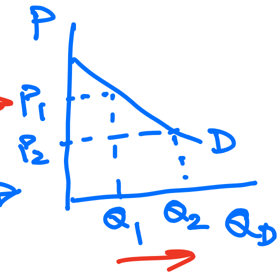
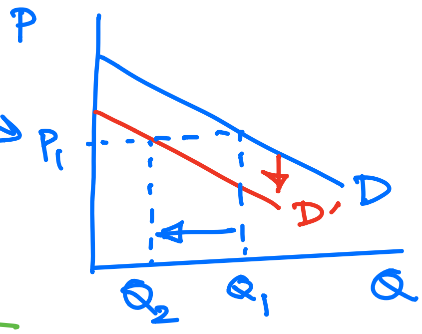


# Chapter 7 Elasticities of Demand

Quantity demanded can change due to price, income or prices of other goods, etc. Thus we have 3 kinds of Elasticities of Demand.

1. Price Elasticity of Demand
2. Income Elasticity of Demand
3. Cross-Price Elasticity of Demand



**1. Price Elasticity of Demand** measures how sensitive the demand is to a change in price.

**Example:**

increase 10%

4 million

Price of Pepsi = 10 bahts, → 5 million cans sold
= 20 bahts → 3 million cans sold

reduces.

Cannot consider absolute change in price

Price of Toyota = 1,000,000 bahts → 1,000 units sold.
= 1,000,010 bahts → 999

~~So Toyota is not as sensitive to change in price compared to Pepsi.~~

Does this means that the demand for Toyota is not as sensitive to change in price compared to the demand for Pepsi?

Have to consider Percentage change in Price 20%

Pepsi increases price by 10% → 1 million reduction in units sold.

Toyota → 10% → 300 reduction in sale.

Also cannot look at absolute change in $Q_D$

−30%

**Price Elasticity of Demand** is the percentage change of the quantity demanded per 1 percentage change in price

must consider percentage change in $Q_D$

$$\eta_D = \frac{\%\Delta Q_D}{\%\Delta P} = \frac{Percentage\ change\ of\ Q_D}{Percentage\ change\ of\ P}$$

$\%\Delta Q_D$

$\Delta$ means "change"

**Example** If the price increases by 10% ($\%\Delta P = 10\%$), the quantity demanded decreases by 25% ($\%\Delta Q_D = -25\%$)

$\eta_D$

$\eta_S$

$$\eta_D = \frac{\%\Delta Q_D}{\%\Delta P} = \frac{-25\cancel{\%}}{+10\cancel{\%}} = -2.5$$

$\%\Delta P = 10\% \rightarrow -25\% = \%\Delta Q_D$

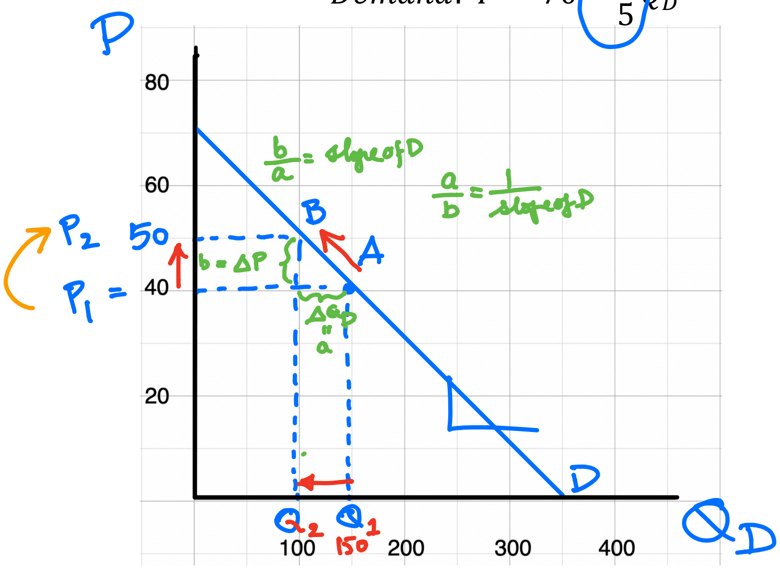
$1\% \rightarrow \frac{-25\%}{10\%} = -2.5 = \eta_D$

- Negative sign reflects the Law of Demand
- $\eta_D$ does not have unit
- $\eta_D = -2.5$ means that if the price increases by 1%, the quantity demanded decreases by 2.5%

~~$\eta_D = -2.5\%$~~

**Example**

$$Demand:\ P = 70 - \frac{1}{5}Q_D$$




$20\% = \frac{20}{100} = 0.2.$

150 reduces to 100.

Consider two points, $A$ and $B$ on the demand

$$A = (150{,}40), Q_1 = 150, P_1 = 40$$
$$B = (100, \cancel{30}\ 50), Q_2 = 100, P_2 = \cancel{30}\ 50.$$

new old

From $A \rightarrow B$,

$\Delta Q_D = Q_2 - Q_1 = 100 - 150 = -50$

$\Delta P = P_2 - P_1 = 50 - 40 = 10.$

$\%\Delta Q_D = -33.33\% = \frac{-50}{150} = \frac{Q_2 - Q_1}{Q_1}$

$\%\Delta P = 25\% = \frac{10}{40} = \frac{P_2 - P_1}{P_1}$

$$\eta_D = \frac{\%\Delta Q_D}{\%\Delta P} = \frac{\overbrace{(Q_2 - Q_1)}^{\Delta Q_D}/Q_1}{\underbrace{(P_2 - P_1)}_{\Delta P}/P_1}$$

Note $A = (Q_1, P_1)$

$$= \boxed{\frac{\Delta Q_D}{\Delta P}} \cdot \frac{P_1}{Q_1}$$

$$= \frac{1}{\underbrace{\text{slope of D}}_{=-\frac{1}{5}}} \cdot \frac{P_1}{Q_1} = -5\left(\frac{40}{150}\right) = -\frac{4}{3} = \eta_D$$

$Q_D$ decreases by 1.33% when price increases by 1%

- $\eta_D$ from $A$ to $B$ is given by: where $P_1, Q_1$ are price & $Q_D$ at $A$.

$$\eta_D = \frac{1}{Slope}\frac{P_1}{Q_1}$$ at A.

where $P_1$ and $Q_1$ are price and quantity demanded at $A$.

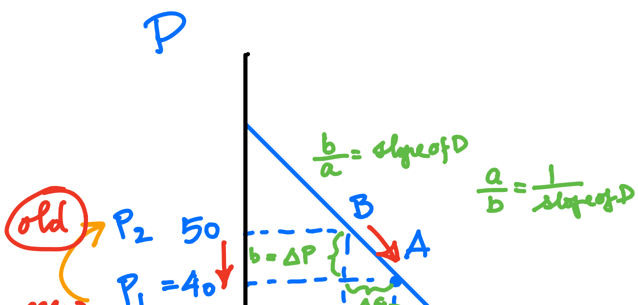
- Is $\eta_D$ the same for $B \rightarrow A$?

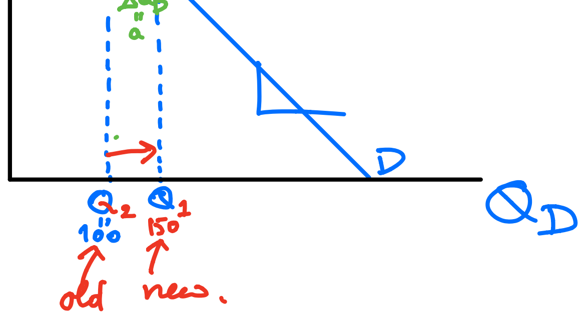
$\Delta P = -10 = 40 - 50 = P_1 - P_2$

$\%\Delta P = -20\% = \frac{-10}{50} = \frac{P_1 - P_2}{P_2}$

$\Delta Q_D = 50 = Q_1 - Q_2$

$\%\Delta Q_D = \frac{50}{\ } = \frac{Q_1 - Q_2}{\ } = 50\%$






From B→A.

$$\eta_D = \frac{\%\Delta Q_D}{\%\Delta P} = \frac{50\%}{-20\%} = -2.5$$

$$= \frac{(Q_1-Q_2)/Q_2}{(P_1-P_2)/P_2} = \frac{\Delta Q_D}{\Delta P}\cdot\frac{P_2}{Q_2}$$

$$= \frac{1}{\text{slope of D}}\cdot\frac{P_2}{Q_2}$$

price and $Q_D$ at B.

- $\eta_D$ from $B \to A$ is given by:

$$\eta_D = \frac{1}{Slope}\frac{P_2}{Q_2} = \frac{1}{-1/5}\cdot\frac{50}{100} = -\frac{5}{2}$$

same as when from A to B because we consider linear Demand.

where $P_2$ and $Q_2$ are price and quantity demanded at $B$.

section of a line.

**Arc Elasticity of Demand**

From $A \to B$, $\eta_D = \frac{1}{Slope}\frac{P_1}{Q_1} = -1.33.$ at A

From $B \to A$, $\eta_D = \frac{1}{Slope}\frac{P_2}{Q_2} = -2.5$ at B

If we want to find, $\eta_D$ between $A$ and $B$ (without specifying from where to where) we use

Average price $= \bar{P} = \frac{P_1+P_2}{2}$

Average Quantity $= \bar{Q} = \frac{Q_1+Q_2}{2}$

$\bar{P} = \frac{P_1+P_2}{2} = \frac{40+50}{2} = 45$

$\bar{Q} = \frac{150+100}{2} = 125$

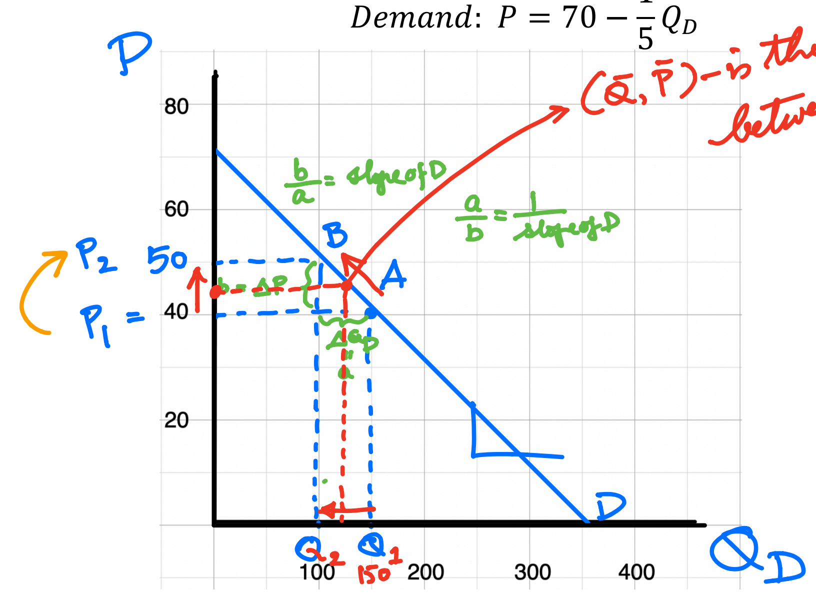
$-\cancel{5}\left(\frac{90}{\cancel{250}}\right) = \frac{9}{5}$

$$\eta_D = \frac{1}{Slope}\frac{\bar{P}}{\bar{Q}} = \frac{1}{Slope}\frac{\bar{P}}{\bar{Q}} = \frac{1}{Slope}\frac{(P_1+P_2)/\cancel{2}}{(Q_1+Q_2)/\cancel{2}}$$

$$= \frac{1}{-1/5}\cdot\frac{(40+50)}{(150+100)} = -\frac{9}{5}.$$

$$Demand: \; P = 70 - \frac{1}{5}Q_D$$

$(\bar{Q}, \bar{P})$ is the midpoint between $(Q_1, P_1)$ and $(Q_2, P_2)$




This is called ***Arc Elasticity by Midpoint method***.

**Point Elasticity of Demand** Price elasticity at a point $A = (Q_1, P_1) = (150,40)$.

$$\eta_D = \left(\frac{1}{Slope\ at\ A}\right)\frac{P_1}{Q_1} \quad \text{at } A$$
$$= \frac{1}{\frac{dP}{dQ_D}}\frac{P_1}{Q_1}$$
$$= \frac{dQ_D}{dP}\frac{P_1}{Q_1} = -1.33$$

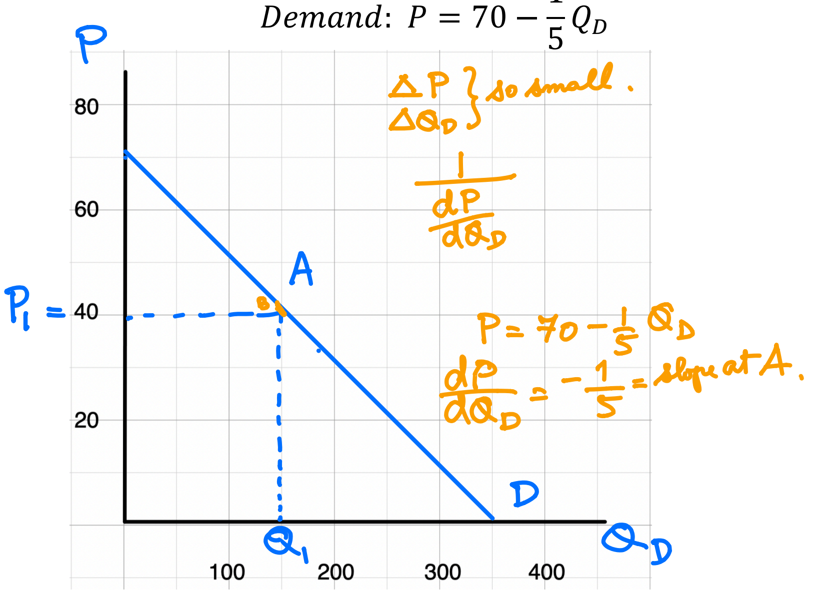
$\frac{1}{-1/5} = -5 = \frac{dQ_D}{dP}$

$P = 70 - \frac{1}{5}Q_D$

$Q_D = 350 - 5P.$

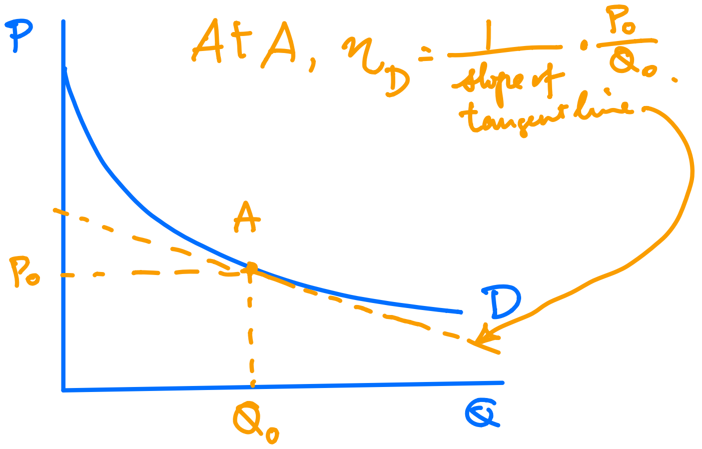
$\frac{dQ_D}{dP} = -5$

$$Demand:\ P = 70 - \frac{1}{5}Q_D$$




- Point Elasticity of a Nonlinear Demand curve.

At A, $\eta_D = \frac{1}{\text{slope of tangent line}} \cdot \frac{P_0}{Q_0}$.



$\eta_D < 0$ negative by Law of Demand.

**Elastic and Inelastic Demand**

Demand has unit elasticity if $|\eta_D| = 1$

Demand is Elastic when $|\eta_D| > 1$

Demand is Inelastic when $|\eta_D| < 1$.

$\eta_D = -2.5 \Rightarrow$ D is elastic.

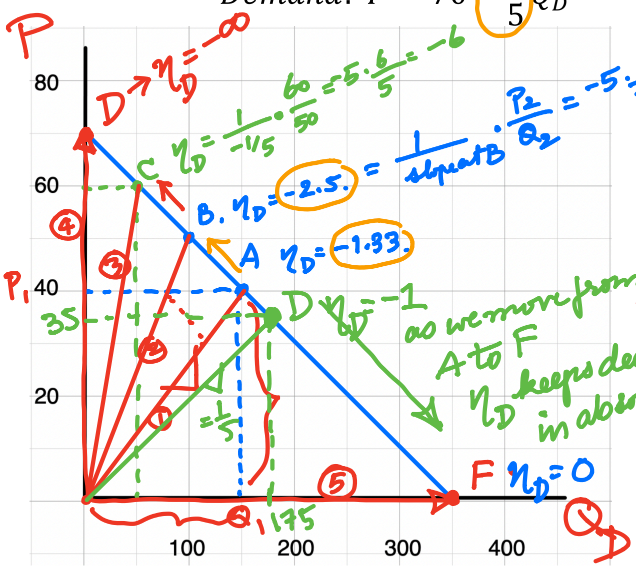
$\eta_D = -\infty$ – Demand is perfectly elastic

$\eta_D = 0$ – " inelastic.

**Point Elasticity of along a Linear Demand Curve**

$\eta_D = -1 = \frac{1}{\text{slope at D}} \cdot \frac{P_D}{Q_D}$ ; $-5$ ; $\frac{1}{5}$.

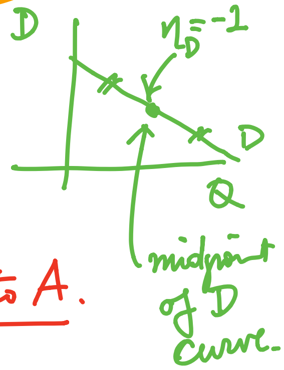
$$Demand: P = 70 - \frac{1}{5}Q_D$$




$$\eta_D = \frac{dQ_D}{dP}\frac{P}{Q_D}$$

at A, $\eta_D = \frac{1}{\text{slope at A}} \cdot \frac{P_1}{Q_1}$

$\eta_D = \frac{\text{slope of straight line from origin to A.}}{\text{slope of D at A.}}$



$\eta_D = -1$ midpoint of D curve.

At D, $\eta_D = -\infty$, because at D, $Q_D = 0$, Price = 70

if you reduce Price by 0.1 ฿ $Q_D$ becomes positive.

At F, $\eta_D = 0 = \frac{\%\Delta Q_D}{\%\Delta P} = \infty = 0$

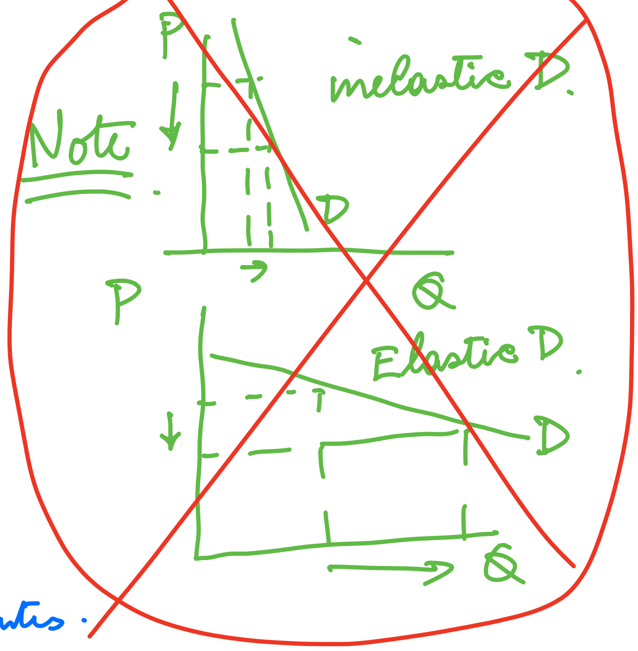
at F, Price = 0, if we increase price by 0.0001 ฿ $\Rightarrow \%\Delta P = \infty$

Question: Where on the D is the $\eta_D = -1$?

why don't we set the price where $|\eta_D|$ is small $|\eta_D| = 0.01$

So Demand is inelastic!

So we raise the price $\Rightarrow Q_D$ reduces by just a little bit.




Determinants

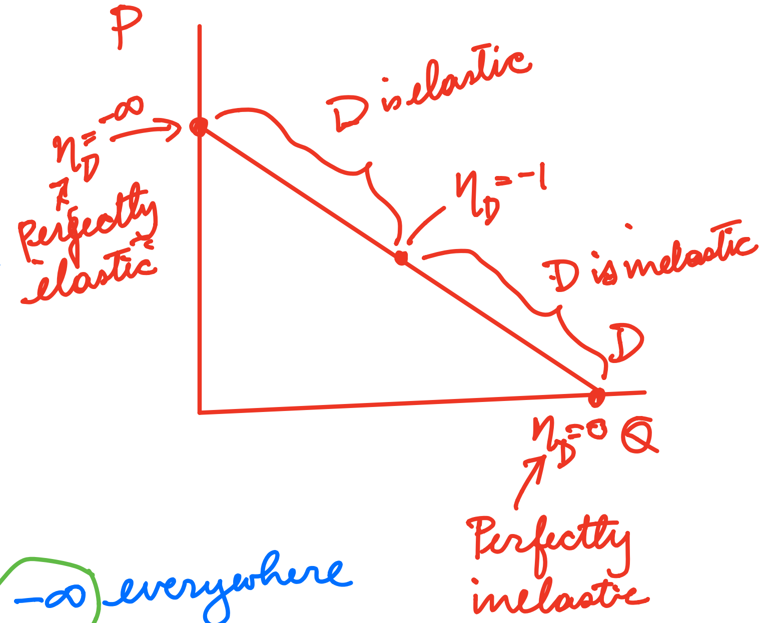
**Price Elasticity of Demand** depends on the availability of substitutes

1. A product and a group of products

D for iPhone
D. for smartphones
more elastic ?
less substitutes.

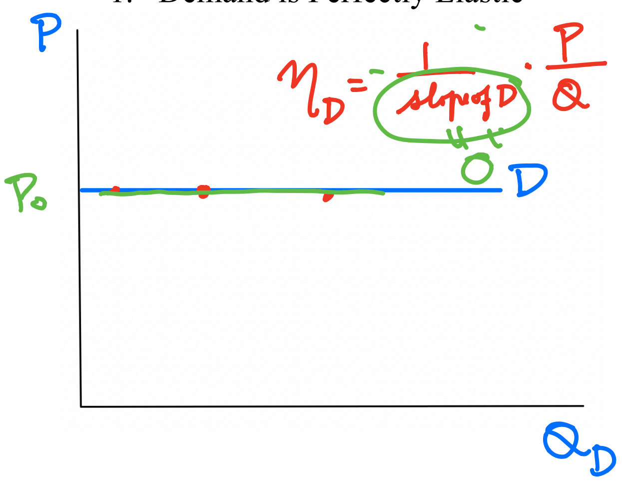
2. Short-run and Long-run adjustments

D for a single product.
less elastic because less time to find substitutes.



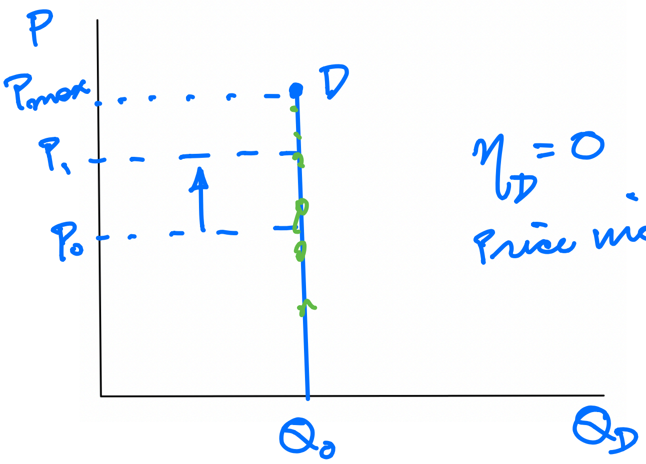

**Extreme Cases**

1. Demand is Perfectly Elastic



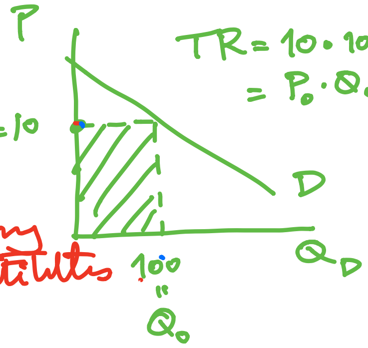

$\eta_D = -\infty$ everywhere
at $P_0$, $Q_D = \infty$
but if price increases to be any amount $> P_0$
then $Q_D = 0$.

2. Demand is Perfectly Inelastic




$\eta_D = 0$
Price increases from $P_0$ to $P_1$
$Q_D$ still remains at $Q_0$.

**Price Elasticity of Demand and Total Revenue** $= P \cdot Q$



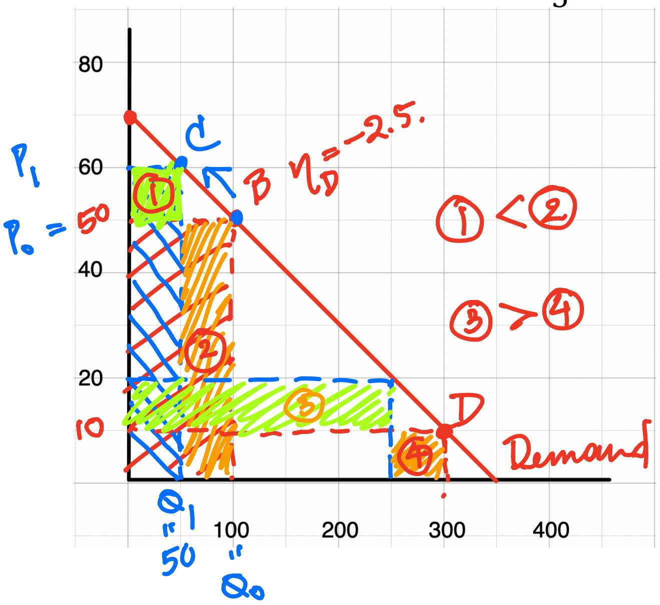

Inelastic $|\eta_D| < 1$

- If BTS Skytrain want to increase its revenue, BTS would want to increase or decrease its fares? } not many substitutes

$|\eta_D| > 1$ Elastic.

- If Toyota wants to increase its revenue, Toyota would increase or decrease its price? many substitutes.

$$Demand\colon\ P = 70 - \frac{1}{5}Q_D$$




$$\eta_D = \frac{\%\Delta Q_D}{\%\Delta P} = -2.5$$

$TR = P \cdot Q$

$\%\Delta P$: $+1$ $\quad \%\Delta Q_D$: $-2.5\%$

new TR $= (1.01)P\, Q(0.975)$

$= (1.01)(0.975)\, P \cdot Q$

$= (0.98475)\, PQ$ old TR

At B, $TR = P_0 Q_0 = 50 \times 100 = 5000$

increase price $P_0 = 50$ to $P_1 = 60$, $Q_1 = 50$ } C

At C, $TR = P_1 \cdot Q_1 = (60)(50) = 3{,}000$ decreases

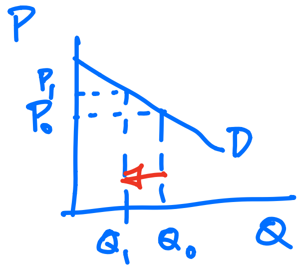
At $D = (Q_3, P_3) = (300, 10)$

$$\eta_D = \frac{dQ_D}{dP}\frac{P_3}{Q_3} = \frac{1}{-1/5}\cdot\frac{10}{300} = -\frac{1}{6}$$

$|\eta_D| = \frac{1}{6} < 1$ inelastic

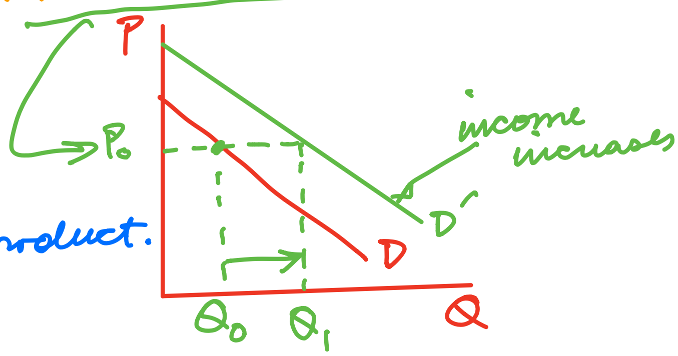
- If the price increase by 1%, the revenue decreases by %. Thus the total revenue

**Example** $P = 10, Q_D = 300 \Longrightarrow TR =$ $10 \times 300 = 3{,}000$
$P = 11, Q_D = 295 \Longrightarrow TR =$ $11 \times 295 = 3{,}245$ } increases.




- Quantity Demand can change due to
  1. Price ⟹ Price Elasticity of Demand $\eta_D = \frac{\%\Delta Q_D}{\%\Delta P}$
  2. Income Income Elasticity of D

     $\eta_I = \frac{\%\Delta Q_D}{\%\Delta I}$ – when P remains unchanged.
  3. Price of other Goods
     - Price of substitute
     - Price of complementary product.




**2. Income Elasticity of Demand** is the percentage change of quantity demanded per one percentage change in income.

$$\eta_I = \frac{\%\Delta Q_D}{\%\Delta I}$$

- While the price is unchanged, income increases from 30,000 to 40,000 bahts a month,

| $I$ | $Q_D$ |
|---|---|
| 30,000 | 100 |
| 40,000 | 120 |

$\%\Delta Q_D =$ 20%

$\%\Delta I =$ 33.33%

$$\eta_I = \frac{\%\Delta Q_D}{\%\Delta I} = \frac{20\%}{33.33\%} = \frac{2/10}{1/3} = \frac{3}{5} > 0$$

(lower).
Higher income leads to higher $Q_D$.
(lower)

- Income Elasticity of Demand will classify a product to be normal (Necessary or Luxury) or Inferior.

normal product

<table>
<tr><th colspan="2">Normal</th><th>Inferior</th></tr>
<tr><td colspan="2">$\eta_I \geq 0$</td><td rowspan="3">$\eta_I < 0$</td></tr>
<tr><td>Necessary</td><td>Luxury</td></tr>
<tr><td>$0 \leq \eta_I \leq 1$</td><td>$\eta_I > 1$</td></tr>
</table>

instant noodles

Higher income ⟹ Lower $Q_D$

Food. (steaks dinner

What is wrong with this statement:

defined by change in income.

LPG 350 ฿. → 400 ฿.

"Even with the higher price of cooking gas, the consumption still remains almost the same because cooking is a *necessary* product." – it is difficult to find substitutes to cooking gas.

We cannot say cooking gas is necessary here because there is no change in income. Here only the price changes.

**3. Cross Price Elasticity of Demand** is the percentage change of the quantity demanded of $X$ per 1 percentage change of price of $Y$.

Let $X$ be Pepsi and $Y$ Coke.

| $Q_X$ Pepsi | $P_Y$ coke |
|---|---|
| 30m | 10 |
| 35m | 12 |

$$\eta_{XY} = \frac{\%\Delta Q_X}{\%\Delta P_Y} = \frac{5/30}{20\%} = \frac{5/30}{2/10} = \frac{5}{6} > 0$$

$\eta_{XY} > 0 \Longrightarrow$ $X$ (Pepsi) is a substitute of $Y$ – coke.

$\eta_{XY} < 0 \Longrightarrow X$ and $Y$ are complementary—$X$ is consumed together with $Y$.

$$\eta_{xy} = \frac{\%\Delta Q_x}{\%\Delta P_y}$$

Y = gasoline. $P_Y \uparrow$

X = car. $Q_X \downarrow$