Long-Run Production

- In the long run, the firm's production function is

$$q = f(L, K)$$

  - where q is output, L is workers, and K is the number of units of capital.
  - Both L and K can be varied.

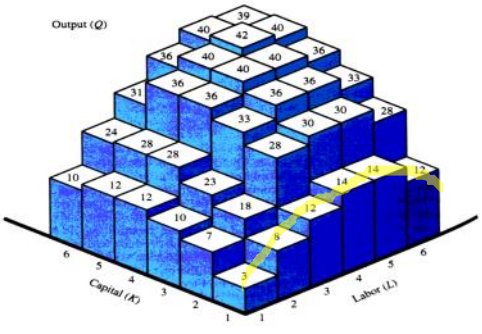
Production Function With Two Inputs

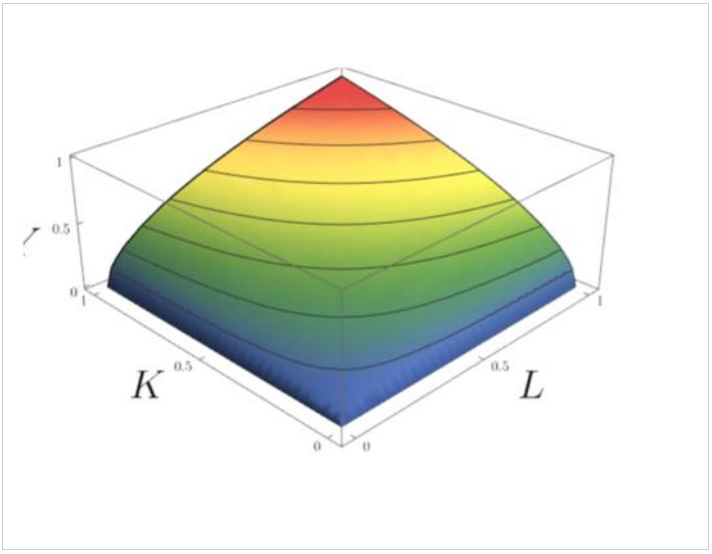
Q = f(L, K)

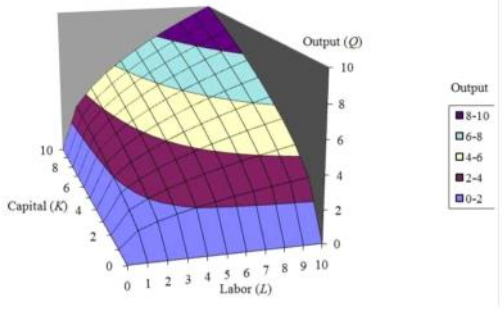
K							Q
6	10	24	31	36	40	39	
5	12	28	36	40	42	40	
4	12	28	36	40	40	36	
3	10	23	33	36	36	33	
2	7	18	28	30	30	28	
1	3	8	12	14	14	12	
	1	2	3	4	5	6	L

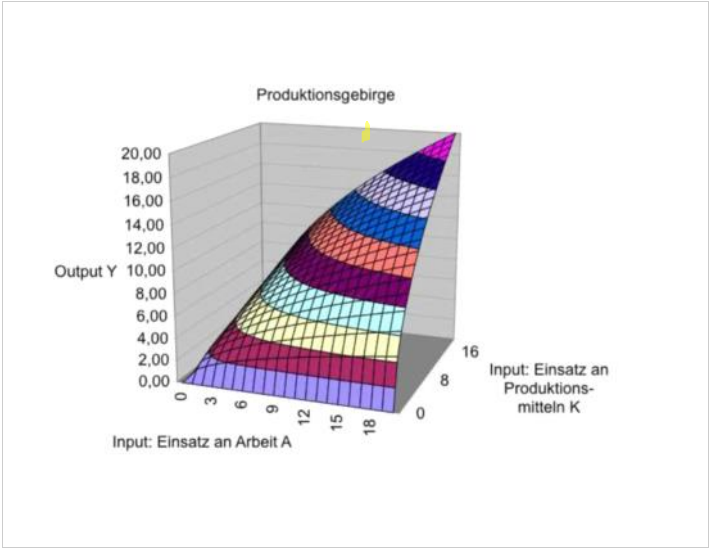
Production Function With Two Inputs

Discrete Production Surface




1
0.5
0
1
0.5
K
0
0
0.5
1
L


Output (Q)
10
8
6
4
2
0
Capital (K)
10
8
6
4
2
0
0 1 2 3 4 5 6 7 8 9 10
Labor (L)
Output
8-10
6-8
4-6
2-4
0-2


Produktionsgebirge
Output Y
20,00
18,00
16,00
14,00
12,00
10,00
8,00
6,00
4,00
2,00
0,00
0
3
6
9
12
15
18
Input: Einsatz an Arbeit A
0
8
16
Input: Einsatz an Produktions-mitteln K

Production Function With Two Inputs
Continuous Production Surface

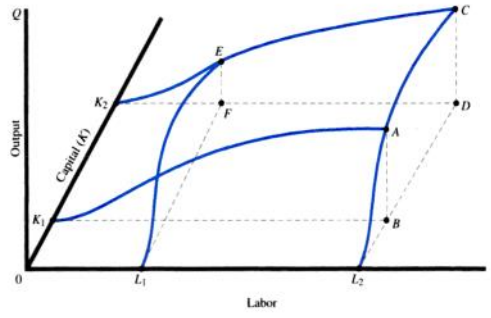


Figure A9-4: The Production Mountain

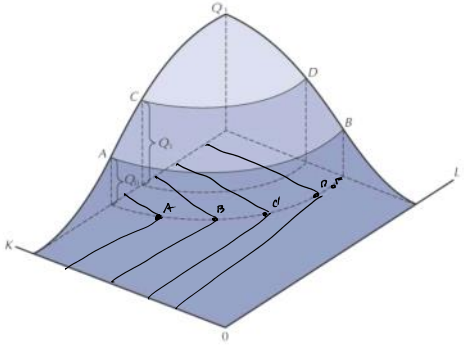
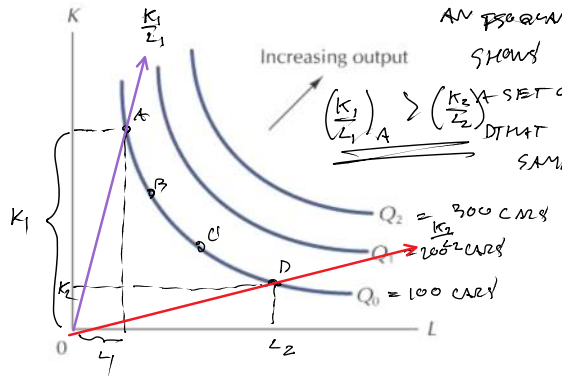


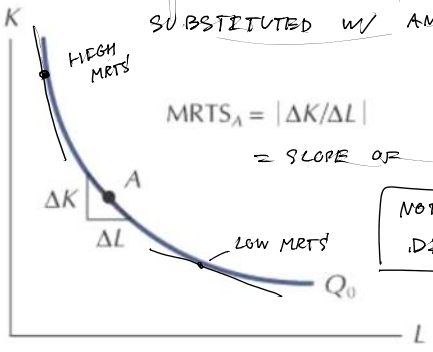
Figure A9-5: The Isoquant Map Derived from the Production Mountain



AN ISOQUANT SHOWS A SET OF INPUT CHOICES $\left(\frac{K_1}{L_1}\right)_A > \left(\frac{K_2}{L_2}\right)_D$ THAT GIVES THE SAME OUTPUT LEVEL

Figure 9-9: The Marginal Rate of Technical Substitution (MRTS)

= RATE AT WHICH ONE INPUT CAN BE SUBSTITUTED W/ ANOTHER INPUT TO GIVE THE SAME LEVEL OF OUTPUT.

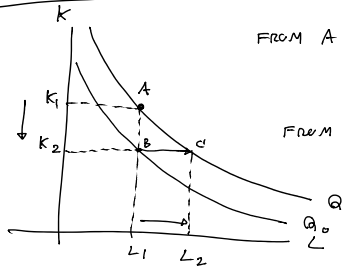


= SLOPE OF ISOQUANT

NOTICE THAT MRTS IS DIMINISHING.

$MP_L = \frac{\Delta Q}{\Delta L}$

$MP_K = \frac{\Delta Q}{\Delta K}$



FROM A → B: LOSS IN OUTPUT OCCURS

$\Delta Q = MP_K \cdot \Delta K$

FROM B → C': GAIN IN OUTPUT OCCURS

$\Delta Q = MP_L \cdot \Delta L$

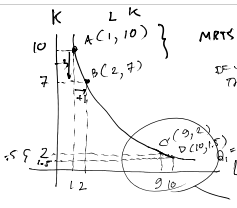
FROM A → C', NO OUTPUT CHANGES

$(\Delta Q = 0)$

$A \to B + B \to C' = A \to C'$

$MP_K \cdot \Delta K + MP_L \cdot \Delta L = 0$

$MRTS = -\frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K}$



$MRTS = \frac{\Delta K}{\Delta L} = \frac{-3}{+1} = -3$

IF YOU ADD A WORKER INTO THE FACTOR, YOU CAN RELEASE YOUR USE OF K BY 3 UNITS AND STILL GET THE SAME OUTPUT (@ = 10)

$MRTS = \frac{\Delta K}{\Delta L} = \frac{-0.5}{+1} = -0.5$!

NOW, ALONG C' → D, YOU CAN RELEASE ONLY 0.5 UNIT OF K WHEN YOU ADD

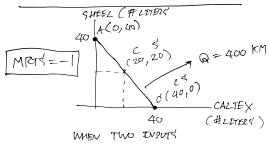
$MRTS = \frac{\Delta K}{\Delta L} = \frac{-0.5}{+1} = -0.5$!

NOW, ALONG C → D, YOU CAN RELEASE ONLY 0.5 UNIT OF K WHEN YOU ADD A WORKER INTO THE FACTORY AND GET THE SAME OUTPUT.

NOTICE THAT ① MRTS VARIES ALONG AN ISOQUANT.

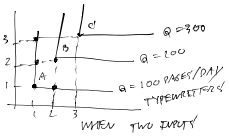
② MRTS IS DIMINISHING, (W/O THE SIGN)

OTHER TYPES OF ISOQUANT



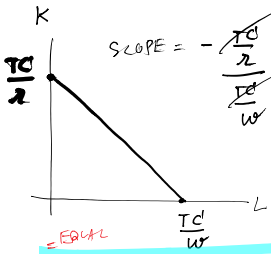
WHEN TWO INPUTS ARE PERFECT SUBSTITUTES, ISOQUANT IS A STRAIGHT LINE

SECRETARY



WHEN TWO INPUTS ARE PERFECT COMPLEMENTS, ISOQUANT IS L-SHAPED.

Isocost



SLOPE $= -\frac{\frac{TC'}{r}}{\frac{TC'}{w}} = -\frac{w}{r}$ OR $= -\frac{P_L}{P_K}$

GIVEN TOTAL COST (TC') = X BAHT/DAY

PRICE OF LABOR (P_L) = WAGE (w) BAHT/ลิ/DAY

PRICE OF CAPITAL (P_K) = rental rate (r) BAHT/MACHINE/DAY

$\frac{TC'}{w}$ = MAXIMUM AMOUNT OF WORKERS CAN BE HIRED IF YOU DON'T BUY CAPITAL

$\frac{TC'}{r}$ = MAXIMUM AMOUNT OF MACHINES CAN BE RENTED IF YOU DON'T BUY LABOR SERVICES.

ISOCOST = A COLLECTION OF ALL INPUT CHOICES THAT CAN BE PURCHASED OR THAT COSTS YOU THE SAME AMOUNT OF MONEY

EX: TC' = 30,000 BAHT/DAY
w = 300 BAHT/ลิ/DAY
r = 1000 BAHT/MACHINE/DAY

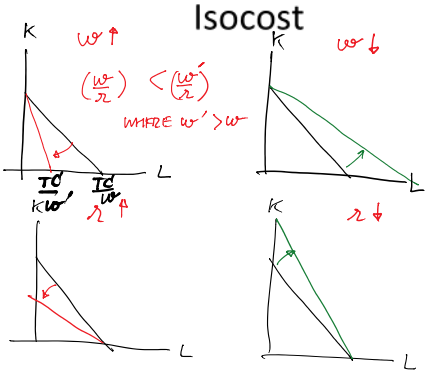
$\frac{w}{r} = \frac{300}{1000} = 0.3$ $\quad w = 0.3r$

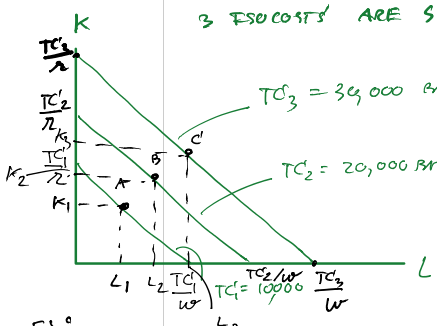
$-\frac{w}{r}$ CALLED "INPUT PRICE RATIO" OR "RELATIVE PRICE OF INPUTS"

$\frac{TC'}{w} = \frac{30,000}{300} = 100$ ลิ/DAY $\quad$ OR $r = \frac{1}{0.3}w$

$\frac{TC'}{r} = \frac{30,000}{1000} = 30$ MACHINES/DAY $\quad r = 3.33w$

Isocost





$TC' = w \cdot L + r \cdot K$

$K = \frac{TC' - w \cdot L}{r}$

EX:
- CHOICE C COSTS MORE THAN CHOICE B.
- CHOICE B COSTS MORE THAN CHOICE A.

OBSERVE THAT $-\frac{w}{r}$ DOES NOT CHANGE.

$K = \frac{TC'}{r} - \frac{w}{r} \cdot L$ ISOCOST EQUATION

Least Cost Combination

GOAL OF A FIRM: MINIMIZE TOTAL COST GIVEN HIS DESIRED LEVEL OF OUTPUT.

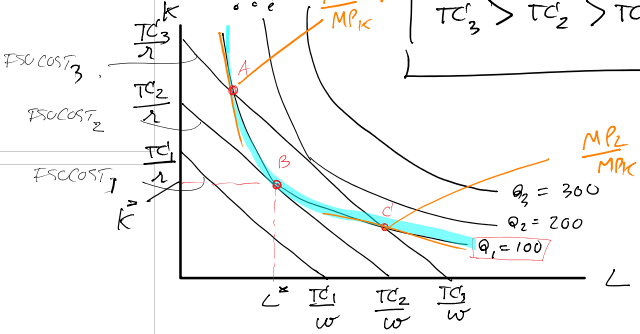
MIN TOTAL COST (TC') = $w \cdot L + r \cdot K$

SUBJECT $Q = \bar{Q}$ (A GIVEN OUTPUT LEVEL CEO ASKED THE MANAGER TO DELIVER)

CHOOSE L^*, K^* THAT MINIMIZE TC !!!

$\frac{MP_L}{w} > \frac{MP_K}{r}$ OR $\frac{MP_L}{MP_K} > \frac{w}{r}$

$TC'_3 > TC'_2 > TC'_1$



$\frac{MP_L}{MP_K} < \frac{w}{r}$ OR $\frac{MP_L}{w} < \frac{MP_K}{r}$

Expansion Path

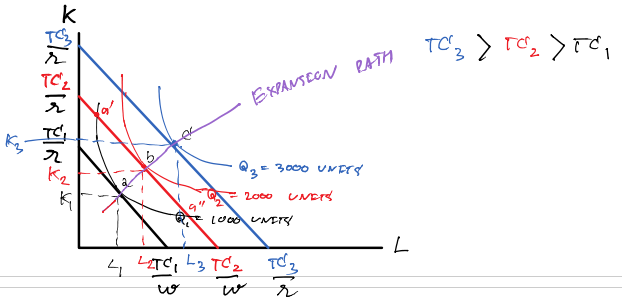
EXPANSION PATH = A COLLECTION OF LEAST-COST COMBINATION

IT TELLS US ABOUT (L, K) THAT MINIMIZES

- INPUT CHOICE B (L^*, K^*) IS COST-MINIMIZING INPUT CHOICE ON

LEAST-COST COMBINATION

IT TELLS US ABOUT (L, K) THAT MINIMIZES TOTAL COST GIVEN A DESIRED OUTPUT LEVEL



7.4

The Inflexibility of Short-Run Production

Figure 7.7

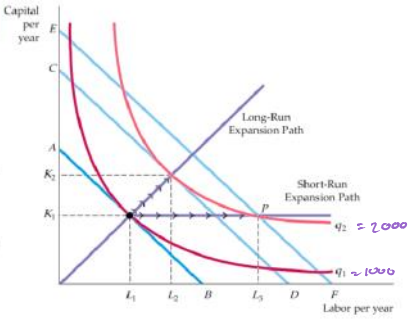
The Inflexibility of Short-Run Production

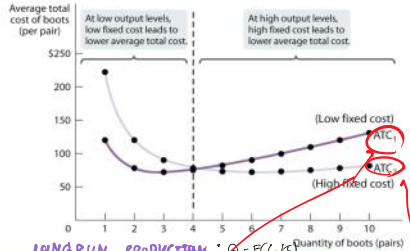
When a firm operates in the short run, its cost of production may not be minimized because of inflexibility in the use of capital inputs.

Output is initially at level q_1.

In the short run, output q_2 can be produced only by increasing labor from L_1 to L_3 because capital is fixed at K_1.

In the long run, the same output can be produced more cheaply by increasing labor from L_1 to L_2 and capital from K_1 to K_2.





Choosing the Level of Fixed Cost for Ben's Boots

There is a trade-off between higher fixed cost and lower variable cost for any given output level, and vice versa.
But as output goes up, average total cost is lower with the higher amount of fixed cost.

LONG RUN PRODUCTION: $Q = F(L, K)$

	Low fixed cost (FC = $108) 1 MACHINE			High fixed cost (FC = $216) 2 MACHINES		
Quantity of boots (pairs)	Variable cost	Total cost	Average total cost of boots ATC_1 (per pair)	Variable cost	Total cost	Average total cost of boots ATC_2 (per pair)
1	$12	$120	$120	56	$222	$222
2	48	156	78	24	240	120
3	108	216	72	54	270	90
4	192	300	75	96	312	78
5	300	408	81.6	150	366	73.2
6	432	540	90	216	432	72
7	588	696	99.4	294	510	72.9
8	768	876	109.5	384	600	75
9	972	1,080	120	486	702	78
10	1,200	1,308	130.8	600	816	81.6

K=1 K=2

- INPUT CHOICE $B(L^*, K^*)$ IS COST-MINIMIZING INPUT CHOICE OR
- A LEAST COST COMBINATION

AT B, SLOPE OF ISOQUANT = SLOPE OF ISOCOST

$$MRTS_{LK} = -\frac{w}{r}$$

$$-\frac{MP_L}{MP_K} = -\frac{w}{r}$$

L

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\boxed{\frac{MP_L}{w} = \frac{MP_K}{r}}$$

COST-MINIMIZING RULE

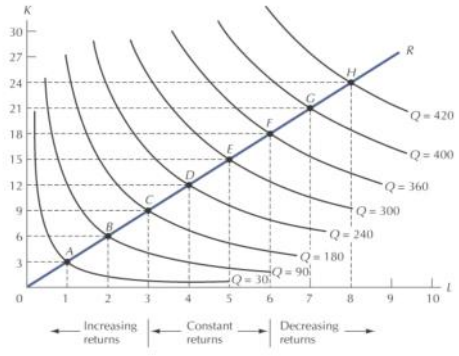
$\frac{MP_L}{w}$ = ADDITIONAL OUTPUT YOU OBTAIN FROM THE LAST BAHT SPENT ON HIRING WORKER

$\frac{MP_K}{r}$ = ADDITIONAL OUTPUT YOU OBTAIN FROM THE LAST BAHT SPENT ON RENTING MACHINE.

IF $\frac{MP_L}{w} > \frac{MP_K}{r}$ (POINT A), USE MORE L AND LESS K

IF $\frac{MP_L}{w} < \frac{MP_K}{r}$ (POINT C), USE MORE K AND LESS L

Figure 9-11: Returns to Scale Shown on the Isoquant Map



EX: USUALLY, AIRLINE INDUSTRY ENJOYS IRS'.

Point	L	% change	K	% change	Q	% change	
A	1		3		30		
B	2	100.00%	6	100.00%	90	200%	IRS
C	3	50.00%	9	50.00%	180	100%	IRS
D	4	33.33%	12	33.33%	240	33.3%	CRS
E	5	25.00%	15	25.00%	300	25%	CRS
F	6	20.00%	18	20.00%	360	20%	CRS
G	7	16.67%	21	16.67%	400	11%	DRS
H	8	14.29%	24	14.29%	420	5%	DRS

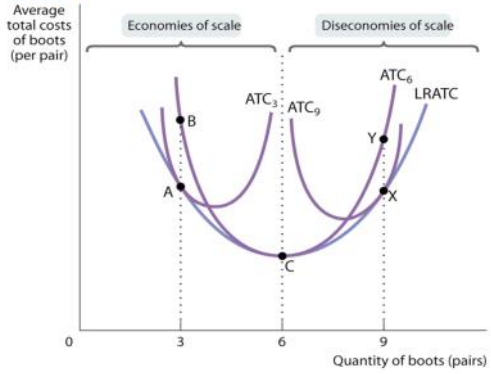
Note: Calculated from Figure 9-11: Returns to Scale Shown on the Isoquant Map, Frank (2006).

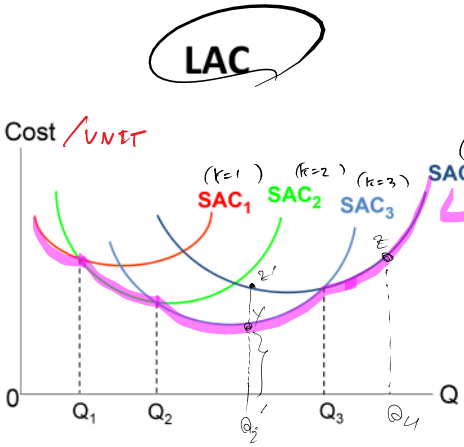
IRS: $\%\Delta Q > \%\Delta L$ AND $\%\Delta K$

CRS: $\%\Delta Q = \%\Delta L = \%\Delta K$

DRS: $\%\Delta Q < \%\Delta L$ AND $\%\Delta K$

Short-Run and Long-Run Average Total Cost Curves





LAC IS AN ENVELOPE OF SHORTRUN AVERAGE COST CURVES (SAC)

Why would a bathroom equipment manufacturer bake the image of a housefly onto the center of its ceramic urinals?

This example is base on Stefan Verhagen, "Fly in the Pot," Cornell Business, April 21, 1992.

$Q = f(L, K, A)$

↓

INNOVATION
TECHNOLOGY
IDEA CREATION