



EE 320 Introductory Mathematical Economics (Section 046402)

Semester 1/2013

Homework 5

Due 14 November 2013

There are four questions in total. Each of them is worth 5 points.

1. Suppose that Mr. Pai's utility depends on two commodities: x_1 and x_2 . His utility function is given by

$$U = 26x_1 + 55x_2 - x_1^2 - x_1x_2 - 2x_2^2$$

- (a) (3 points) Determine the values x_1^* and x_2^* that maximize Pai's utility, and determine the corresponding utility level.
- (b) (2 points) Show the second-order sufficient condition for utility maximization.

2. Suppose that a monopolistic firm sells a single product in three separate markets (say, three different countries), where the firm can set different prices. The inverse demand functions in the three markets are:

$$P_1 = a_1 - b_1Q_1$$

$$P_2 = a_2 - b_2Q_2$$

$$P_3 = a_3 - b_3Q_3$$

where a_i and b_i , $i = 1, 2, 3$, are positive constants. The total cost function is given by: $TC = C_0 + cQ$, where $Q = Q_1 + Q_2 + Q_3$. C_0 is the total fixed cost, and c is the variable cost per unit.

- (a) (3 points) Find the profit-maximizing output levels of Q_1 , Q_2 , and Q_3 , and determine the maximum profit.
- (b) (2 points) Write down the Hessian matrix, and verify that the second-order sufficient condition is met.

3. Suppose that there are two firms in the industry, and they are competing in quantities. The amount of the commodity sold by firm i is q_i , $i = 1, 2$. The market demand function is given by $P = 50 - 3q$, where $q = q_1 + q_2$. The cost functions for each firm is given by $TC_i = 25 + 5q_i$, $i = 1, 2$.

- (a) (2 points) Find the profit-maximizing quantity for each firm, and determine each firm's profit level.
- (b) (3 points) Suppose that both firms merge. Compute the new profit-maximizing quantity and the new profit of the merged firm. Do firms have incentive to merge, and why?

4. Given the production function

$$Q = f(K, L) = 8K^{1/2}L^{1/4}$$

Suppose that the price per unit of Q is $\$P$, and the per unit input prices for K and L are $\$r$ and $\$w$, respectively. (P , w , and r are positive constants.)

- (a) (3 points) Solve for the values K^* and L^* that maximizes the profit. Verify that the second-order sufficient condition is met.
- (b) (2 points) Find the comparative statics derivatives $\frac{\partial K^*}{\partial r}$ and $\frac{\partial L^*}{\partial P}$, evaluate the signs, and interpret their economic meanings.