

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

$$\begin{aligned} \text{a. } \sum_{i=1}^5 (a + bx_i) &= \sum_{i=1}^5 a + \sum_{i=1}^5 bx_i \\ &= 5a + b \sum_{i=1}^5 x_i \\ &= 5a + b(x_1 + x_2 + x_3 + x_4 + x_5) \end{aligned}$$

$$\text{b. } \sum_{y=0}^5 f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$$

$$\begin{aligned} \text{c. } \sum_{i=1}^{10} i^2 &= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 \\ &= 385 \end{aligned}$$

$$\begin{aligned} \text{d. } \sum_{x=1}^2 \sum_{y=2}^3 (2x+y) &= \sum_{x=1}^2 \left(\sum_{y=2}^3 (2x+y) \right) = \sum_{x=1}^2 ((2x+2) + (2x+3)) \\ &= \sum_{x=1}^2 (4x+5) = (4(1)+5) + (4(2)+5) = 22 \end{aligned}$$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

- a. Find the value of b

$$\begin{aligned} \sum_x f(x) &= 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b \\ &= 8b \\ 1 &= 8b \quad b = \frac{1}{8} \end{aligned}$$

- b. Find the answer for $P(X \leq 2)$

$$\begin{aligned} P(X \leq 2) &= P(-2) + P(-1) + P(0) + P(1) + P(2) \\ &= 0.5b + b + 2.25b + 2b + 1.5b \\ &= 7.25b = 7.25\left(\frac{1}{8}\right) = 0.90625 \end{aligned}$$

- c. Find the answer for $P(-2 \leq X \leq 3)$

$$\begin{aligned} P(-2 \leq X \leq 3) &= P(-2) + P(-1) + P(0) + P(1) + P(2) + P(3) \\ &= 0.5b + b + 2.25b + 2b + 1.5b + 0.5b \\ &= 7.75b = 7.75\left(\frac{1}{8}\right) = 0.96875 \end{aligned}$$

- d. Find the answer for $P(X \geq 1)$

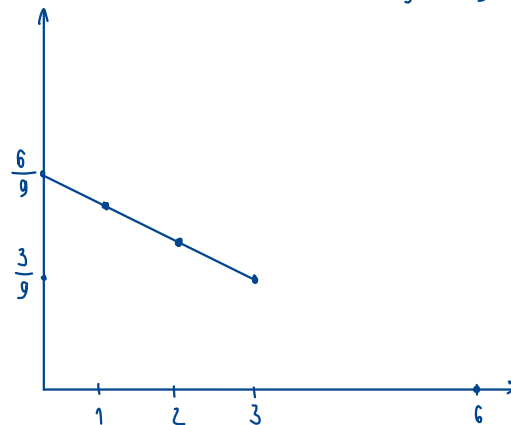
$$\begin{aligned} P(X \geq 1) &= P(1) + P(2) + P(3) + P(4) \\ &= 2b + 1.5b + 0.5b + 0.25b \\ &= 4.25b = 4.25\left(\frac{1}{8}\right) = 0.53125 \end{aligned}$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$

$$f(x) = -\frac{1}{9}x + \frac{6}{9}$$



$$f(0) = -\frac{1}{9}(0) + \frac{6}{9} = \frac{6}{9}$$

$$f(1) = -\frac{1}{9}(1) + \frac{6}{9} = \frac{5}{9}$$

$$f(2) = -\frac{1}{9}(2) + \frac{6}{9} = \frac{4}{9}$$

$$f(3) = -\frac{1}{9}(3) + \frac{6}{9} = \frac{3}{9}$$

- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} \int_1^3 -\frac{1}{9}x + \frac{6}{9} dx &= \left(-\frac{1}{9}x^2 \right) \frac{1}{2} + \frac{6}{9}x \Big|_1^3 \\ &= \left(-\frac{1}{9}(3)^2 \right) \frac{1}{2} + \frac{6}{9}(3) - \left(-\frac{1}{9}(1)^2 \right) \frac{1}{2} + \frac{6}{9}(1) \\ &= \left(-\frac{1}{2} + 2 \right) - \left(-\frac{1}{18} + \frac{6}{9} \right) = \frac{3}{2} - \frac{11}{18} = \frac{16}{18} \# \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} \int_2^3 -\frac{1}{9}x + \frac{6}{9} dx &= \left(-\frac{1}{9}x^2 \right) \frac{1}{2} + \frac{6}{9}x \Big|_2^3 \\ &= \left(-\frac{1}{9}(3)^2 \right) \frac{1}{2} + \frac{6}{9}(3) - \left(-\frac{1}{9}(2)^2 \right) \frac{1}{2} + \frac{6}{9}(2) \\ &= \left(-\frac{1}{2} + 2 \right) - \left(-\frac{2}{9} + \frac{12}{9} \right) = \frac{3}{2} - \frac{10}{9} = \frac{27-20}{18} = \frac{7}{18} \# \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} E(X) &= \int_0^3 x f(x) dx \\ &= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x dx \\ &= \left(-\frac{1}{9}x^3 \right) \frac{1}{3} + \left(\frac{6}{9}x \right) \frac{1}{2} \Big|_0^3 \\ &= \left(-\frac{1}{27} + \frac{6}{18} \right) - \left(-\frac{1}{27} + \frac{6}{18} \right) \\ &= (-1 + 3) - 0 \\ &= 2 \# \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of X and Y

$\begin{matrix} X \\ Y \end{matrix}$	1	2	3	4	5	6	$P(X=x)$
0	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{6}{12}$
1	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$	$\frac{6}{12}$
$P(Y=y)$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	

- b. Find the marginal probability distribution function (PDF) of X

$$\begin{aligned}
 f(X=1) &= \sum f(X=1, Y=y) = f(X=1, Y=0) + f(X=1, Y=1) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6} \\
 f(X=2) &= \sum f(X=2, Y=y) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6} \\
 f(X=3) &= \sum f(X=3, Y=y) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6} \\
 f(X=4) &= \sum f(X=4, Y=y) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6} \\
 f(X=5) &= \sum f(X=5, Y=y) = \frac{1}{6} \\
 f(X=6) &= \sum f(X=6, Y=y) = \frac{1}{6}
 \end{aligned}$$

- c. Find the marginal probability distribution function (PDF) of Y

$$\begin{aligned}
 f(Y=0) &= \sum f(X=x, Y=0) = 6 \left(\frac{1}{12} \right) = \frac{1}{2} \\
 f(Y=1) &= \sum f(X=x, Y=1) = 6 \left(\frac{1}{12} \right) = \frac{1}{2}
 \end{aligned}$$

- d. Find the conditional probability distribution function (PDF) of

$$\begin{aligned}
 P(X=x | Y=1) &= \frac{f(X=x, Y=1)}{f(Y=1)} \\
 P(X=1 | Y=1) &= \frac{f(X=1, Y=1)}{f(Y=1)} = \frac{1}{12} \cdot 2 = \frac{1}{6} \\
 P(X=2 | Y=1) &= \frac{1}{6} \\
 P(X=3 | Y=1) &= \frac{1}{6} \\
 P(X=4 | Y=1) &= \frac{1}{6} \\
 P(X=5 | Y=1) &= \frac{1}{6} \\
 P(X=6 | Y=1) &= \frac{1}{6}
 \end{aligned}$$

- e. Find the expected value of X given Y is equal to 1

$$\begin{aligned}
 E(X | Y=1) &= \sum x f(X=x | Y=1) = 1 \left(\frac{f(X=1, Y=1)}{f(Y=1)} \right) + 2 \left(\frac{f(X=2, Y=1)}{f(Y=1)} \right) + 3 \left(\frac{f(X=3, Y=1)}{f(Y=1)} \right) + 4 \left(\frac{f(X=4, Y=1)}{f(Y=1)} \right) \\
 &\quad + 5 \left(\frac{f(X=5, Y=1)}{f(Y=1)} \right) + 6 \left(\frac{f(X=6, Y=1)}{f(Y=1)} \right) \\
 &= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = 3.5
 \end{aligned}$$

- f. Find the variance of X given Y is equal to 1

$$\begin{aligned}
 \text{Var}(X | Y=1) &= \sum [X - E(X | Y=1)]^2 \cdot f(X=x | Y=1) \\
 &= (1-3.5)^2 \left(\frac{1}{6} \right) + (2-3.5)^2 \left(\frac{1}{6} \right) + (3-3.5)^2 \left(\frac{1}{6} \right) + (4-3.5)^2 \left(\frac{1}{6} \right) + (5-3.5)^2 \left(\frac{1}{6} \right) + (6-3.5)^2 \left(\frac{1}{6} \right) \\
 &= 6.25 \left(\frac{1}{6} \right) + 2.25 \left(\frac{1}{6} \right) + 0.25 \left(\frac{1}{6} \right) + 0.25 \left(\frac{1}{6} \right) + 2.25 \left(\frac{1}{6} \right) + 6.25 \left(\frac{1}{6} \right) \\
 &= 17.5 \left(\frac{1}{6} \right) = 2.91667
 \end{aligned}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left(\frac{X_1 + X_2 + X_3}{3}\right)$$

$$= \frac{1}{3}(E(X_1) + E(X_2) + E(X_3))$$

$$= \frac{1}{3}(\mu_x + \mu_x + \mu_x)$$

$$= \frac{1}{3} 3\mu_x$$

$$= \mu_x \quad \#$$

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{3} \sum_{i=1}^3 X_i\right)$$

$$= \left(\frac{1}{3}\right)^2 \text{Var}(X_1 + X_2 + X_3)$$

$$= \frac{1}{9} \left[\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + 2\text{Cov}(X_1, X_2) + 2\text{Cov}(X_1, X_3) + 2\text{Cov}(X_2, X_3) \right]$$

$$= \frac{1}{9} \left[\sigma^2 + \sigma^2 + \sigma^2 + 2\left(\frac{1}{4}\sigma^2\right) + 2\left(\frac{1}{4}\sigma^2\right) + 2\left(\frac{1}{4}\sigma^2\right) \right]$$

$$= \frac{1}{9} \left[3\sigma^2 + 3\left(\frac{1}{2}\sigma^2\right) \right]$$

$$= \frac{1}{9} (3) \left[\sigma^2 + \frac{1}{2}\sigma^2 \right]$$

$$= \frac{1}{9} \left(\frac{3}{2} \sigma^2 \right) = \frac{\sigma^2}{2} \quad \#$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{X}) = E\left(\frac{X_1 + X_2 + X_3 + X_4}{4}\right)$$

$$= \frac{1}{4}(E(X_1) + E(X_2) + E(X_3) + E(X_4))$$

$$= \frac{1}{4}(\mu_x + \mu_x + \mu_x + \mu_x)$$

$$= \frac{1}{4}(4\mu_x)$$

$$= \mu_x$$

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{4} \sum_{i=1}^4 X_i\right)$$

$$= \left(\frac{1}{4}\right)^2 \text{Var}(X_1 + X_2 + X_3 + X_4)$$

$$= \frac{1}{16} (\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + \text{Var}(X_4))$$

$$= \frac{1}{16} (\sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \sigma_x^2)$$

$$= \frac{1}{16} 4\sigma_x^2$$

$$= \frac{\sigma_x^2}{4}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that $\tilde{X}$ is an unbiased estimator of μ

$$\begin{aligned}
 E(\tilde{X}) &= E\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right) & \text{Var}(\tilde{X}) &= \text{Var}\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right) \\
 &= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4) & &= \left(\frac{1}{8}\right)^2 \text{Var}(X_1) + \left(\frac{1}{4}\right)^2 \text{Var}(X_2) + \left(\frac{1}{8}\right)^2 \text{Var}(X_3) + \left(\frac{1}{2}\right)^2 \text{Var}(X_4) \\
 &= \frac{1}{8}\mu_x + \frac{1}{4}\mu_x + \frac{1}{8}\mu_x + \frac{1}{2}\mu_x & &= \frac{1}{64}\sigma_x^2 + \frac{1}{16}\sigma_x^2 + \frac{1}{64}\sigma_x^2 + \frac{1}{4}\sigma_x^2 \\
 &= \frac{8}{8}\mu_x & &= \frac{22}{64}\sigma_x^2 \\
 & & &= \frac{11}{32}\sigma_x^2
 \end{aligned}$$

$$E(\tilde{X}) = \mu_x$$

$\tilde{X}$ is an unbiased estimator of μ_x .

- c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$$\text{Var}(\bar{X}) = \frac{\sigma^2}{4} \quad \text{Var}(\tilde{X}) = \frac{11}{32}\sigma^2$$

$$\text{Var}(\tilde{X}) > \text{Var}(\bar{X})$$

so, $\bar{X}$ is more efficient estimator of μ_x than $\tilde{X}$ because it has smaller variance.