

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.), 2020

Use 4 decimal places for numerical answers

1. In Table 1.a, X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\bar{Y} = 3.2125$$

$$\bar{X} = 77.625$$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1 . (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^n (X_i - \bar{X})^2}$$

$$\hat{\beta}_1 = \frac{(2.8 - 3.2125)(63 - 77.625) + (3.4 - 3.2125)(72 - 77.625) + (3 - 3.2125)(78 - 77.625) + (3.5 - 3.2125)(81 - 77.625) + (3.6 - 3.2125)(87 - 77.625) + (3 - 3.2125)(75 - 77.625) + (2.7 - 3.2125)(75 - 77.625) + (3.7 - 3.2125)(90 - 77.625)}{(2.8 - 3.2125)^2 + (3.4 - 3.2125)^2 + (3 - 3.2125)^2 + (3.5 - 3.2125)^2 + (3.6 - 3.2125)^2 + (3 - 3.2125)^2 + (2.7 - 3.2125)^2 + (3.7 - 3.2125)^2}$$

$$\hat{\beta}_1 = 0.0340659 \#$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_0 = 3.2125 - 0.0340659(77.625)$$

$$\hat{\beta}_0 = 0.5681319 \#$$

1.2 For each observation i , find $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^N \hat{u}_i = 0$.

$$\text{from } \hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$$

$$\hat{Y}_1 = 2.9143$$

$$\hat{u}_1 = 0.0857$$

$$\hat{Y}_2 = 3.0209$$

$$\hat{u}_2 = 0.3791$$

$$\hat{Y}_3 = 3.2252$$

$$\hat{u}_3 = -0.225$$

$$\hat{Y}_4 = 3.3295$$

$$\hat{u}_4 = 0.195$$

$$\hat{Y}_5 = 3.5319$$

$$\hat{u}_5 = 0.0681$$

$$\hat{Y}_6 = 3.1231$$

$$\hat{u}_6 = -0.1231$$

$$\hat{Y}_7 = 3.1231$$

$$\hat{u}_7 = -0.4237$$

$$\hat{Y}_8 = 3.6340$$

$$\hat{u}_8 = 0.066$$

$$\sum_{i=0}^N \hat{u}_i = 0.0024$$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

$$var(\hat{u}_i) = \sigma^2 = \frac{\sum \hat{u}_i^2}{n-2}$$

$$var(\hat{u}_i) = \sigma^2 = 0.0854^2 + 0.3391^2 + (-0.225)^2 + 0.195^2 + 0.0681^2 + (-0.1231^2) + (-0.4231^2) + 0.066^2$$

$$var(\hat{u}_i) = \sigma^2 = \frac{0.4355}{6} = 0.0726 \#$$

$$var(\hat{\beta}_1) = \frac{\sigma^2}{\sum x_i^2} = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = \frac{0.0726}{511.875} = 0.0001 \#$$

$$var(\hat{\beta}_0) = \frac{\sum x_i^2}{n \sum x_i^2} \sigma^2 = \sum x_i^2 \cdot \frac{\sigma^2}{n \sum x_i^2} = 48,717 \cdot 0.0001 = 48,717 \#$$

2. Data is listed in the table

X_i	Y_i	$(x_i - \bar{x})$	$(y_i - \bar{y})$	
10	0	(-10)	(-9.1)	= 91
12	2	(-8)	(-7.1)	= 56.8
14	5	(-6)	(-4.1)	= 24.6
16	6	(-4)	(-3.1)	= 12.4
18	7	(-2)	(-2.1)	= 4.2
22	10	2	(0.9)	= 1.8
24	10	4	(0.9)	= 3.6
26	15	6	(5.9)	= 35.4
28	16	8	(6.9)	= 55.2
30	20	10	(10.9)	= 109

$$\bar{x} = \frac{200}{10} = 20$$

$$\bar{y} = \frac{91}{10} = 9.1$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$= \frac{394}{440}$$

$$= 0.8955 \#$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 9.1 - 0.8955(20)$$

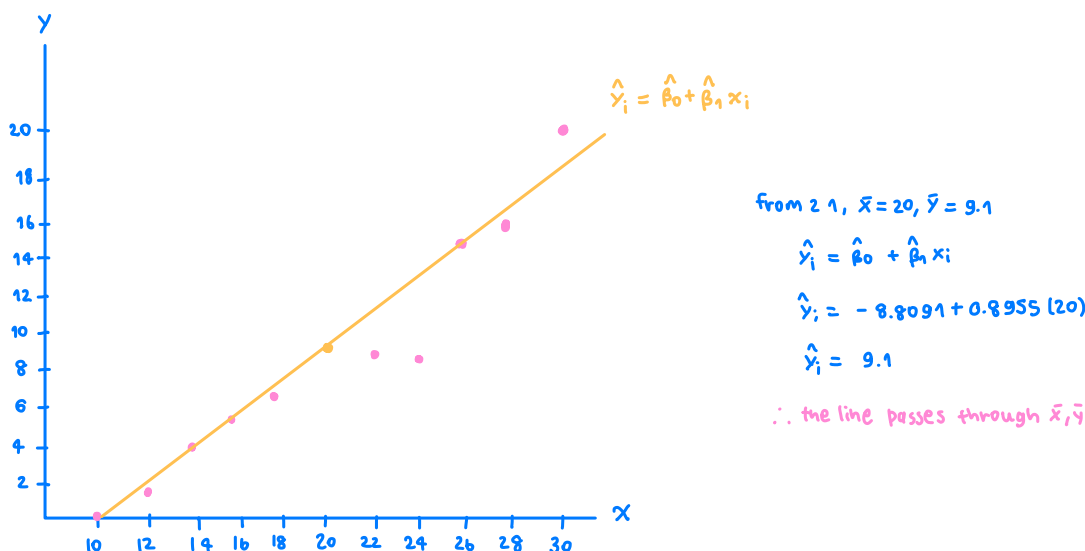
$$= -8.8091 \#$$

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^N \hat{u}_i = 0$.

from $\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$
 $\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{Y}_1 = 0.1455$	$\hat{Y}_5 = 7.3091$	$\hat{Y}_9 = 16.2636$
$\hat{u}_1 = -0.1455$	$\hat{u}_5 = -0.3091$	$\hat{u}_9 = -0.2636$
$\hat{Y}_2 = 1.9364$	$\hat{Y}_6 = 10.8909$	$\hat{Y}_{10} = 18.0545$
$\hat{u}_2 = 0.0636$	$\hat{u}_6 = -0.8909$	$\hat{u}_{10} = 1.9455$
$\hat{Y}_3 = 3.7273$	$\hat{Y}_7 = 12.6818$	$\downarrow$
$\hat{u}_3 = 1.2727$	$\hat{u}_7 = -2.6818$	$\sum \hat{u}_i = 0$
$\hat{Y}_4 = 5.5182$	$\hat{Y}_8 = 14.4727$	
$\hat{u}_4 = 0.4818$	$\hat{u}_8 = 0.5273$	

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



2.4 If $X_i = 16$, what is the predicted Y?

$$\begin{aligned} \hat{Y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_i \\ &= -8.8091 + 0.8955(16) \\ &= 5.5189 \end{aligned}$$

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_0), var(\hat{\beta}_1)$

$$var(\hat{u}_i) = 6^2 = \frac{\sum \hat{u}_i^2}{n-2}$$

$$\begin{aligned} &= (-0.1455)^2 + (0.0636)^2 + (1.2727)^2 + (0.4818)^2 + (-0.3091)^2 + (-0.8909)^2 + (-2.6818)^2 \\ &\quad + (0.5273)^2 + (-0.2636)^2 \end{aligned}$$

$$= \frac{14.0909}{10-2}$$

$$= 1.7614 \neq$$

$$var(\hat{\beta}_1) = \frac{6^2}{\sum x_i^2} = \frac{1.7614}{440}$$

$$= 0.004 \neq$$

$$var(\hat{\beta}_0) = \frac{\sum x_i^2}{n \sum x_i^2} 6^2$$

$$= \frac{4400 \cdot 0.004}{10}$$

$$= 1.7614 \neq$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum_i (y_i - \bar{y})(x_i - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$

$$\text{let } A = x_i - \bar{x}$$

$$B = \frac{x_i - \bar{x}}{\sum_i (x_i - \bar{x})^2}$$

$$\begin{aligned}\hat{\beta}_1 &= \sum_i (y_i - \bar{y}) B_i \\ &= \sum_i (\beta_1 x_i + u_i - \beta_1 \bar{x}) B_i + \sum_i u_i B_i \\ &= \beta_1 \sum_i (x_i - \bar{x}) B_i + \sum_i u_i B_i \\ &= \beta_1 \sum_i A \cdot \frac{A}{\sum_i A^2} + \sum_i u_i B_i \\ &= \beta_1 \cdot \frac{\sum_i A^2}{\sum_i A^2} + \sum_i u_i B_i\end{aligned}$$

$$\begin{aligned}E(\hat{\beta}_1) &= E(\beta_1 + \sum_i u_i B_i) \\ &= \beta_1 + E(\sum_i u_i B_i)\end{aligned}$$

SLR4 make it to be 0

$$E(\hat{\beta}_1) = \sum_i B_i E(u_i)$$

$$\text{, so } E(\hat{\beta}_1) = \beta_1$$

$\therefore \hat{\beta}_1$ is an unbiased estimator under assumption SLR 1-4