Name:________________
ID:__________________

BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

Solution

EE320(sec 046402)
Semester 1/2020

Quiz#4

This quiz has two questions. Each question will be treated as if they are separated quizzes which will be added to the pool of your quizzes throughout the semester that you can drop the lowest score.

Company beCake sells cake in a competitive market. The price of cake is P. The firm needs two factors of production, labor (L) and capital(K). Market wage for labor and rent for capital are w and r, respectively.

To produce cake, company beCake uses production function $Q = F(L, K) = L^{\frac{1}{4}}K^{\frac{1}{2}}$.

Question 1 How many labor and capital should beCake employ in order to maximize the profit? Please state the optimization problem, find FOCs and solve for *unconditional factor demand functions*. You can skip checking SOC.

Solution:

$$\max_{L,K} PQ - wL - rK$$

$$\max_{L,K} PL^{\frac{1}{4}}K^{\frac{1}{2}} - wL - rK$$

FOCs:

$$\pi_L = \frac{\partial \pi}{\partial L} = P\frac{1}{4}L^{\frac{-3}{4}}K^{\frac{1}{2}} - w = 0 \quad \text{Hence,} \quad P\frac{1}{4}L^{\frac{-3}{4}}K^{\frac{1}{2}} = w \qquad (1.)$$

$$\pi_K = \frac{\partial \pi}{\partial K} = P\frac{1}{2}L^{\frac{1}{4}}K^{\frac{-1}{2}} - r = 0 \quad \text{Hence,} \quad P\frac{1}{2}L^{\frac{1}{4}}K^{\frac{-1}{2}} = r \qquad (2.)$$

(1.)/(2.)

$$\frac{K}{L} = 2\frac{w}{r}$$

We have:

$$K = 2\frac{w}{r}L \qquad (3.)$$

(1.) & (3.)

$$\frac{1}{4}L^{\frac{-3}{4}}(2\frac{w}{r}L)^{\frac{1}{2}} = w$$

Name:________________
ID:__________________

Thus, we have that the unconditional labor demand function is:

$$L^* = \frac{P^4}{64r^2w^2} \qquad \#$$

Use L^* and (3.), we have:

$$K^* = 2\frac{w}{r} \times \frac{P^4}{64r^2w^2}$$

Thus, we have that the unconditional capital demand function is:

$$K^* = \frac{P^4}{32r^3w} \qquad \#$$

Question 2 Now suppose that beCake has an advanced order that it needs to bake the cake Q_0 units.

(a.) How many labor and capital should beCake employ in order to minimize its cost? Please state the optimization problem, write the Lagrangian function, find FOCs and solve for *conditional factor demand functions*. You can skip checking SOC.

Solution:

$$\min_{L,K} wL + rK$$
$$\text{St. } L^{\frac{1}{4}}K^{\frac{1}{2}} = Q_0$$

The lagragian function is:

$$\min_{\lambda,L,K} Z = wL + rK + \lambda[Q_0 - L^{\frac{1}{4}}K^{\frac{1}{2}}]$$

FOCs:

$$\frac{\partial z}{\partial \lambda} = Q_0 - L^{\frac{1}{4}}K^{\frac{1}{2}} = 0 \qquad \text{Hence, } L^{\frac{1}{4}}K^{\frac{1}{2}} = Q_0 \qquad (1.)$$

$$\frac{\partial z}{\partial L} = w - \lambda\frac{1}{4}\,L^{\frac{-3}{4}}K^{\frac{1}{2}} = 0 \qquad \text{Hence, } \lambda\frac{1}{4}\,L^{\frac{-3}{4}}K^{\frac{1}{2}} = w \qquad (2.)$$

$$\frac{\partial z}{\partial K} = r - \lambda\frac{1}{2}\,L^{\frac{1}{4}}K^{\frac{-1}{2}} = 0 \qquad \text{Hence, } \lambda\frac{1}{2}\,L^{\frac{1}{4}}K^{\frac{-1}{2}} = r \qquad (3.)$$

(3.)/(4.)

Name:________________
ID:__________________

$$\frac{K}{L} = 2\frac{w}{r}$$

We have:

$$K = 2\frac{w}{r}L \qquad (4.)$$

Use (4.) in (1.),

$$L^{\frac{1}{4}}(2\frac{w}{r}L)^{\frac{1}{2}} = Q_0$$

Thus, we have that the conditional labor demand function is:

$$L^* = \left(\frac{r}{2w}\right)^{\frac{2}{3}} Q_0^{\frac{4}{3}} \qquad \#$$

Use L^* and (4.),

$$K = 2\frac{w}{r}\left(\frac{r}{2w}\right)^{\frac{2}{3}} Q_0^{\frac{4}{3}}$$

Thus, we have that the conditional capital demand function is:

$$K^* = \left(\frac{2w}{r}\right)^{\frac{1}{3}} Q_0^{\frac{4}{3}} \qquad \#$$

(b.) How does rent relate to unconditional demand for labor ? How does rent relate to conditional demand for labor? Are the two relations in the same direction ? Briefly explain the intuition behind the patterns of these relations.

Solution:

From above solutions, we have that:

$$L^*_{unconditional} = \frac{P^4}{64r^2w^2} \qquad \#$$

$$L^*_{conditional} = \left(\frac{r}{2w}\right)^{\frac{2}{3}} Q_0^{\frac{4}{3}} \qquad \#$$

This say that when rent decreases, unconditional labor demand increases, while conditional labor demand decreases.

Name:________________
ID:__________________

Unconditional labor demand increases because when rent decreases firm has incentive to use more capital to produce more output. Without the need to keep the level of output constant, firm demands more labor to use them with additional employed capital as well.
Conditional labor demand decreases because firm now substitute capital for labor (turn to use cheaper capital instead) to produce the same level of output.