EE 421 Mathematical Economics I

Chapter 1 Introduction

There are two approaches of learning Mathematical Economics:

a. Economics explained mathematically, see Mas-Colell , et. al., [1995], Varian [1992], Luenberger [1995] and Jehle [2001].

Calculus
~~Real Analysis~~
~~Linear Algebra~~.

b. Mathematics with Economics examples, see Simon and Blume [1994], Sydsaeter and Hammond [1995] and Chiang and Wainwright [2005].

best choice of $x + y$.

$\max\ u(x,y)$
$\text{st. } P_x x + P_y y = B$

$x + y$ are endogeneous

We will adopt the second approach and use Baldani, et. al., [2005] as the textbook for this class.

1.1 Mathematical Economic Model is a collection of assumptions about:

1) **Who are the Economic Agents:** Consumers, workers, firms, nations and governments--those who can make decisions and pursue goals (objective function).
2) **Which are Endogeneous Variables:** Endogeneous variables are economic values directly controlled (***decision variables***) or indirectly affected (***auxiliary variables***) by the agents' decisions. These variables are determined endogeneously

   variables whose values will be determined inside the model.

3) **Which are Exogeneous Variables:** Exogeneous variables are the economic values that are not changed or controlled by agents' decision. Sometimes called ***parameters*** and are determined outside the model.

   outside. → just given constants

4) **How the Endogeneous and Exogeneous are interrelated.** The relationship among these variables in functional forms. This relationship could be
   a) structural equations.
   b) equality or inequality constraints.

Note: The assumptions are not expected to be a complete representation of reality. The purpose of model building is to specify the simplest model that can explain a given economic phenomenon.

1.2 Use of Economic Model

1) Use mathematical solution method to find the best decision the agents can make. This is represented as

$$\max\ u(x,y)$$
$$\text{s.t. } P_x x + P_y y = B$$
x & y are endogeneous

the best values of the decision variables--optimal solution.

2) Perform the sensitivity analysis. How a change in the value of an exogeneous variable affects

a) the endogeneous variables (**Implicit Function Theorem**)

b) objective function. (**Envelope Theorem**)

how the best choice of X would change when P_x increases

- the relationship between P_x and the best choice of X given P_y and B fixed.

→ demand of X.

how the highest utility changes when P_x increases.

1.3 An Example of Mathematical Models

Consider a firm in a perfectly competitive market. We have the following assumptions:

1. Perfect competition
2. Technology is fixed
3. Each output level q is produced with cost minimization choices of inputs K and L
4. No tax or government intervention

Each firm is the agent who tries to maximize its profit as given by the objective function

$$\boldsymbol{max}\ \pi(q) = TR(q) - TC(q)$$

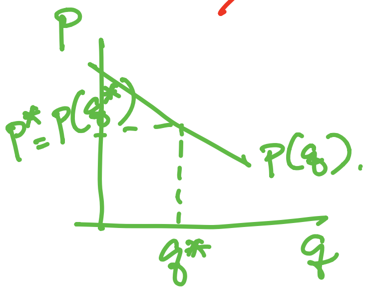
endog - q.

exog - P.

$$\max \pi(q) = P(q)\cdot q - TC(q)$$

- $TR(q) = pq$ by which assumption?
- $TC(q)$ is the minimal cost for any given quantity level q. This means the firm always select the best choice of inputs quantities at constant input prices. The total cost depends only on the quantity q.
- The quantity q is chosen by the firm. (agent.) This is the decision variable that are determined endogeneously.
- The price p is exogeneous beyond the control of the firm—thus the model is not complete.
- The optimal solution q^* will be a function of the (exog.) exogeneous variable p. That is, $q^* = q(p)$.
- Sensitivity Analysis—Comparative Static Analysis: What-if kind of questions. When the market price of the product is increased, the optimal solution q^* and maximal profit $\pi(q^*)$ will change according to the derivatives,

Once q^* is found then we can find $P(q^*)$



$\hat{q}, q_0$ $\quad \pi(q) = TR(q) - TC(q)$

$$\frac{dq^*}{dp} = q'(p) \quad ①$$

$$\frac{d\pi(q^*)}{dp} = \frac{d\pi\big(q(p)\big)}{dp} \quad ②$$

$$\pi'(q^*) = \frac{dTR(q^*)}{dq} - \frac{dTC(q^*)}{dq} = 0$$

$\pi'(q)$ is not equal to 0 at every q.

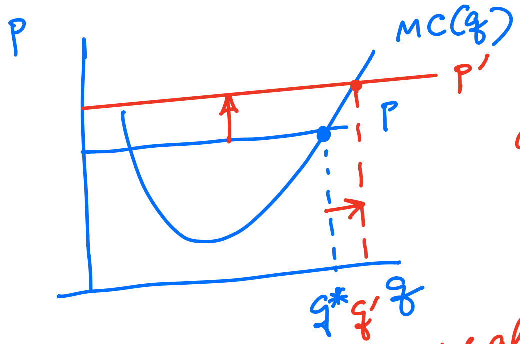
$\pi'(q^*) = 0$ only at q^*

- This sensitivity analysis can be performed with respect to any other parameter or exogeneous variable.

$q^* = q(d)$

Eg $\quad TR(C) = c + dq.$

$c + d$ are exogeneous (parameters)



$q'(p) = ?$

we have not only the sign (direction)
we also have the quantity of the change.