EE 320 Introductory Mathematical Economics

Semester 1/2015

Homework 4

Due 17 November 2015

Question 1: optimal factor inputs decision I

Suppose that the output Q of a firm depends on two inputs: x and y. The output level is determined by the production function

$$Q = f(x, y) = 8x + 12y - x^2 - 2y^2$$

Suppose that the output price is \$3 per unit, and the input prices for x and y are \$6 and \$12 per unit, respectively.

a. Determine whether the firm's production function is convex or concave by the derivative conditions.
b. Find the levels of x^* and y^* that maximize the firm's profit, and verify the answer by using the second-order sufficient conditions.

Question 2: optimal factor inputs decision II

Suppose that the output Q of a firm depends on two inputs of the quantities K and L. The output level is determined by the production function

$$Q = 32K + 24L - 4K^2 - 2KL - 2L^2$$

a. Is the firm's production function strictly concave? Explain.
b. Write down the firm's profit function when the price of Q is \$1 and the per-unit factor prices of K and L are r and w, respectively, where both r and

w are positive numbers. Find the levels of K^* and L^* that maximize the firm's profits.

c. Verify that the second-order sufficient conditions for maximum profits are satisfied.

d. Determine the effect of an increase in r on the firm's use of each input. (i.e. determine $\frac{\partial K^*}{\partial r}$ and $\frac{\partial L^*}{\partial r}$).

Question 3: Multi-product problem

The demand for a monopolist's two products are determined by the equations

$$p_1 = 30 - q_1 \qquad \text{and} \quad p_2 = 28 - 2q_2$$

where p_1 and p_2 are prices per unit of the two goods, and q_1 and q_2 are the corresponding quantities. The costs of producing and selling q_1 units of the first good and q_2 units of the second good are

$$C(q_1, q_2) = 2q_1^2 + 4q_1q_2 + q_2^2 \ .$$

a) Find the monopolist's profit $\pi(q_1, q_2)$ from producing and selling q_1 units of the first good and q_2 units of the second good.

b) Find the values q_1 and q_2 that maximize $\pi(q_1, q_2)$. Show the sufficient conditions for profit maximization.

Question 4

Consider the function f defined for all (x,y) such that

$$f(x,\ y; a) = \tfrac{1}{2}x^2 - x + ay(x-1) - \tfrac{1}{3}y^3 + a^2y^2 \ ,$$

where a is a constant.

a. Prove that $(x^*,\ y^*) = (1 - a^3, a^2)$ is a stationary point of $f(x,\ y; a)$.

b. Given that $G(a) = f(x^*,\ y^*; a)$, show the derivative of G with respect to a.

c. Calculate $\frac{\partial f(x, y; a)}{\partial a}$ and evaluate its value where $(x^*,\ y^*) = (1 - a^3, a^2)$. Compare your answer with the answer obtained in b. Are they the same?

d. Where in the xy-plan is f convex.

Question 5: Price discrimination

Consider a monopolist producer of a product whose technology can be given by a constant marginal cost function, i..e MC = 4. The demand curves for the two market segments of this product are given below.

Segment A: P = 100 - 2Q
Segment B: P = 50 - .5Q

a.) If a monopolist can practice third-degree price discrimination, what price will they set in the two markets?
(Hint: your work should start from defining the objective function, and state down all the first-order conditions. You need to check for the sufficient condition to warrant that your answer is truly a maximum point.)

b) Now suppose the monopolist *cannot* price discriminate. Instead, they must charge a single price in both markets. What price will they charge?
(Hint: you should start from discussing implication of demand curve faced by the monopolist, when the monopolist cannot distinguish type of the buyer. Then, start the optimization problem using that assumption.)

Question 6

Consider a simple two-market model where demand and supply for each market is given below. (Notationally, let's name the two markets as A and B, respectively.)

Market A:

Demand: $p_A = 10 - 2Q_A$
Supply: $p_A = 1 + Q_A$

Market B:

Demand: $p_B = 20 - Q_B$
Supply: $p_B = 2 + 2Q_B$

a. Derive the market equilibrium

b. Suppose the government imposes unit tax on consumers in both markets at the rate of t_A and t_B. Solve for the after-tax equilibrium as the function of t_A and t_B.
c. How much revenue can the government collect from the taxation?
d. Determine the level of t_A and t_B that maximizes government's revenue.