

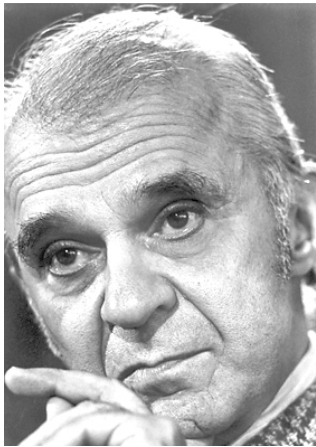
# Input-Output Multiplier

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EE 459

# Input-Output Model

- Prof. Wassily Leontief (1906-1999) developed an “input-output” method for estimating economic impacts and tracing the flows of dollars. Leontief later won the **Nobel Prize in 1973**, largely related to this work.



*The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1973 was awarded to Wassily Leontief "for the development of the input-output method and for its application to important economic problems".*

[http://www.nobelprize.org/nobel\\_prizes/economics/laureates/1973/leontief.html](http://www.nobelprize.org/nobel_prizes/economics/laureates/1973/leontief.html)

## Main advantages of IO Model

- 1) Many different industries/sectors
- 2) Ripple (multiplier) effects contained in the inter-industry transactions
- 3) Analyzes changes and impacts at a sector by sector level, tracing flows of dollars between industries

# Input-Output Model

- The IO model is centered on the idea of inter-industry transactions:
  - Industries use the products of other industries to produce their own products.
  - For example - automobile producers use steel, glass, rubber, and plastic products to produce automobiles.
  - Outputs from one industry become inputs to another.
  - When you buy a car, you affect the demand for glass, plastic, steel, etc.

*Taken from a Power Point presentation prepared by  
Pam Perlich at the University of Utah.  
<http://www.business.utah.edu/~bebrpsp/IO/IO.ppt>*

# ***IO Conceptualization of the Economy***

- The major conceptual step is to divide the economy into “purchasers” and “suppliers”.
- Primary Suppliers*: They sell primary inputs (labor, land, capital, etc.) to other industries. Payments to these suppliers are “primary inputs” because they generate no further sales. (example: Households)
- Intermediate Suppliers*: They purchase inputs for processing into outputs they supply to other firms or to final purchasers. (example: Automaker)
- Intermediate Purchasers*: They purchase outputs of suppliers for use as inputs for further processing. (example: Automaker)
- Final Purchasers*: Purchase the outputs of suppliers in their final form and for final use. (example: Households)
- *Intermediate Suppliers* and *Intermediate Purchasers* are the same thing!
- *Primary Suppliers* and *Final Purchasers* may or may not be the same entities. When they are the same (households), these activities are understood as separate activities.

# *The Problems with IO Analysis*

## **Practical Issues**

- *Data needs and complexity*: IO models are *tremendously complex* and *very data hungry*. This typically places these models in the hands of experts.

## **Theoretical Issues**

- *Time/Data issues*: Usually a single year's data are used to develop the Total Requirements Table. But 1) purchases may actually reflect a longer term investment and 2) short term trends may impact the data.
- *Stability of the technical coefficients over time*: Technology changes, prices change, and demand changes, all affecting the coefficients in the Tot Reqs Table. This can impact the results if the coefficients are "out of date".
- *IO assumes a linear relationship between increasing demand for inputs and outputs*: This assumes away 1) externalities and 2) increasing/decreasing returns to scale.
- *Industrial categorization*: IO models still assume that each industry 1) has a single, homogeneous production function and 2) each produces one product. These assumptions do not reflect the real economy very well.

# *The Power of IO Models*

- Despite these problems IO analysis is a tremendously popular and powerful analytical tool.
- “The chief value of regional input-output analysis is in its descriptive analytical power.” (Bendavid-Val, p.113)
- “As a descriptive tool, input-output tables:
  - present an enormous quantity of information in a concise, orderly, and easily understood fashion;*
  - provide a comprehensive picture of the inter-industry structure of the regional economy;*
  - point up the strategic importance of various industries and sectors;*
  - highlight possible opportunities for strengthening regional income and employment multiplication.”* (Bendavid-Val, p.113)
- Urban Planners should be capable of understanding the structure, assumptions, and data requirements of Input-Output Analysis. While you may not be performing this analysis in your jobs, you almost certainly will come across this type of work sometime in your career.

# Predictive Use of Input-Output Analysis

- Impacts are tracked throughout the economy
- **Multipliers** are derived from regional economic accounts

# What are Multipliers?

Multipliers measure total change throughout the economy from a one unit change for a given sector.

# Input-Output (Matrix Accounting)

Because of the accounting conventions adopted in the construction of an I/O transactions table, the following will always be true:

1. For each industry: Total output  $\equiv$  Total input, that is, the sum of the elements in any row is equal to the sum of the elements in the corresponding column.
2. For the table as a whole: Total intermediate sales  $\equiv$  Total intermediate purchases, and Total final demand  $\equiv$  Total primary input

Note the use here of the identity sign,  $\equiv$ , reflecting the fact that these are accounting identities, which always hold in an I/O transactions table.

Reading across rows the necessary equality of total output with the sum of its uses for each industry or sector can be written as a set of 'balance equations':

$$X_i \equiv \sum_j X_{ij} + Y_i, \quad i = 1, \dots, n$$

*where*

$X_i$  = total output of industry  $i$

$X_{ij}$  = sales of commodity  $i$  to industry  $j$

$Y_i$  = sales of commodity  $i$  to final demand

$n$  = the number of industries

# Input-Output (Matrix Accounting)

To go from accounting to analysis, the basic assumption is

$$X_{ij} = a_{ij} X_j \quad (\text{eq.2})$$

where  $a_{ij}$  is a constant.

Substituting (eq.2) into (eq.1) gives

$$X_i = \sum_j a_{ij} X_j + Y_i, i=1, \dots, n \quad (\text{eq.3})$$

as a system of  $n$  linear equations in  $2n$  variables, the  $X_i$  and  $Y_i$ , and  $n^2$  coefficients, the  $a_{ij}$ . If the  $Y_i$  – the final demand levels – are specified, there are  $n$  unknown  $X_i$  – the gross output levels – which can be solved for using the  $n$  equations.

# Input-Output Multiplier

In matrix notation, (eq.3) is

$$X = AX + Y$$

which can be written

$$X - AX = Y \quad (\text{eq.4})$$

where  $X$  is an  $n \times 1$  vector of gross outputs,  $A$  is an  $n \times n$  matrix of coefficients  $a_{ij}$ , and  $Y$  is an  $n \times 1$  vector of final demands,  $Y_i$ . With  $I$  as the identity matrix, (8.4) can be written

$$(I - A)X = Y$$

which has the solution

$$X = (I - A)^{-1}Y \quad (\text{eq.5})$$

where  $(I - A)^{-1}$  is the inverse of  $(I - A)$ . This can be written

$$X = LY \quad (\text{eq. 6})$$

$L$  is often known as the **Leontief inverse**, in recognition of inventor of i-o analysis

### Simplified China IO Table , 1997

	Agriculture	MineMetal	ConsGood	ChemMet	ManuConst	TranServ	HH	Investment	Rest of World	Total
Agriculture	39.64	0.68	77.35	7.45	1.50	7.51	103.61	9.26	4.08	251.07
MineMetal	0.51	5.19	2.45	48.33	13.65	1.96	0.91	-0.19	3.90	76.71
ConsGood	17.98	1.00	111.12	21.29	9.35	35.36	117.15	22.06	58.48	393.79
ChemMet	25.47	12.84	32.76	226.41	126.48	51.14	25.23	64.51	39.21	604.05
ManuConst	2.65	5.62	8.94	37.67	53.67	34.30	23.25	176.17	32.78	375.03
TranServ	13.10	7.28	30.52	52.42	37.26	80.59	187.01	7.52	13.86	429.55
Capital	29.66	24.77	64.15	90.79	69.23	121.41				
Labor	113.43	7.29	19.12	26.84	24.50	42.70				
TAX	4.69	4.40	25.00	34.42	13.48	47.32				
Rest of World	3.94	7.66	22.39	58.44	25.91	7.25				
<b>Total</b>	<b>251.07</b>	<b>76.71</b>	<b>393.79</b>	<b>604.05</b>	<b>375.03</b>	<b>429.55</b>				

A	0.1579	0.0088	0.1964	0.0123	0.0040	0.0175
	0.0020	0.0677	0.0062	0.0800	0.0364	0.0046
	0.0716	0.0130	0.2822	0.0352	0.0249	0.0823
	0.1015	0.1673	0.0832	0.3748	0.3373	0.1191
	0.0105	0.0732	0.0227	0.0624	0.1431	0.0798
	0.0522	0.0949	0.0775	0.0868	0.0993	0.1876

I	1	0	0	0	0	0
	0	1	0	0	0	0
	0	0	1	0	0	0
	0	0	0	1	0	0
	0	0	0	0	1	0
	0	0	0	0	0	1

I-A	0.8421	-0.0088	-0.1964	-0.0123	-0.0040	-0.0175
	-0.0020	0.9323	-0.0062	-0.0800	-0.0364	-0.0046
	-0.0716	-0.0130	0.7178	-0.0352	-0.0249	-0.0823
	-0.1015	-0.1673	-0.0832	0.6252	-0.3373	-0.1191
	-0.0105	-0.0732	-0.0227	-0.0624	0.8569	-0.0798
	-0.0522	-0.0949	-0.0775	-0.0868	-0.0993	0.8124

inv(I-A)	1.231106783	0.040323116	0.35481605	0.065425762	0.052510136	0.077424573
	0.030902989	1.117251949	0.04613091	0.164664268	0.119254938	0.047457571
	0.154680674	0.074257207	1.475291452	0.133380542	0.120689196	0.184658845
	0.283968326	0.434661437	0.358489363	1.813540332	0.788994732	0.388216194
	0.055091771	0.14865949	0.093853841	0.173788421	1.263393643	0.161165353
	0.134514781	0.204798155	0.218681483	0.251132596	0.267585062	1.320246956

Sum = 1.82160642 (Forward Linkage Multiplier)

Sum = 1.890265324 (Backward Linkage Multiplier)