Solution: Quiz 1

1. Define the truth table for a connective operator $\oplus$ as follow.

p	q	$p \oplus q$
T	T	T
T	F	T
F	T	F
F	F	F

(a) Determine whether or not $p \oplus (q \oplus \sim p) \equiv (\sim p \oplus q) \oplus p$.

(b) Suppose $(p \oplus q) \oplus p$ is false. Determine the possible truth value(s) of $\sim p \oplus \sim (q \oplus p)$.

Solution:

(a) $p \oplus (q \oplus \sim p)$ and $(\sim p \oplus q) \oplus p$ are not equivalent. . From the truth table,

p	q	$\sim p$	$q \oplus \sim p$	$\sim p \ \oplus q$	$p \oplus (q \oplus \sim p)$	$(\sim p \ \oplus q) \oplus p$
T	T	F	T	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	F	T	F	T	F	T

the truth values of $p \oplus (q \oplus \sim p)$ and $(\sim p \oplus q) \oplus p$ in the last two columns are different. Therefore, they are not equivalent.

(b) When

$$(p \oplus q) \oplus p$$

is false, the given truth table for "$\oplus$" implies the followings.

(i) $p \oplus q$ is false (since the compound statement with two statement variables connected with the operator "$\oplus$" will be false whenever the first statement variable $p \oplus q$ is false) and the second statement variable p can be either true or false.

(ii) From (i), since $p \oplus q$ is always false, then p is false. However, q still can be either true or false.

The truth table of possible values for $\sim p \oplus \sim (q \oplus p)$ is shown below.

p	q	$\sim p$	$q \oplus p$	$\sim (q \oplus p)$	$\sim p \oplus \sim (q \oplus p)$
F	T	T	T	F	T
F	F	T	F	T	T

Hence, $\sim p \oplus \sim (q \oplus p)$ is always **true** as shown in the last column of the truth table.

Remark: For (b), instead of using the truth table, it is also possible to argue that the truth value of the compound statement with two statement variables connected with the operator "$\oplus$" always depends on the truth value of the **first** statement variable: it will be **true** if the first statement variable is **true**, and it will be **false** if the first statement variable is **false**. Since p **is false** and $\sim p$ **is true**, then $\sim p \oplus \sim (q \oplus p)$ is always **true**.