3.3.2) Multi-market model (General equilibrium model)

- So far, we have only analyzed the market equilibrium concept when we assume that there is only single market in the economy.

- In fact, the economy comprises of several markets.

- Equilibrium concept that we use so far can be extended to the more realistic environment where we have N markets.

Example 3.K Let demand and supply of market for goods A and B be

$$D_a = 20 - 3Pa - 2Pb$$
$$S_a = 12 + 2Pa + 5Pb$$
$$D_b = 4 - Pa - 3Pb$$
$$S_b = -1 + 2Pa + 4Pb$$

- State equilibrium conditions of this multimarket model.

$\underbrace{D_a = \cancel{D_b} S_a}_{\#a}$; $\underbrace{D_b = S_b}_{\#b}$

- Find the equilibrium price and quantity of the market for goods A and the equilibrium price and quantity of the market for goods B.

$\underline{P_a, P_b} \Rightarrow \underline{D_a = S_a}$ and $\underline{D_b = S_b}$

Price Equilibrium in Multi-market environment.

$20 - 3P_a - 2P_b = 12 + 2P_a + 5P_b \Rightarrow \#a$

$4 - P_a - 3P_b = -1 + 2P_a + 4P_b \Rightarrow \#b$

$\Rightarrow 5P_a + 7P_b = 8$

$\Rightarrow 3P_a + 7P_b = 5$

$2P_a = 3 \Rightarrow P_a^* = 3/2$

$P_b^* = 1/14$

Plus this Eqm Price into De/Su.

3.4) Simple macroeconomics models

- **Keynesian cross model (partial** equilibrium **model)**
    - Analyze equilibrium determined of output in the goods market.
    - Treating financial sector as given, i.e treating market interest rate as given!
        - Making sense under the liquidity trap environment
    - Implications: emphasizing at the role of fiscal policy

- **IS-LM model (General** equilibrium **model)**
    - Endogenize the market interest rate.
    - Introduce the feedback/interactive effect between financial market and goods market.
    - Implications: emphasizing at the role of both fiscal and monetary policy.

Where we are headed!

- Solving for equilibrium solution of endogenous variables.
- Some comparative static analysis: analyze behavior of endogenous variables.
- Policy issues
    - *Multipliers*
    - *Crowding-out effects*
    - *Policy effectiveness*

3.4.1) The basic workhorse for the Keynesian cross model

From example 1.B, the simplest workhorse for the Keynesian cross model is the one that assumes a closed economy with no government sector. The model can be explained by 3 equations followed.

$Y = C + I$ ➔ equilibrium condition

$C = \mathrm{C(Y)} = \mathrm{a} + \mathrm{bY}$ ➔ behavior of consumption

$I \; = I_0$ ➔ exogenous investment (take it as given.)

- Exogenous: I_0.
- Endogenous: C and Y.
- Parameters: a $\left(\text{autonomous consumption}\right)$, b (mpc)

Static analysis: Equilibrium solution of endogenous variables takes the form

$$\mathrm{Y}^* = \frac{\mathrm{a} + \mathrm{I}_0}{1 - \mathrm{b}}$$

$$\mathrm{C}^* \; = \mathrm{a} + \mathrm{b}\left(\frac{\mathrm{a} + \mathrm{I}_0}{1 - \mathrm{b}}\right)$$

Sensitivity analysis: suppose that $\mathrm{I}_0 \rightarrow \mathrm{I}_0'$, what would happen to the equilibrium?

$$\Delta Y^* = Y^{*\prime} - Y^* = \frac{1}{1 - b} * (I_0' - I_0)$$

$$\Delta C^* = C^{*\prime} - C^* = \frac{b}{1-b} * (I_0' - I_0)$$

Extensions: *Simplicity and reality*

One can enrich the basic model in several directions; this allows us to yield the model that betters reflecting how the world/economy actually works. An exhaustive list of possible extension includes:

- Endogenous investment
  - Investment also depends on level of income, i.e. I(y)
  - Income-accelerator framework, i.e. $I(\Delta y)$ (Dynamic)

- Government sector
  - Government spending
    - Typically treated as an exogenous variable.
    - Alternatively, G might be tied to the level of GDP.
    - Automatic stabilizer, i.e. counter-cyclical government spending rule.
  - Taxation
    - Consumption depends on disposable income, i.e. C(Y-T)
    - Income tax, i.e. T(y)

- Effect of interest rate on private spending
  - Consumption and investment could also depend on interest rate, i.e. C(y,r) and I(y,r)
    - This extension reflects the idea that cost of borrowing matters to both households' and firms' spending.
  - Also, introducing the role of interest rate allows for the channel of linkage between real and financial sector!

- Open economy
  - Equilibrium condition must include net export, i.e. X – M.
  - Need to model the behavior of "X" and "M".

Example 3.L: **A full-version of Keynesian cross model**

No X "0" ; No M "0"
$X - M = 0$
NX

Equilibrium (1)	$Y = C + I + G$
Consumption function (2)	$C = a + bY_d$
Disposable income (3)	$Y_d = Y - T$
Tax rule (4)	$T = T_0 + t * Y$
Investment function (5)	$I = I_0 - I_1 r$
Government spending (6)	$G = G_0$

- Solve for the equilibrium output function?

\# 6 equation: / ~~6~~ 5 ~~unknown~~ Endogenous unknown

Y, C, Y_d, T, I, G (Exo)

Y^*

$$Y = C + I + G$$

$$= a + bY_d + I_0 - I_1 \cdot r + G_0$$

$$Y_d = Y - T$$

$$= Y - T_0 - t \cdot Y = -T_0 + (1-t) \cdot Y$$

$$Y = a + b\left(-T_0 + (1-t) \cdot Y\right) + I_0 - I_1 r + G_0$$

$$Y(1 - b(1-t)) = a - bT_0 + I_0 - I_1 r + G_0$$

$$Y^* = \frac{a - bT_0 + I_0 - I_1 r + G_0}{1 - b(1-t)}$$

○ Multiplier of autonomous expenditures

"r" → Exogenous

$$\Delta y^* = \Delta\left(\frac{a - bT_0 + I_0 - I_1 r + G_0}{1-b(1-t)}\right).$$

$a\,;\ I_0\,;\ G_0\,;\ T_0$

a: $\dfrac{\Delta Y^*}{\Delta a}$

$$\Delta y^* = \frac{\Delta a - \Delta(bT_0) + \Delta I_0 - \Delta I_1 r + \Delta G_0}{1-b(1-t)}$$

all other changes set equal to zero, except Δa

$$\Delta y^* = \frac{1}{1-b(1-t)} \cdot \Delta a$$

>0 → multiplier of "a".

I_0:

$$\Delta y^* = \frac{1}{1-b(1-t)} \cancel{\Delta a}\,\Delta I_0$$

>0

G_0:

$$\Delta y^* = \frac{1}{1-b(1-t)} \cdot \Delta G_0$$

>0

T_0:

$$\Delta y^* = -\frac{b}{1-b(1-t)} \cdot \Delta T_0$$

<0

multiplier of G > multiplier of T $\left|\dfrac{1}{1-b(1-t)}\right| > \left|-\dfrac{b}{1-b(1-t)}\right|$

3.4.2) The IS-LM model

- So far, we have seen a simple Keynesian cross model that "r" is exogenously given.
  - An increase in "r" reduces aggregate expenditure, and hence the level of income in the equilibrium.
- Is the assumption that "r" an exogenous variable sensible? Not really!..
- IS-LM model is an extension of the Keynesian cross.
  - The model *endogenizes* the value of interest rate by modeling how interest rate is determined in the equilibrium.
- Conceptually, interest rate is determined from the equilibrium in money market, where money demand is equal to money supply.
- In economics, we typically assume that money demand depends on (i) income and (ii) interest rate.
- Since money demand also depends on income, equilibrium level of interest rate in the money market also depends on the level of income.
- As a result, income and interest rate will have to be simultaneously determining each other in the general equilibrium.

Money market modeling

Equilibrium condition	$M^d = M^s$
Money demand equation	$M^d = \mathrm{L}_0 + \mathrm{L}_1\mathrm{Y} - \mathrm{L}_2\mathrm{r}$ $\quad L_0, L_1, L_2 > 0$
Money supply	$M^s = M_0^s$

Characterizing equilibrium in the money market

- Putting everything into the equilibrium condition

$$L_0 + L_1Y - L_2r = M_0^s$$

- Suppose that "Y" is given for now. Write "r" in terms of "Y" and "M_0^s", we yield **the LM equation,**

↳ Equation eqm$^=$ in money market

$$r = \frac{1}{L_2}\left(L_0 + L_1Y - M_0^s\right)$$

General Equilibrium in the IS-LM model

The equilibrium solution for Y* and r* simultaneously satisfy the following two equations

IS equation: $Y = \frac{1}{1-b(1-t)}[a - bT_0 + I_0 - I_1r + G_0]$

LM equation: $r = \frac{1}{L_2}\left(L_0 + L_1Y - M_0^s\right)$

Example 3.M Consider the following model equations that explain the behavior of goods and money market.

$C = 0.8(Y - T)$ → $C^* = 0.8(4000 - 1000)$

$T = 1{,}000$ $= 2{,}400$

$I = 800 - 20r$

$G = 1{,}000$ $I^* = 800 - 20(10) = 600$

$Y = C + I + G$ $S = Y_d - C$

$M^d = 0.4Y - 40r$ Eq^n in good market

$M^S = 1{,}200$ $= 3000 - 2{,}400$

$= 600$ → $(y; r)$

Write numerical formula for IS equation

$y = C + I + G$

$y = 0.8(y - 1000) + 800 - 20r + 1{,}000$

$y = 0.8y - 800 + 800 - 20r + 1{,}000$

$0.2y = 1{,}000 - 20.r$

$y = 5(1{,}000 - 20.r)$ — $\boxed{\text{Is Equation}}$

Write numerical formula for LM equation

$M^d = M^s$

$0.4y - 40 \cdot r = 1{,}200$

$40r = 0.4y - 1{,}200$

$r = \frac{1}{40}(0.4y - 1200)$

$r = 0.01y - 30$ → LM Equation

Solve for endogenous equilibrium solution of "Y", "C", "I" and "saving".

Equ: (y^*, r^*) → Solve IS/LM Equations

$y = 5(1000 - 20 \cdot r) = 5000 - 100r$

$r = 0.01y - 30$

$y = 5000 - 100(0.01y - 30)$

$= 5000 - (y) + 3{,}000$

$y^* = \frac{8000}{2} = 4{,}000$; $r^* = 40 - 30 = 10$

general Equ for IS-LM Model