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Group Homework 7 & 8

Semester 2/2021 EE320 Introductory mathematical economics

Due date: May 7th 2022 (before midnight /B.E. moodle).

1. *Theory of firm*

Suppose that production function is given by $Q = f(K, L) = \alpha\sqrt{K} + \beta L$ where K and L are the unit of capital installed and the number of employees hired, respectively. Assume that price of K and L are set equal to "r" and "w", respectively. Consider the following problems.

a) The firm wants to minimize cost and seek for combination of the two factor inputs to produce output level Q_0. Derive the factor inputs demand.

b) Confirm your result with the second order derivative test.

c) State the condition under which demand for capital and labor are both strictly positive.

d) Suppose that the condition required for strictly positive solution holds, derive the long-run optimal cost function, and show that marginal cost function is equal to the LaGrange multiplier of your cost minimization problem.

2. *Theory of consumer*

Consider a household with the utility function given by,

$$U(x, y) = [x^2 + y^2]^{\frac{1}{2}}$$

where x *and* y are two different consumption goods, i.e. good x and good y. Suppose that (i) *the prices for each of the two consumption goods are* p_x *and* p_y *respectively*, and (ii) *household's income is equal to M*. Consider the following problems

a) Calculate the total differential of the utility function.

b) Set up the constrained optimization problem and derive the Marshallian demand function.

c) Does the demand function satisfy *the law of demand*? Mathematically, how do you know that?

d) How does the demand for good y respond to price of good x?

e) What is the numerical value of λ^* when $M = \$300,\ p_x = 1,\ p_y = 1$?

f) Without redoing the optimization problem, what would be the new optimized level of maximum utility when income increases to \$310.

3. Suppose that a monopolist has its marginal cost function given by $MC = 16 + 6q^2$ where q is the amount of output produced. The monopolist faces the market demand function given by $P = 160 - 10q^2$ where P is the price per unit of output. Consider the following problem.

a) Suppose that fixed cost is equal to \$240. Calculate the total and the average cost when q = 9 units.
b) Determine the profit-maximizing level of output for the monopolist. Also, confirm your result by using the second derivative test.
c) Calculate the social welfare under the monopoly environment.
d) Calculate the social welfare loss under the monopoly environment.

4. Suppose the demand and supply curves are $P = \frac{6000}{Q+50}$ and $P = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.

5. Let $MR = 25 - 5x - 2x^2$ and $MC = 10 - 3x - x^2$, where x is the unit of output. Assume that fixed cost is \$7. Determine the level of production that contributes to maximum profit and determine the level of maximized profit.

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1. *Theory of firm*

Suppose that production function is given by $Q = f(K, L) = \alpha\sqrt{K} + \beta L$ where K and L are the unit of capital installed and the number of employees hired, respectively. Assume that price of K and L are set equal to "r" and "w", respectively. Consider the following problems.

a) The firm wants to minimize cost and seek for combination of the two factor inputs to produce output level Q_0. Derive the factor inputs demand.

min Cost : $wL + rK$

s.t. production function $\bar{Q} = f(K,L) = \alpha\sqrt{K} + \beta L$

$$\mathcal{L} = wL + rK + \lambda(\bar{Q} - \alpha\sqrt{K} - \beta L)$$

F.O.C.s

$[K] \Rightarrow \mathcal{L}_K = r - \lambda \frac{1}{2}\alpha K^{-\frac{1}{2}} = 0$ —— ①

$[L] \Rightarrow \mathcal{L}_L = w - \lambda\beta = 0$ —— ② $\rightsquigarrow \lambda^* = \frac{w}{\beta}$

$[\lambda] \Rightarrow \mathcal{L}_\lambda = \bar{Q} - \alpha\sqrt{K} - \beta L$ —— ③

$\frac{①}{②}$; $\frac{r}{w} = \frac{\cancel{\lambda}\frac{1}{2}\alpha K^{-\frac{1}{2}}}{\cancel{\lambda}\beta} = \frac{\alpha}{2\beta\sqrt{K}}$ —— ④

Taking square over ④; $\left(\frac{r}{w}\right)^2 = \left(\frac{\alpha}{2\beta\sqrt{K}}\right)^2$

$$\left(\frac{r}{w}\right)^2 = \frac{\alpha^2}{4\beta^2 K}$$

$$= \frac{\alpha^2}{4\beta^2\left(\frac{r}{w}\right)^2}$$

$K^* = \left(\frac{w\alpha}{2\beta r}\right)^2$ sub into ③;

$$\bar{Q} - \alpha\sqrt{\left(\frac{w\alpha}{2\beta r}\right)^2} - \beta L = 0$$

$$\bar{Q} - \alpha\left(\frac{w\alpha}{2\beta r}\right) - \beta L = 0$$

$$\frac{\bar{Q} - \alpha\left(\frac{w\alpha}{2\beta r}\right)}{\beta} = L^*$$

$$K^* = \left(\frac{W\alpha}{2\beta r}\right)^2 \quad \# \qquad L^* = \frac{\bar{Q} - \alpha\left(\frac{W\alpha}{2\beta r}\right)}{\beta} \quad \#$$

b) Confirm your result with the second order derivative test.

$$H = \begin{bmatrix} \mathcal{L}_{\lambda\lambda} & \mathcal{L}_{\lambda K} & \mathcal{L}_{\lambda L} \\ \mathcal{L}_{K\lambda} & \mathcal{L}_{KK} & \mathcal{L}_{KL} \\ \mathcal{L}_{L\lambda} & \mathcal{L}_{LK} & \mathcal{L}_{LL} \end{bmatrix} = \begin{bmatrix} 0 & -\alpha\frac{1}{2}K^{-\frac{1}{2}} & -\beta \\ -\alpha\frac{1}{2}K^{-\frac{1}{2}} & \frac{1}{4}\lambda\alpha K^{-\frac{3}{2}} & 0 \\ -\beta & 0 & 0 \end{bmatrix}$$

$$\left(-\beta \cdot -\beta \cdot \frac{1}{4}\lambda\alpha K^{-\frac{3}{2}}\right) \qquad 0 \qquad 0$$

$$= \left[\begin{array}{ccc} 0 & -\alpha\frac{1}{2}K^{-\frac{1}{2}} & -\beta \\ -\alpha\frac{1}{2}K^{-\frac{1}{2}} & \frac{1}{4}\lambda\alpha K^{-\frac{3}{2}} & 0 \\ -\beta & 0 & 0 \end{array}\right] \begin{array}{cc} 0 & -\alpha\frac{1}{2}K^{-\frac{1}{2}} \\ -\alpha\frac{1}{2}K^{-\frac{1}{2}} & \frac{1}{4}\lambda\alpha K^{-\frac{3}{2}} \\ -\beta & 0 \end{array}$$

$$0 \qquad 0 \qquad 0$$

$$= (0+0+0) - \left[\beta^2 \cdot \left(\frac{1}{4}\lambda\alpha K^{-\frac{3}{2}}\right)\right]$$

$$= -\left[\beta^2 \cdot \left(\frac{1}{4}\lambda\alpha K^{-\frac{3}{2}}\right)\right] < 0 \quad \#$$

- convex function
- confirmed minimized cost function

c) State the condition under which demand for capital and labor are both strictly positive.

$$K^* = \boxed{\left(\frac{W\alpha}{2\beta r}\right)}^2$$

must be more than zero, and
for the condition to exist β and r must not equal zero

$$L^* = \frac{\boxed{\bar{Q} - \frac{W\alpha^2}{2\beta r}}}{\beta}$$

This value must be more than zero.

for the condition to exist
β and r must not equal zero

d) Suppose that the condition required for strictly positive solution holds, derive the long-run optimal cost function, and show that marginal cost function is equal to the LaGrange multiplier of your cost minimization problem.

$$K^* = \left(\frac{W\alpha}{2\beta r}\right)^2 \qquad L^* = \frac{\bar{Q} - \alpha\left(\frac{W\alpha}{2\beta r}\right)}{\beta}$$

$$C = w \cdot L + r \cdot K$$

$$C^* = C(K^*, L^*)$$

$$C^* = W\left[\frac{\bar{Q} - \alpha\left(\frac{W\alpha}{2\beta r}\right)}{\beta}\right] + r \cdot \left(\frac{W\alpha}{2\beta r}\right)^2$$

Prove that $\lambda^* = \dfrac{\partial C^*}{\partial \bar{Q}}$

$$\boxed{\begin{aligned} &[K] \Rightarrow \mathcal{L}_K = r - \lambda \tfrac{1}{2} \alpha K^{-\frac{1}{2}} = 0 && \text{—— ①} \\ &[L] \Rightarrow \mathcal{L}_L = W - \lambda\beta = 0 && \text{—— ②} \rightsquigarrow \lambda^* = \frac{W}{\beta} \\ &[\lambda] \Rightarrow \mathcal{L}_\lambda = \bar{Q} - \alpha\sqrt{K} - \beta L && \text{—— ③} \end{aligned}}$$

Marginal cost: $\dfrac{\partial C^*}{\partial \bar{Q}} = \dfrac{W}{\beta}$ and $\lambda^* = \dfrac{W}{\beta}$ #

2. *Theory of consumer*

Consider a household with the utility function given by,

$$U(x,y) = [x^2 + y^2]^{\frac{1}{2}}$$

where x *and* y are two different consumption goods, i.e. good x and good y. Suppose that (i) *the prices for each of the two consumption goods are p_x and p_y respectively*, and (ii) *household's income is equal to M.* Consider the following problems

a) Calculate the total differential of the utility function.

$$dU(x,y) = \frac{\partial U(x,y)}{\partial x} dx + \frac{\partial U(x,y)}{\partial x} dy$$

$$= \frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}} \cdot \cancel{2}x \cdot dx + \frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}} \cdot \cancel{2}y \cdot dy$$

$$= (x^2+y^2)^{-\frac{1}{2}} dx + (x^2+y^2)^{-\frac{1}{2}} dy \quad \#$$

b) Set up the constrained optimization problem and derive the Marshallian demand function. $\rightarrow$ Find x^*, y^*

Marshallian demand function: $\max \; U(x,y) = (x^2+y^2)^{\frac{1}{2}}$

$\quad$ s.t. $Px \cdot x + Py \cdot y = M$

$$\mathcal{L} = (x^2+y^2)^{\frac{1}{2}} + \lambda (M - Px \cdot x - Py \cdot y)$$

F.O.C.s

$$\frac{\partial \mathcal{L}}{\partial x} = 0 \; ; \quad \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x + \lambda(-Px) = 0 \quad —① $$

$$\frac{\partial \mathcal{L}}{\partial y} = 0 \; ; \quad \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2y + \lambda(-Py) = 0 \quad —② $$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \; ; \quad M - Px \cdot x - Py \cdot y = 0 \quad —③ $$

$$① \rightarrow \quad (x^2+y^2)^{-\frac{1}{2}} \cdot x = \lambda Px \quad —④$$

$$② \rightarrow \quad (x^2+y^2)^{-\frac{1}{2}} \cdot y = \lambda Py \quad —⑤$$

$$\frac{④}{⑤} \Rightarrow \frac{\cancel{(x^2+y^2)^{-\frac{1}{2}}} \cdot x}{\cancel{(x^2+y^2)^{-\frac{1}{2}}} \cdot y} = \frac{\cancel{\lambda} Px}{\cancel{\lambda} Py} \Rightarrow \frac{x}{y} = \frac{Px}{Py}$$

$$y = x \cdot \frac{Py}{Px} \quad —⑥$$

sub ⑥ into ③ ; $M - Px \cdot x - Py \cdot x \cdot \frac{Py}{Px} = 0$

$$M = P_x \cdot x + P_y^2 \cdot \frac{x}{P_x}$$

$$M = x\left(P_x + \frac{P_y^2}{P_x}\right)$$

$$\frac{M}{\left(P_x + \frac{P_y^2}{P_x}\right)} = x$$

$$x^* = \frac{M \cdot P_x}{P_x^2 + P_y^2}$$

From ⑥: $x = \frac{P_x}{P_y} \cdot y$ sub into ③ ; $M - P_x \cdot \left(\frac{P_x}{P_y} \cdot y\right) - P_y \cdot y = 0$

$$M - \frac{P_x^2}{P_y} \cdot y - P_y \cdot y = 0$$

$$M = \frac{P_x^2}{P_y} \cdot y + P_y \cdot y$$

$$M = y\left(\frac{P_x^2}{P_y} + P_y\right)$$

$$\frac{M}{\left(\frac{P_x^2}{P_y} + P_y\right)} = y$$

$$\frac{M \cdot P_y}{P_x^2 + P_y^2} = y^*$$

Marshallian demand function $(x^*, y^*) = \left(\frac{M \cdot P_x}{P_x^2 + P_y^2}, \frac{M \cdot P_y}{P_x^2 + P_y^2}\right)$ #

c) Does the demand function satisfy *the law of demand*? Mathematically, how do you know that?

$$(x^*, y^*) = \left(\frac{M \cdot P_x}{P_x^2 + P_y^2}, \frac{M \cdot P_y}{P_x^2 + P_y^2}\right)$$

$$\frac{dx^*}{dP_x} = \frac{(P_x^2 + P_y^2)(M) - (M \cdot P_x)(2P_x)}{(P_x^2 + P_y^2)^2}$$

$$= \frac{MP_x^2 + MP_y^2 - 2MP_x^2}{(P_x^2 + P_y^2)^2}$$

$$= \frac{MP_y^2 - MP_x^2}{(P_x^2 + P_y^2)^2} = -\frac{M(P_x^2 - P_y^2)}{(P_x^2 + P_y^2)^2}$$

Law of demand:

#

1.) x^* and y^* are positive

2.) If goods are normal, demand for goods will decrease as price increases.

3.) slope $= -\frac{\Delta y}{\Delta x} = \frac{\cancel{M} \cdot P_y}{\cancel{P_x^2 + P_y^2}} \times \frac{\cancel{P_x^2 + P_y^2}}{\cancel{M} \cdot P_x} = -\frac{P_y}{P_x}$

4.) Price of x should be more than price of y for equation to satisfy.

d) How does the demand for good y respond to price of good x?

✓ y^* ✓ P_x

$$y^* = \frac{M \cdot P_y}{P_x^2 + P_y^2} \qquad \frac{dy^*}{dP_x} = \frac{(P_x^2 + P_y^2)(0) - (M \cdot P_y)(2P_x)}{(P_x^2 + P_y^2)^2}$$

$$= \frac{-2MP_xP_y}{(P_x^2 + P_y^2)^2}$$

When price of good x increases by 1 \$/unit, demand for good y decreases by $\frac{2MP_xP_y}{(P_x^2+P_y^2)^2}$. #

Thus, good x and good y are complement products. #

e) What is the numerical value of λ^* when $M = \$300,\ p_x = 1,\ p_y = 1$?

$$\frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}} \cdot \cancel{2}x + \lambda(-P_x) = 0$$

$$x^* = \frac{M \cdot P_x}{P_x^2 + P_y^2} = \frac{300 \cdot 1}{1^2 + 1^2} = 150$$

$$y^* = \frac{M \cdot P_y}{P_x^2 + P_y^2} = \frac{300 \cdot 1}{1^2 + 1^2} = 150$$

$$\frac{x}{\sqrt{x^2+y^2}} = \lambda(1)$$

$$\frac{150}{\sqrt{150^2+150^2}} = \lambda$$

$$\lambda^* = 0.7071$$ #

f) Without redoing the optimization problem, what would be the new optimized level of maximum utility when income increases to \$310.

$$\lambda^* = \frac{\Delta U}{\Delta M} \quad ; \quad \Delta M = 310 - 300 = 10$$

$$\lambda^* \cdot \Delta M = \Delta U$$

$$0.7071\,(10) = \Delta U$$

$$7.071 = \Delta U$$

$$\text{Old } U = (x^2+y^2)^{\frac{1}{2}} = (150^2+150^2)^{\frac{1}{2}} = 212.1320$$

$$\therefore \text{New } U = \text{Old } U + \Delta U = 212.1320 + 7.071 = 219.2030$$ #

3. Suppose that a monopolist has its marginal cost function given by $MC = 16 + 6q^2$ where q is the amount of output produced. The monopolist faces the market demand function given by $P = 160 - 10q^2$ where P is the price per unit of output. Consider the following problem.

a) Suppose that fixed cost is equal to $240. Calculate the total and the average cost when q = 9 units.

$$TC = \int MC\, dq = \int (16 + 6q^3)\, dq$$

$$= 16q + \frac{6q^3}{3} + C$$

fixed cost $= 240$

$$= 16q + \frac{6q^3}{3} + 240$$

$$TC(q=9) = 16(9) + \frac{6 \cdot 9^3}{3} + 240$$

$$= 1{,}842 \ \#$$

$$AC(q=9) = \frac{TC}{q}$$

$$= \frac{1{,}842}{9} = 204.6667 \ \#$$

b) Determine the profit-maximizing level of output for the monopolist. Also, confirm your result by using the second derivative test.

$$\pi = TR - TC$$

$$= P(q) \cdot q - C(q)$$

$$= (160 - 10q^2)q - \left(16q + \frac{6q^3}{3} + 240\right) = 160q - 10q^3 - 16q - 2q^3 - 240$$

$\max_q \pi \Rightarrow$ F.O.C.

$$\frac{d\pi}{dq} = 160 - 30q^2 - 16 - 6q^2 = 0$$

$$144 = 36q^2$$

$$4 = q^2$$

$$q^* = 2 \text{ units}$$

$$p^* = 160 - 10(2^2) = 120 \ \$/\text{unit}$$

S.O.C.

$$\frac{d^2\pi}{dq^2} = -60q - 12q$$

$$= -72q < 0 \quad \forall q$$

π is globally concave ;

Thus $q^* = 2$ is the profit maximization solution. #

c) Calculate the social welfare under the monopoly environment.

Equilibrium for monopoly : $MR = MC$

$$MR = \frac{dTR}{dq} = \frac{d(160q - 10q^3)}{dq} = 160 - 30q^2$$

$$MC = 16 + 6q^2$$

To find q^m

- Setting $MR - MC = 0$

$$160 - 30q^2 - (16 + 6q^2) = 0$$

$$160 - 30q^2 = 16 + 6q^2$$

$$144 = 36q^2$$

$$q^m = 2$$

$$\text{Thus, } p^m = 160 - 10(2^2) = 120$$

$$C.S. = \int_0^{q^m} (\overbrace{160 - 10q^2}^{\text{Demand}})\,dq - p^M \cdot q^m$$

$$= \int_0^2 (160 - 10q^2)\,dq - (120)(2)$$

$$= 160q - \frac{10q^3}{3} + C \Big]_0^2 - 240$$

$$= \left[\left(160(2) - \frac{10(2^3)}{3} \cancel{+C}\right) - \left(160(0) - \frac{10(0^3)}{3} \cancel{+C}\right)\right] - 240$$

$$= \frac{880}{3} - 240 = \underline{53.33}$$

$$\underline{P.S.} = p^M \cdot q^m - \int_0^{q^m} (\overbrace{16 + 6q^2}^{MC})\,dq$$

$$= (120)(2) - 16q + \frac{6q^3}{3} + C \Big]_0^2$$

$$= 240 - \left[\left(16(2) + \frac{6(2^3)}{3} \cancel{+C}\right) - \left(16(0) + \frac{6(0^3)}{3} \cancel{+C}\right)\right]$$

$$= 240 - 48$$

$$= \underline{192}$$

Total welfare under monopoly $= C.S.^M + P.S.^M$

$= 53.33 + 192$

$= 245.33$ #

d) Calculate the social welfare loss under the monopoly environment.

Welfare loss or D.W.L. = Total welfare^d − Total welfare^m

Equilibrium under Perfect competition: $P = MC$

$$160 - 10q^2 = 16 + 6q^2$$
$$144 = 16q^2$$
$$9 = q^2$$
$$q^d = 3$$
Thus, $P^d = 160 - 10(3^2) = 70$

$$C.S.^d = \int_0^{q^c} (160 - 10q^2)\,dq - P^d \cdot q^d$$
$$= 160q - \frac{10q^3}{3} + C \Big]_0^3 - (70)(3)$$
$$= \left[\left(160(3) - \frac{10(3^3)}{3} + \cancel{C}\right) + \left(160(0) - \frac{10(0^3)}{3} + \cancel{C}\right)\right] - 210$$
$$= 390 - 210$$
$$= 180$$

$$P.S.^d = P^d \cdot q^d - \int_0^{q^c} (16 + 6q^2)\,dq$$
$$= (70)(3) - 16q + \frac{6q^3}{3} + C \Big]_0^3$$
$$= 210 - \left[\left(16(3) + \frac{6(3^3)}{3} + \cancel{C}\right) + \left(16(0) + \frac{6(0^3)}{3} + \cancel{C}\right)\right]$$
$$= 210 - 102$$
$$= 108$$

Total welfare under perfect competition = $C.S.^d + P.S^d = 180 + 108 = 288$

$$D.W.L. = \text{Total welfare}^d - \text{Total welfare}^m$$
$$= 288 - 245.33$$
$$= 42.67 \; \#$$

4. Suppose the demand and supply curves are $P = \frac{6000}{Q+50}$ and $P = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.

Equilibrium: $D = S$

$$\frac{6{,}000}{Q+50} = Q+10$$
$$6{,}000 = (Q+10)(Q+50)$$
$$6{,}000 = Q^2+50Q+10Q+500$$
$$0 = Q^2+60Q-5500$$
$$0 = (Q-50)(Q+110)$$
$$Q^* \Rightarrow 50, \cancel{-110} \longrightarrow P^* = \frac{6{,}000}{Q+50} = \frac{6{,}000}{50+50} = 60$$

$\therefore$ Equilibrium $(P^*, Q^*) = (60, 50)$ #

$$C.S. = \int_0^{50} \left(\frac{6{,}000}{Q+50}\right) dQ - P^* \cdot Q^*$$; let $Q+50 = u$, $du = 1\, dQ$

$$\Rightarrow \left[6{,}000 \int_0^{50} \frac{1}{u}\, du\right] - (60)(50)$$; since $\int x^{-1} dx = \int \frac{1}{x} dx = \int \frac{dx}{x} = \ln x + C$

$$= 6{,}000\left(\int_0^{50} \frac{du}{u}\right) - 3{,}000$$
$$= 6{,}000 \left[\ln(Q+50) + C \Big|_0^{50}\right] - 3{,}000$$
$$= 6{,}000 \left\{\ln(50+50) \cancel{+C} - [\ln(0+50) \cancel{+C}]\right\} - 3{,}000$$
$$= 6{,}000 \, [\ln(100) - \ln(50)] - 3{,}000$$

$\log(a) - \log(b) = \log\left(\frac{a}{b}\right)$

$$= 6{,}000 \ln\left(\frac{100}{50}\right) - 3{,}000$$
$$= 6{,}000 \ln(2) - 3{,}000 = 1{,}158.88$$ #

$$P.S. = P^* \cdot Q^* - \int_0^{50} (Q+10)\, dQ$$
$$= (60)(50) - \frac{Q^2}{2} + 10Q + C \Big|_0^{50}$$
$$= 3{,}000 - \left[\left(\frac{50^2}{2} + (10\times 50) \cancel{+C}\right) - \cancel{\left(\frac{0^2}{2} + (10\times 0) + C\right)}\right]$$
$$= 1{,}250$$ #

$\therefore$ consumer surplus $= 1{,}158.88$ #

producer surplus $= 1{,}250$ #

5. Let $MR = 25 - 5x - 2x^2$ and $MC = 10 - 3x - x^2$, where x is the unit of output. Assume that fixed cost is $7. Determine the level of production that contributes to maximum profit and determine the level of maximized profit.

$$\max_x \pi = TR - TC$$

$$MR = \frac{dTR}{dx} = 25 - 5x - 2x^2$$

$$\int MR = TR = \int (25 - 5x - 2x^2)dx$$

$$= 25x - \frac{5x^2}{2} - \frac{2x^3}{3} + c$$

$$MC = \frac{dTC}{dq} = 10 - 3x - x^2$$

$$\int MC = TC = \int (10 - 3x - x^2)dx$$

$$= 10x - \frac{3x^2}{2} - \frac{x^3}{3} + C \xrightarrow{} \text{fixed cost} = 7$$

$$TC = 10x - \frac{3x^2}{2} - \frac{x^3}{3} + 7$$

$$\pi = TR - TC$$

$$= 25x - \frac{5x^2}{2} - \frac{2x^3}{3} + c - \left(10x - \frac{3x^2}{2} - \frac{x^3}{3} + 7\right)$$

$\max_x \pi$: F.O.C. : $\pi'(x^*) = 0$

$$25 - 5x - 2x^2 - 10 + 3x + x^2 = 0$$

$$15 - 2x - x^2 = 0$$

$$0 = x^2 + 2x - 15$$

$$0 = (x-3)(x+5)$$

$$x^* \Rightarrow 3, \cancel{-5}$$

level of maximized profit : $\pi(3) = 25x - \frac{5x^2}{2} - \frac{2x^3}{3} + c - \left(10x - \frac{3x^2}{2} - \frac{x^3}{3} + 7\right)$

$$= 25(3) - \frac{5 \cdot 3^2}{2} - \frac{2 \cdot 3^3}{3} + C - 10(3) + \frac{3 \cdot 3^2}{2} + \frac{3^3}{3} - 7$$

$$= 20 + C \; \#$$