Simultaneous Equations Models

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Part 1

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Nature of simultaneous equations models (SEMs)

- Another form of endogeneity problem is simultaneity. This happens when one or more explanatory variables is jointly determined with the dependent variable, typically through an equilibrium mechanism, e.g. supply and demand equations of commodity or input.
  - We only observe the outcomes in equilibrium.
- Example of an SEM: labor supply and demand

Nature of simultaneous equations models

- A labor supply function: $h_s = \alpha_1 w + \beta_1 z_1 + u_1$ (1)
  - α_1 measures how labor supply changes when the wage changes.
  - h_s labor supply could be number of hours that individual desires to work
  - z_1 can be the manufacturing wage. Then, we expect $\beta_1 \leq 0$: other factors equal, if the manufacturing wage increases, more workers will go into manufacturing than into agriculture.
  - When we graph labor supply, we have hours on x axis and wage on y axis, with z_1 and u_1 held fixed.
  - A change in z_1 or u_1 shifts the labor supply fuction: z_1 - observed supply shifter, u_1 - unobserved supply shifter.

Nature of simultaneous equations models

- A labor demand function: $h_d = \alpha_2 w + \beta_2 z_2 + u_2$ (2)
  - For a demand function, $\alpha_2 < 0$.
  - When h_d is hours demanded, we might think z_2 as agricultural land area. Since labor and land are complements in production, we expect $\beta_2 > 0$.
  - z_2 is obserable demand shifter, and u_2 is unobservable demand shifter.
- We call these supply-demand functions as the structural equations since they can be obtained from (utility-profit) maximization problems.
  - labor supply = a behavioral equation for workers; labor demand = a behavioral equation for farmers

Nature of simultaneous equations models

- We have observed data on wages and hours that are already determined by the intersection of supply and demand:
  - $h_{is} = h_{id}$, i - district or province
  - That is, we observe only h_i
  - Same with w, equilibrium wage that clear the market
- Now, we have a simultaneous equations model (SEM)
  - $h_i = \alpha_1 w_i + \beta_1 z_{1i} + u_{1i}$ (3), and
  - $h_i = \alpha_2 w_i + \beta_2 z_{2i} + u_{2i}$ (4)
  - h_i and w_i are endogenous variables. (We assume that $\alpha_1 \neq \alpha_2$)
  - What about z_{1i} and z_{2i}?

Nature of simultaneous equations models

- For SEMs, each equation should have a behaviroal, ceteris paribus interpretation on its own
- An example of an inappropriate use of SEMs:
  $housing = \alpha_1 saving + \beta_{10} + \beta_{11} inc + \beta_{12} educ + \beta_{13} age + u_1$
  $saving = \alpha_2 housing + \beta_{20} + \beta_{21} inc + \beta_{22} educ + \beta_{23} age + u_2$
- Here, two endogenous variabls are chosen by the same household. Hence, neither equation can stand on its own

Simultaneity bias in OLS

- Consider the two-equation structural model:
  $y_1 = \alpha_1 y_2 + \beta_1 z_1 + u_1$ (5)
  $y_2 = \alpha_2 y_1 + \beta_2 z_2 + u_2$ (6)
- Substitute y_2 into y_1:
  $(1 - \alpha_2\alpha_1)y_2 = \alpha_2\beta_1 z_1 + \beta_2 z_2 + \alpha_2 u_1 + u_2$
  Reduced form equation for y_2: $\Rightarrow y_2 = \pi_{21} z_1 + \pi_{22} z_2 + v_2$ (7)
- π_{21}, π_{22} are nonlinear functions of the structural parameters; v_2 is a linear function of u_1, u_2
- OLS estimation of (5) (and (6)) will be biased and inconsistent.

Simultaneity bias in OLS

- v_2 is uncorrelated with z_1 and z_2. We can use OLS to estimate eq(7).
- $cov(y_2, u_1) \neq 0$ iff $cov(v_2, u_1) \neq 0$
  - What if assume $cov(u_1, u_2) = 0$?
  - What if $\alpha_2 = 0$?
  - What if $\alpha_2 = 0$ and $cov(u_1, u_2) = 0$?

Example 1

Murder rates and size of the police force

- Q: How much additional law enforcement will decrease their murder rates? or If a city exogenously increases its police force, will that increase lower the murder rate, on average?
- $murdpc = \alpha_1 polpc + \beta_{10} + \beta_{11} incpc + u_1$ (i)
  - murdpc = murders per capita, polpc = #police officers per capita, incpc = income per capita
- Why can't we run OLS on the above equation?
  - A city's spending on police force size is partly determined by its expected murder rate.
  - $polpc = \alpha_2 murdpc + \beta_{20} + other\ factors$ (ii)
  - We expect that $\alpha_2 > 0$
- (i) describes the actions of potential murderers ; (ii) describes behavior by city officials. Each has a clear ceteris paribus interpretation.

Example 2

The simple Keynesian model of income determination

- Consumption function: $C_t = \beta_0 + \beta_1 Y_t + u_t$ (iii)
- Income identity: $Y_t = C_t + I_t, I_t = S_t$ (iv)
- C_t and Y_t are interdependent. When u_t shifts, C_t also shifts and it affects Y_t
- We can find that $cov(Y_t, u_t) = \frac{\sigma^2}{1-\beta_1} \neq 0$, and hence OLS of (iii) will make β_1 inconsistent
  - $plim(\hat{\beta}_1) = \beta_1 + \frac{1}{1-\beta_1}\left(\frac{\sigma^2}{\sigma_Y^2}\right)$
  - $\hat{\beta}_1$ will overestimate the true β_1