	Closeness index	Eigen index	Diameter	Transitivity index	Average shortest path length
2012	Board Network	0.0081	0.0355	0.0026	0.9199	15.0000	0.3878	4.2342
2013	Board Network	0.0094	0.0349	0.0027	0.8993	15.0000	0.3835	3.9568
2014	Board Network	0.0094	0.0350	0.0028	0.9043	12.0000	0.3999	4.0182
2015	Board Network	0.0093	0.0340	0.0027	0.9061	11.0000	0.3950	3.9478
2016	Board Network	0.0083	0.0413	0.0026	0.9340	13.0000	0.3984	4.1595

4.1 Quantitative Characteristics of Networks (cont'd)

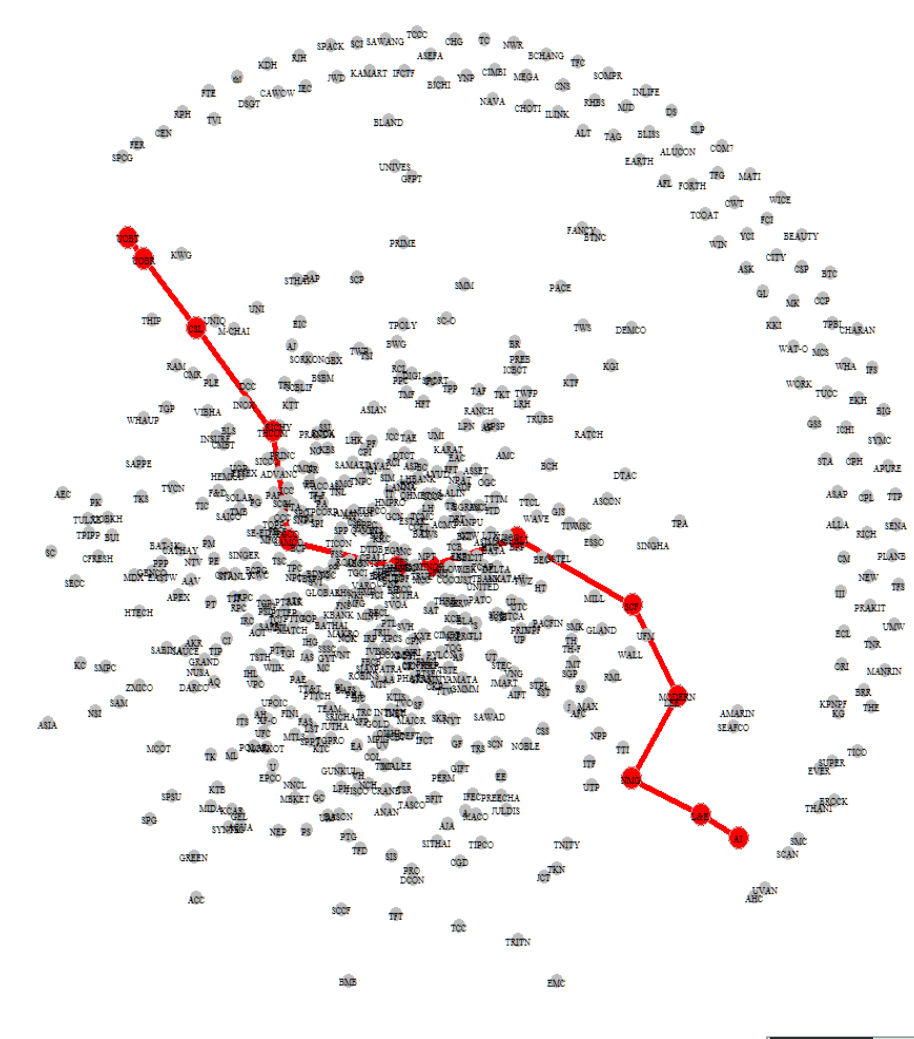
Diameter of network of directors

(computationally formulated based on Betweenness index of directors in 2016)



Diameter of network of listed companies

(computationally formulated based on Betweenness index of boards in 2016)



Ranking of Betweenness index (network of directors)**

Betweenness index		Betweenness index		Betweenness index		Betweenness index		Betweenness index	
2012		2013		2014		2015		2016	
Mr. CHACKCHAI PANICHAPAT	0.039	Mr. MANU LEOPAIROTE	0.045	Mr. CHATCHAVAL JIARAVANON	0.050	Mr. MANU LEOPAIROTE	0.046	Mr. MANU LEOPAIROTE	0.064
Mr. PRAKIT PRADIPASEN	0.037	Miss POTJANEE THANAVARANIT	0.036	Mr. MANU LEOPAIROTE	0.041	Mr. SUCHIN WANGLEE	0.043	Mr. PRASERT BUNSUMPUN	0.049
Mrs. CHANTRA PURNARIKSHA	0.033	Mr. SUCHIN WANGLEE	0.036	Mr. SUCHIN WANGLEE	0.040	Miss POTJANEE THANAVARANIT	0.037	Mr. SUCHIN WANGLEE	0.040
Mr. SUCHIN WANGLEE	0.032	Mr. CHATCHAVAL JIARAVANON	0.036	Miss POTJANEE THANAVARANIT	0.037	Mr. CHATCHAVAL JIARAVANON	0.036	Mr. ARSA SARASIN	0.038
Mr. WEERAWONG CHITTMITTRAPAP	0.031	Mr. PRASERT BUNSUMPUN	0.033	Mr. PRAKIT PRADIPASEN	0.032	Mr. PRAKIT PRADIPASEN	0.034	Mr. PRAKIT PRADIPASEN	0.036
Mr. MANU LEOPAIROTE	0.030	Mr. WISSANU KREA - NGAM	0.028	Mr. PRASERT BUNSUMPUN	0.031	Mr. WEERAWONG CHITTMITTRAPAP	0.033	Mr. RAWAT CHAMCHALERM	0.032
Mr. WISSANU KREA - NGAM	0.030	Mrs. NUALPHAN LAMSAM	0.028	Mr. SUVARN VALAISATHIEN	0.028	Mr. PRASERT BUNSUMPUN	0.032	Mr. VIRACH APHIMETEETAMRONG	0.031
MR. PONG SARASIN	0.029	Mr. PRAKIT PRADIPASEN	0.028	Mr. SIRI GANJARERNDDEE	0.026	Mr. SIRIPOL YODMUANGCHAROEN	0.027	Mr. SUVARN VALAISATHIEN	0.030
Mr. CHAI SOPHONPANICH	0.025	Mr. JOTI BHOKAVANIJ	0.025	Mr. SATIT CHANJAVANAKUL	0.024	Mr. ARSA SARASIN	0.026	Mr. CHAINOI PUANKOSOOM	0.026
Mr. CHATCHAVAL JIARAVANON	0.025	Mrs. KULPATRA SIRODOM	0.025	Mr. CHAI SOPHONPANICH	0.023	Mr. CHAI SOPHONPANICH	0.024	Mr. CHATCHAVAL JIARAVANON	0.026

** This is an outcome of scientific computation. With all best regards, the author has no intention of creating any negative consequences to all directors listed in this table.

Ranking of Betweenness index (network of boards of listed companies)

Betweenness index		Betweenness index		Betweenness index		Betweenness index		Betweenness index	
2012		2013		2014		2015		2016	
SSC	0.036	SCB	0.033	TRUE	0.031	TOC	0.032	SSC	0.044
SCC	0.032	MFC	0.030	THRE	0.029	DTC	0.032	DTC	0.043
KBANK	0.032	SCC	0.028	ERW	0.029	SCC	0.031	TOC	0.036
BH	0.030	LOXLEY	0.027	FBCB	0.028	SCB	0.029	MINOR	0.032
SAMCO	0.026	FBCB	0.027	SCB	0.027	ERW	0.029	SCC	0.031
LOXLEY	0.026	TRUE	0.026	LOXLEY	0.026	THRE	0.028	ERW	0.027
FBCB	0.025	BKI	0.024	SCC	0.025	TRUE	0.028	SPALI	0.027
ERW	0.025	THAI	0.024	TOC	0.024	BKI	0.025	TRUE	0.025
TRUE	0.025	THRE	0.023	DTC	0.023	MFC	0.024	MTI	0.025
CSC	0.024	TOC	0.022	MC	0.023	BAY	0.024	BH	0.024

Ranking of standardized average degree (network of directors)**

Std. Avg. Degree		Std. Avg. Degree		Std. Avg. Degree		Std. Avg. Degree		Std. Avg. Degree	
2012		2013		2014		2015		2016	
Mr. SUCHIN WANGLEE	0.042	Mr. SUCHIN WANGLEE	0.051	Mr. SUCHIN WANGLEE	0.054	Mr. SUCHIN WANGLEE	0.050	Mr. ARSA SARASIN	0.054
Mr. BOONSITHI CHOKWATANA	0.035	Mr. PRASERT BUNSUMPUN	0.049	Mr. PRASERT BUNSUMPUN	0.048	Mr. PRASERT BUNSUMPUN	0.046	Mr. SUCHIN WANGLEE	0.051
Mr. WEERAWONG CHITTMITTRAPAP	0.034	Miss POTJANEE THANAVARANIT	0.044	Mr. MANU LEOPAIROTE	0.043	Mr. ARSA SARASIN	0.042	Mr. PRASERT BUNSUMPUN	0.051
Mr. APIPORN PASAWAT	0.034	Mr. MANU LEOPAIROTE	0.042	Mr. CHATCHAVAL JIARAVANON	0.042	Mr. MANU LEOPAIROTE	0.040	Mr. RAWAT CHAMCHALERM	0.046
Mr. BOONCHAI CHOKWATANA	0.033	Mr. BOONCHAI CHOKWATANA	0.041	Miss POTJANEE THANAVARANIT	0.040	Mr. WEERAWONG CHITTMITTRAPAP	0.038	Mr. MANU LEOPAIROTE	0.044
Mr. CHACKCHAI PANICHAPAT	0.033	Mr. CHATCHAVAL JIARAVANON	0.038	Mr. ARSA SARASIN	0.038	Miss POTJANEE THANAVARANIT	0.037	Mr. SITHICHAI CHAIKRIANGKRAI	0.042
Mr. PRAJYA PHINYAWAT	0.033	Mr. BOONSITHI CHOKWATANA	0.038	Mr. BOONSITHI CHOKWATANA	0.038	Mr. CHATCHAVAL JIARAVANON	0.036	Mr. WEERAWONG CHITTMITTRAPAP	0.038
Mr. CHATCHAVAL JIARAVANON	0.032	Mr. ARSA SARASIN	0.037	Mr. AVIRUTH WONGBUDDHAPITAK	0.037	Mr. CHOKCHAI AKSARANAN	0.036	Mr. THAPANA SIRIVADHANABHAKDI	0.038
Mr. MANU LEOPAIROTE	0.031	Mr. AVIRUTH WONGBUDDHAPITAK	0.037	Mrs. KULPATRA SIRODOM	0.037	Mrs. KULPATRA SIRODOM	0.035	Mr. BOONSITHI CHOKWATANA	0.036
Mr. WISSANU KREA - NGAM	0.031	Mr. PRAJYA PHINYAWAT	0.037	Mr. JOTI BHOKAVANIJ	0.037	Mr. BOONSITHI CHOKWATANA	0.034	Mr. CHATCHAVAL JIARAVANON	0.035

** This is an outcome of scientific computation. With all best regards, the author has no intention of creating any negative consequences to all directors listed in this table.

Ranking of standardized average degree (network of boards of listed companies)

Std. Avg. Degree		Std. Avg. Degree		Std. Avg. Degree		Std. Avg. Degree		Std. Avg. Degree	
2012		2013		2014		2015		2016	
SSC	0.043	ERW	0.044	ERW	0.044	THRE	0.043	SSC	0.049
ERW	0.042	SSC	0.044	THRE	0.042	ERW	0.041	SCC	0.037
HIPRO	0.040	HIPRO	0.042	HIPRO	0.041	HIPRO	0.040	TRUE	0.037
TRUE	0.037	THRE	0.041	SSC	0.041	TRUE	0.040	DTC	0.036
SPI	0.035	TRUE	0.041	TRUE	0.041	SCC	0.038	HIPRO	0.036
SCC	0.034	MINOR	0.036	SCB	0.038	SSC	0.038	TOC	0.036
MINOR	0.032	SCB	0.036	MINOR	0.036	DTC	0.037	ERW	0.034
MPT	0.032	LOXLEY	0.035	SCC	0.035	SCB	0.037	MINOR	0.033
TOC	0.031	TOC	0.035	DRT	0.032	TOC	0.037	LOXLEY	0.031
DRT	0.029	DRT	0.032	MPT	0.032	MINOR	0.035	SPI	0.028

Ranking of Pagerank index (network of directors)**

Pagerank index		Pagerank index		Pagerank index		Pagerank index		Pagerank index	
2012		2013		2014		2015		2016	
Mr. SUCHIN WANGLEE	0.004	Mr. SUCHIN WANGLEE	0.004	Mr. SUCHIN WANGLEE	0.004	Mr. SUCHIN WANGLEE	0.004	Mr. PRASERT BUNSUMPUN	0.004
Mr. BOONSITHI CHOKWATANA	0.003	Mr. PRASERT BUNSUMPUN	0.003	Mr. PRASERT BUNSUMPUN	0.004	Mr. PRASERT BUNSUMPUN	0.003	Mr. ARSA SARASIN	0.003
Mr. PRAJYA PHINYAWAT	0.003	Mr. BOONSITHI CHOKWATANA	0.003	Mr. BOONSITHI CHOKWATANA	0.003	Mr. ARSA SARASIN	0.003	Mr. SITHICHA CHAIKRIANGKRAI	0.003
Mr. SITHICHA CHAIKRIANGKRAI	0.003	Mr. PRAJYA PHINYAWAT	0.003	Mr. MANU LEOPAIROTE	0.003	Mr. BANYONG PONGPANICH	0.003	Mr. SUCHIN WANGLEE	0.003
Mr. APIPORN PASAWAT	0.003	Mr. BOONCHAI CHOKWATANA	0.003	Mr. BANYONG PONGPANICH	0.003	Mr. BOONSITHI CHOKWATANA	0.003	Mr. WEERAWONG CHITTMITRAPAP	0.003
Mr. PRAKIT PRADIPASEN	0.003	Miss POTJANEE THANAVARANIT	0.003	Mr. CHATCHAVAL JIARAVANON	0.003	Mr. CHATCHAVAL JIARAVANON	0.003	Mr. PRAJYA PHINYAWAT	0.003
Mr. BOONCHAI CHOKWATANA	0.003	Mr. SITHICHA CHAIKRIANGKRAI	0.003	Mr. PRAKIT PRADIPASEN	0.003	Mr. CHUMPOL NALAMLIENG	0.003	Mr. THAPANA SIRIVADHANABHA KDI	0.003
Mr. WISSANU KREA - NGAM	0.003	Mr. THAPANA SIRIVADHANABHAKDI	0.003	Mr. SIRI GANJARERNDDEE	0.003	Mr. MANU LEOPAIROTE	0.003	Mr. BOONSITHI CHOKWATANA	0.003
Mr. CHATCHAVAL JIARAVANON	0.003	Mr. MANU LEOPAIROTE	0.003	Miss POTJANEE THANAVARANIT	0.003	Miss POTJANEE THANAVARANIT	0.003	Mr. MANU LEOPAIROTE	0.003
Mr. THAPANA SIRIVADHANABHAKDI	0.003	Mr. CHATCHAVAL JIARAVANON	0.003	Mr. CHUMPOL NALAMLIENG	0.003	Mr. SIRI GANJARERNDDEE	0.002	Mr. VUDHIPHOL SURIYABHIVADH	0.003

** This is an outcome of scientific computation. With all best regards, the author has no intention of creating any negative consequences to all directors listed in this table.

Ranking of Pagerank index (network of boards of listed companies)

Pagerank index		Pagerank index		Pagerank index		Pagerank index		Pagerank index	
2012		2013		2014		2015		2016	
SPI	0.006	SSC	0.005	TRUE	0.005	TRUE	0.005	SSC	0.006
SSC	0.005	TRUE	0.005	ERW	0.005	TOC	0.005	BJC	0.005
BJC	0.005	SPI	0.005	SPI	0.005	THRE	0.004	BMCL	0.005
SFP	0.005	TF	0.005	SSC	0.005	SPI	0.004	BTC	0.005
ERW	0.005	BJC	0.005	THRE	0.004	ERW	0.004	SPI	0.005
TF	0.005	ERW	0.005	TF	0.004	TF	0.004	TOC	0.005
TRUE	0.005	TOC	0.004	CPN	0.004	SSC	0.004	TRUE	0.005
TOC	0.005	SFP	0.004	BJC	0.004	KSL	0.004	KSL	0.004
NPC	0.004	THRE	0.004	KSL	0.004	BJC	0.004	ERW	0.004
HIPRO	0.004	HIPRO	0.004	SCB	0.004	HIPRO	0.004	MINOR	0.004

4.2 Validation of Small-World Networks

- The first classification of network is the **Random-Graph** class or the **Erdős–Rényi model**.
- This model assumes that **each pair of nodes** is **randomly connected**.
- Therefore, this model is the hypothetical case.
- Furthermore, most of empirical evidences documented that the **actual structure of human's society has a different property** from that of Erdős–Rényi model.

4.2 Validation of Small-World Networks (cont'd)

- Particularly, it is conventionally found that the **actual network** has the **substantially higher degree of transitivity** (i.e. the **high value of clustering coefficient**).
- Following Humphries and Gurney (2008) and Sankowska and Siudak (2016), the Watts-Strogatz Statistics (S^{WS}) can be computed from the following equation.

$$S^{WS} = \frac{\gamma^{sw}}{\lambda},$$

$$\gamma^{sw} = \frac{\text{Cluster Coefficient of Actual Data}}{\text{Cluster Coefficient of Random Graph}}$$

$$\lambda = \frac{\text{Avg Shortest Path Length of Actual Data}}{\text{Avg Shortest Path Length of Random Graph}}$$

Based on the model developed by Watts and Strogatz (1998), it is expected that $\gamma^{sw} \gg 1.0$ and $\lambda \approx 1.00$, leading to the case of $S^{WS} \gg 1.00$, which is the **main indicator of the small-world case**.

4.2 Validation of Small-World Networks (cont'd)

Result of Watts-Strogatz Statistics (network of directors)

2012				2013				2014				2015				2016			
Model	Transitivity	Avg. path length	Density	Model	Transitivity	Avg. path length	Density	Model	Transitivity	Avg. path length	Density	Model	Transitivity	Avg. path length	Density	Model	Transitivity	Avg. path length	Density
Random-graph model	0.008	3.488	0.008	random-graph model	0.011	3.097	0.011	random-graph model	0.011	3.114	0.011	random-graph model	0.009	3.183	0.011	random-graph model	0.011	3.103	0.012
Actual data	0.641	4.105	0.008	actual data	0.600	3.813	0.012	actual data	0.590	3.846	0.011	actual data	0.638	3.928 3	0.011	actual data	0.613	4.031	0.012
$\gamma^{sw} = 80.125, \lambda = 1.176, S^{ws} = 68.081$				$\gamma^{sw} = 54.545, \lambda = 1.231, S^{ws} = 44.303$				$\gamma^{sw} = 53.636, \lambda = 1.235, S^{ws} = 43.427$				$\gamma^{sw} = 70.888, \lambda = 1.234, S^{ws} = 57.439$				$\gamma^{sw} = 55.727, \lambda = 1.299, S^{ws} = 42.897$			

- This finding indicates the **small-world pattern** in the **network of directors**.

4.2 Validation of Small-World Networks (cont'd)

Result of Watts-Strogatz Statistics (network of listed firms)

2012				2013				2014				2015				2016			
Model	Transitivity	Avg. path length	Density	Model	Transitivity	Avg. path length	Density	Model	Transitivity	Avg. path length	Density	Model	Transitivity	Avg. path length	Density	Model	Transitivity	Avg. path length	Density
Random-graph model	0.006	4.146	0.008	Random-graph model	0.007	3.796	0.009	Random-graph model	0.008	3.813	0.009	Random-graph model	0.010	3.833	0.009	Random-graph model	0.013	4.103	0.008
Actual data	0.388	4.234	0.008	Actual data	0.383	3.957	0.009	Actual data	0.400	4.018	0.009	Actual data	0.395	3.948	0.009	Actual data	0.398	4.159	0.008
$\gamma^{sw} = 64.666, \lambda = 1.021, S^{ws} = 63.322$				$\gamma^{sw} = 54.714, \lambda = 1.042, S^{ws} = 52.448$				$\gamma^{sw} = 50.000, \lambda = 1.053, S^{ws} = 47.448$				$\gamma^{sw} = 39.500, \lambda = 1.030, S^{ws} = 38.349$				$\gamma^{sw} = 30.615, \lambda = 1.013, S^{ws} = 30.203$			

- Also this outcome identifies the **small-world pattern** in the **network of listed firms**.

4.2 Validation of Small-World Networks(cont'd)

- In the case of small-world network, the magnitude of **Transitivity** (i.e. the clustering coefficient) is **substantially larger** than that of the **random-graph model**.
- However, the value of **average shortest paths** of both **small-world** and **random-graph** networks are **within the same range**.
- This property resembles the **actual social structure**, which is **highly clustering**.

4.3 Community Detection

- In this study, in order to deeply examine the structure of network, the computational method of community detection was applied to the network of directors and the network of boards of listed companies.
- Specifically, the **Walktrap algorithm** was the method used.
- Developed by Pons and Latapy (2005), the Walktrap algorithm introduces the idea that for any network, the **distance between two nodes** can be measured based on the concept of **random walk and probability**.
- Because most of social networks are highly clustered, the community detection can **decompose the subtle affiliation** and **unveil the connection among sub-communities** constituting the network.

4.3 Community Detection (cont'd)

- Mathematically, it is defined that P^t is the probability of randomly walking from a randomly picked node in the community C to reach node j within t steps, denoted as:

$$P^t(Cj) = \frac{1}{|C|} \cdot \sum_{i \in C} P_{i,j}^t$$

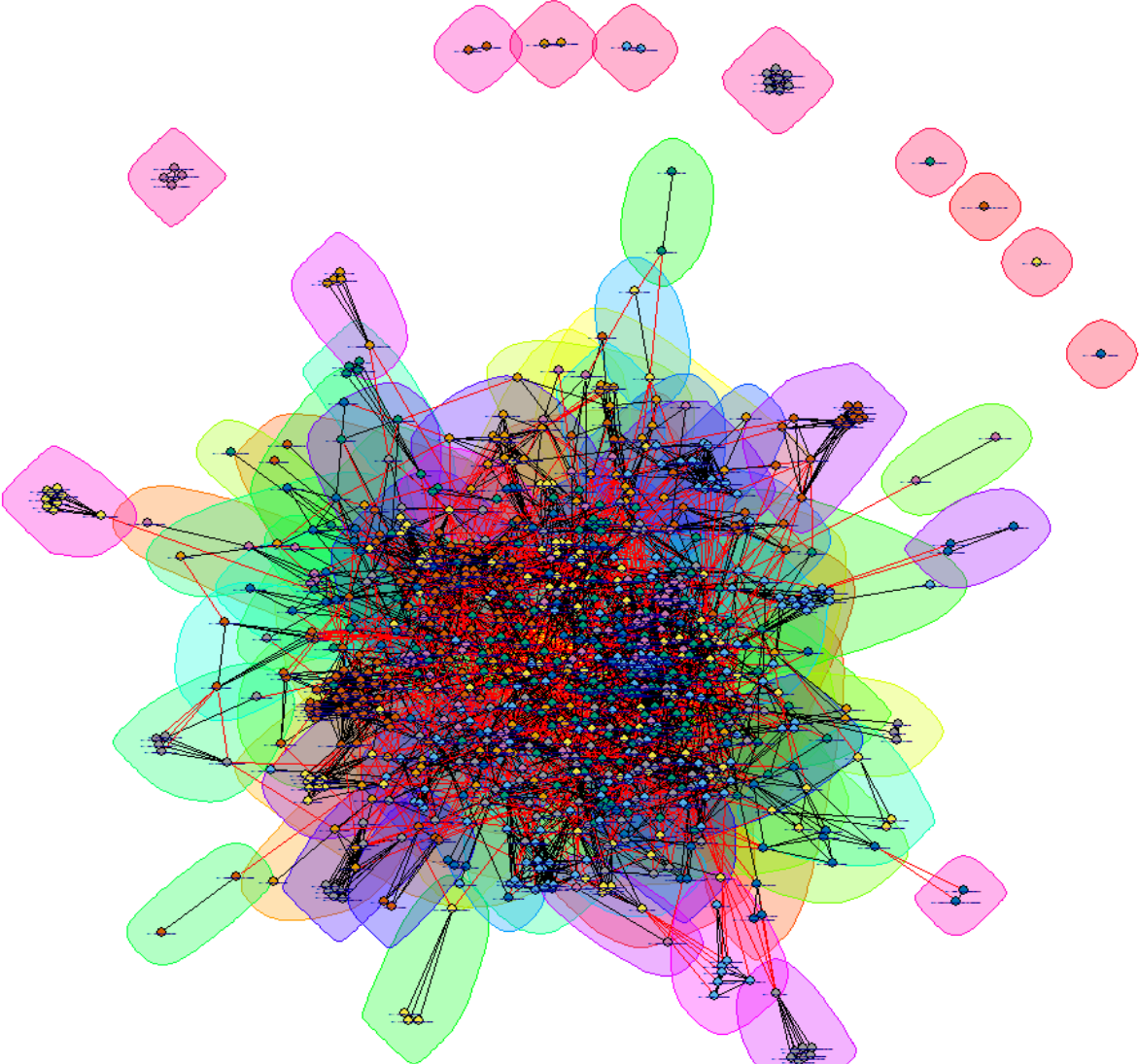
- The distance between two communities, C_1 and C_2 , of the network V is defined as r_t :

$$r_t(C_1, C_2) = \sqrt{\sum_{k=1}^n \frac{(P_{C_1k}^t - P_{C_2k}^t)^2}{d(k)}}$$
$$C_1, C_2 \in V$$

- Fundamentally, the Walktrap algorithm proposes the main idea that it is highly likely that **the random walker can get “trapped” in the densely connected parts (i.e. the communities) of the network.**
- Hence the **network can be separated into communities** based on this concept of **distance measured** in terms of P^t and r_t

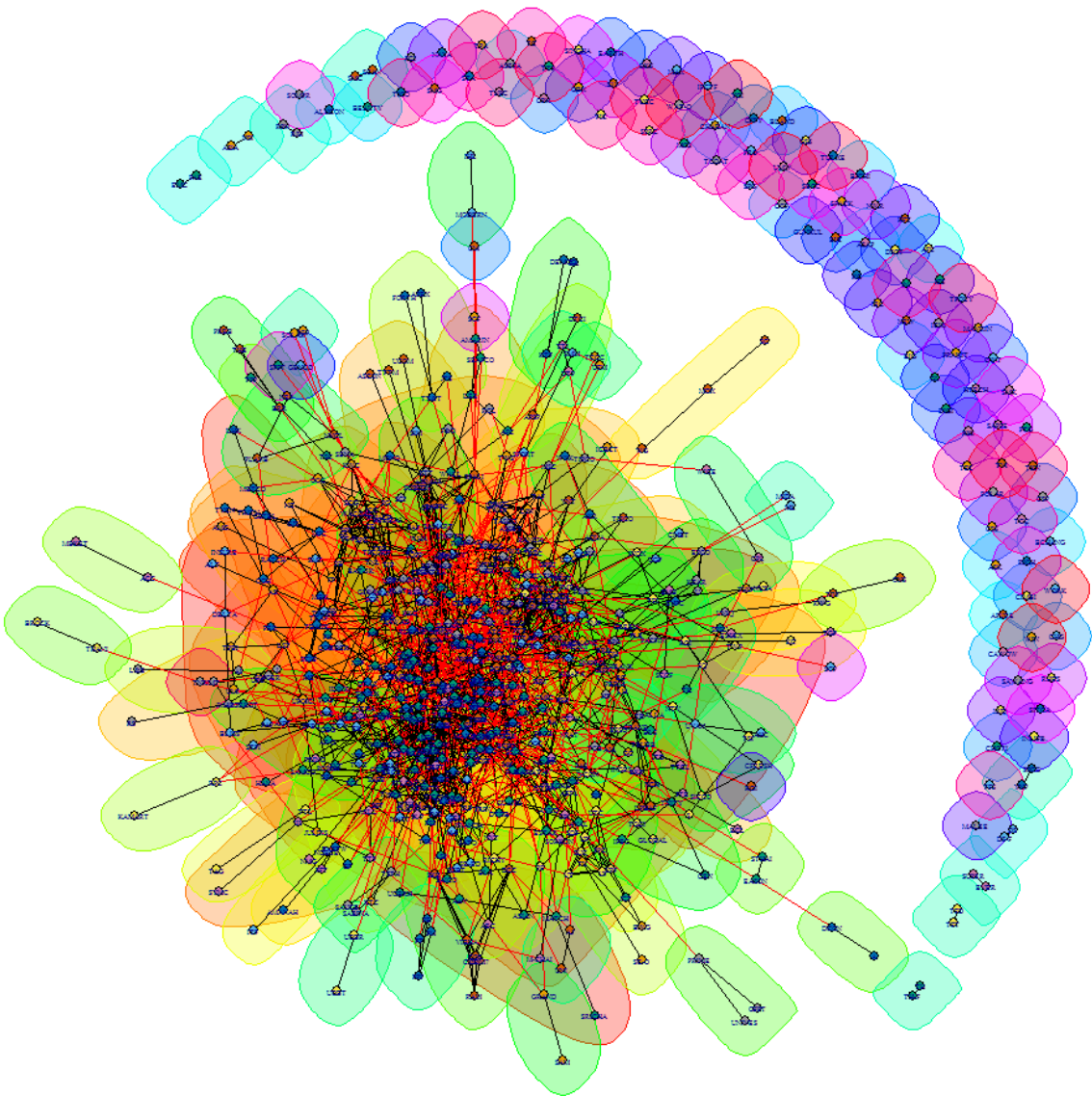
4.3 Community Detection (cont'd)

Subgroups of network of directors (data of 2016)



4.3 Community Detection (cont'd)

Subgroups of network of listed firms (data of 2016)



4.3 Community Detection (cont'd)

Characteristics of subgroups of directors

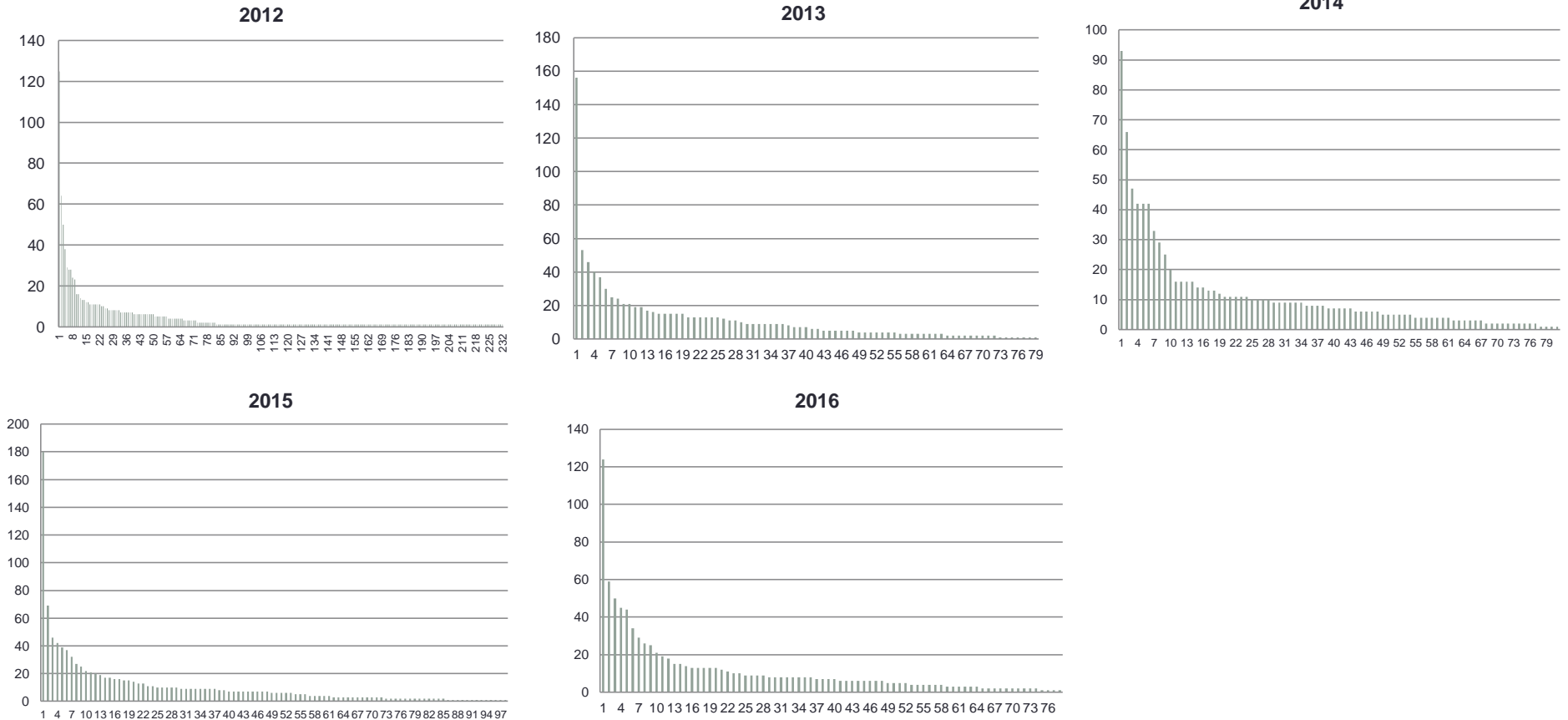
	Number. of interlocking directors	Number. of subgroups	Number. of subgroups having more than 2 directors	Number o. of subgroups having only one director	Total directors in top 10% of subgroups
2012	1,030	232	82	150	581
2013	1,072	79	72	7	411
2014	1,047	81	77	4	394
2015	1,089	99	85	14	472
2016	1,123	78	73	5	385

Characteristics of subgroups of boards of listed companies

	Total firms	Number. of subgroups	Number. of subgroups having more than 2 firms	Number. of subgroups having only one firm	Total firms in top 10% of subgroups
2012	643	192	91	101	328
2013	655	185	91	94	238
2014	655	170	89	81	249
2015	671	182	89	93	221
2016	666	187	92	95	239

4.3 Community Detection (cont'd)

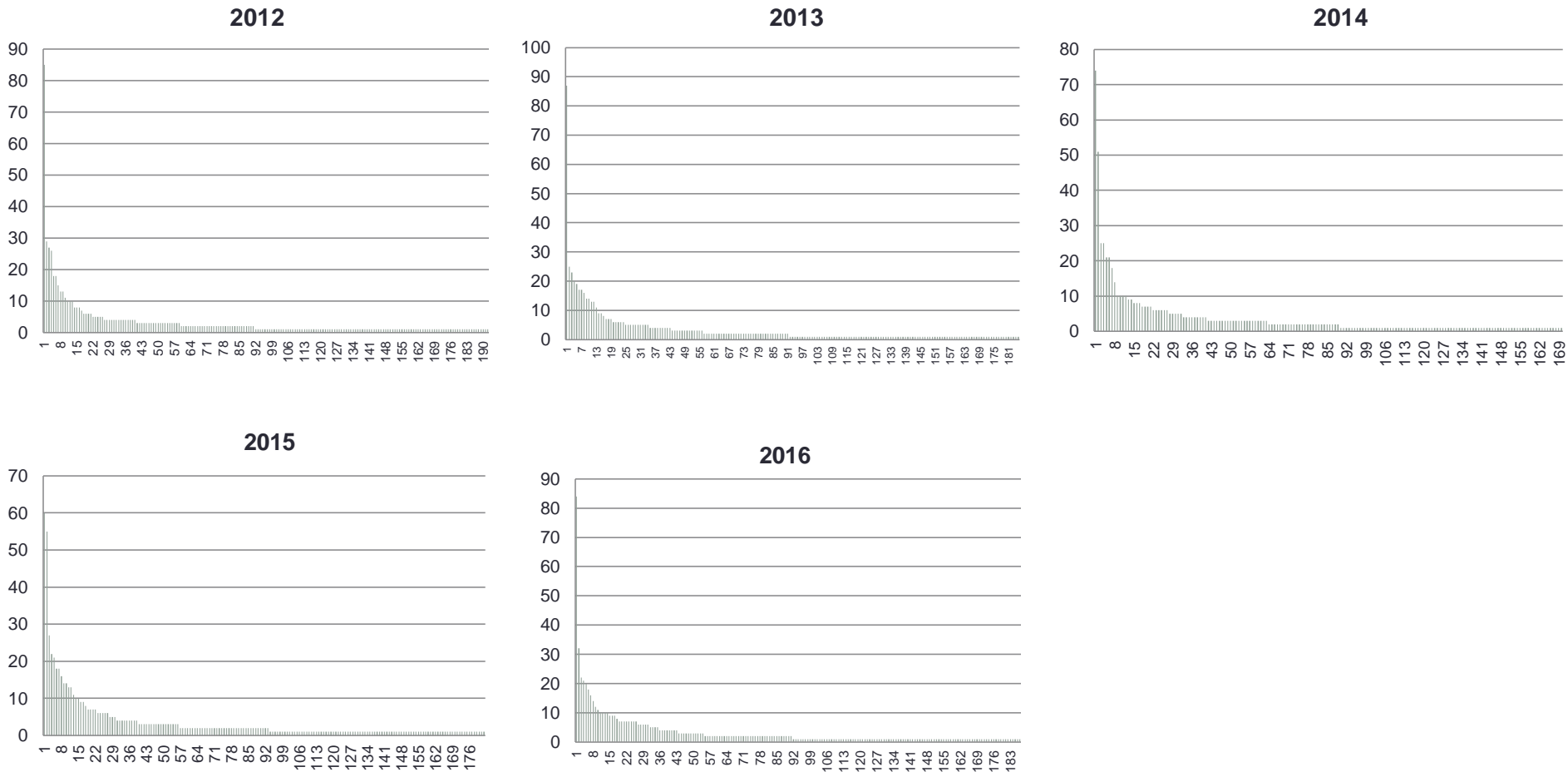
Distributions of members of director subgroups (2012-2016)



Note: the value on vertical axis is the number of members in each subgroup. The horizontal axis lists the identification number of each subgroup.

4.3 Community Detection (cont'd)

Distributions of members of board subgroups (2012-2016)



Note: the value on vertical axis is the number of members in each subgroup. The horizontal axis lists the identification number of each subgroup.

4.3 Community Detection (cont'd)

- The outcomes indicate that for both networks of directors and boards, there are only **a small number of subgroups constituting the majority of the network.**
- These unveiled properties of **highly clustering** and **unequal distribution of subgroups' members.**

5. Conclusion

5. Conclusion

- The outcomes show that the **structural properties** of both director network and that of boards of listed companies were **stable during 2012-2016**.
- The analysis also **confirmed** the case of **small-world networks** of locking boards based on the computed Watts-Strogatz Statistics. This result was **analogous** to **most of international empirical evidences**.
- The outcomes of community detection indicate that for both networks of directors and boards, there are **only a small number of subgroups constituting the majority of the network**.
- These findings suggest the future studies examining the consequences of these features on the performance of listed companies.

Thank you

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