1.

```
. dfuller y, trend lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)              1.000            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 1.0000

------------------------------------------------------------------------------
D.y          |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |   .0001178   .0001179     1.00   0.318    -.0001137    .0003494
         LD. |   .6997015   .0248993    28.10   0.000     .6507799    .7486231
      _trend |   2.897751   1.159296     2.50   0.013      .619992    5.175511
       _cons |   1811.233   147.0426    12.32   0.000     1522.327    2100.139
------------------------------------------------------------------------------

. dfuller y, lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             10.999            -3.440            -2.870            -2.570
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 1.0000

------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |   .0003983   .0000362    11.00   0.000     .0003272    .0004695
         LD. |   .7218985   .0233849    30.87   0.000     .6759527    .7678444
             |
       _cons |    1773.23   147.0277    12.06   0.000     1484.355    2062.105
------------------------------------------------------------------------------

. dfuller d.y, trend lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       497

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -10.554            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
D2.y         |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         D.y |
         L1. |  -.2787856   .0264156   -10.55   0.000    -.3306866   -.2268845
         LD. |    -.32127   .0373756    -8.60   0.000    -.3947051   -.2478349
      _trend |   3.631984   .3708318     9.79   0.000     2.903379     4.36059
       _cons |   1678.082   154.4788    10.86   0.000     1374.564      1981.6
------------------------------------------------------------------------------
```

```
. dfuller x, trend lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)              0.601            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.9970

------------------------------------------------------------------------------
D.x          |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |
         L1. |   .0001061   .0001764     0.60   0.548    -.0002405    .0004526
         LD. |     .46018   .0349881    13.15   0.000     .3914361    .5289239
      _trend |   4.166909    1.14105     3.65   0.000     1.924999    6.408818
       _cons |   2128.626   140.0551    15.20   0.000     1853.449    2403.803
------------------------------------------------------------------------------

. dfuller x, lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             13.875            -3.440            -2.870            -2.570
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 1.0000

------------------------------------------------------------------------------
         D.x |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |
         L1. |   .0007222   .0000521    13.88   0.000       .00062    .0008245
         LD. |   .5003968    .033621    14.88   0.000     .4343393    .5664543
             |
       _cons |   2103.573   141.6193    14.85   0.000     1825.324    2381.822
------------------------------------------------------------------------------

. dfuller d.x, trend lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       497

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -10.657            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
D2.x         |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         D.x |
         L1. |  -.4007114   .0376021   -10.66   0.000    -.4745915   -.3268312
         LD. |   -.414172   .0358603   -11.55   0.000    -.4846298   -.3437141
      _trend |   3.508317   .3506567    10.00   0.000     2.819351    4.197283
       _cons |   1596.429   147.0971    10.85   0.000     1307.415    1885.444
------------------------------------------------------------------------------
```

We test unit root test to see whether the dependent variable x and y are stationary or not. Hence, we perform the hypothesis test, H0 : $\alpha 1 = 0$,from the test, p-value is 1.0000 which is > 0.05, therefore, H0 is rejected and there is unit root and the model is nonstationary. Such that we perform the test without trend, H0 : $\gamma = 0$, from the test, p-value > 0.05 and H0 is rejected, and there exists unit root and the model is nonstationary. Then, we perform the test with integrated level 1, from the test, MacKinnon approximate p-value for Z(t) is 0.0000 > 0.05 such that there is no unit root and the model is stationary at X~I(1) and Y~I(1), hence, there is a long run relationship between these two variables.

2.

(i)

```
. vecrank y x,trend(t) lags(1/1) max

                    Johansen tests for cointegration
Trend: trend                                       Number of obs =     499
Sample:  2 - 500                                            Lags =       1
-------------------------------------------------------------------------
                                                        5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0       4    -7387.4577         .       790.3357    18.17
    1       7    -6992.3441      0.79477      0.1085*    3.74
    2       8    -6992.2899      0.00022
-------------------------------------------------------------------------
                                                        5%
maximum                                       max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0       4    -7387.4577         .       790.2272    16.87
    1       7    -6992.3441      0.79477      0.1085     3.74
    2       8    -6992.2899      0.00022
-------------------------------------------------------------------------
```

(ii)

```
. vecrank y x,trend(rt) lags(1/1) max

                    Johansen tests for cointegration
Trend: rtrend                                      Number of obs =     499
Sample:  2 - 500                                            Lags =       1
-------------------------------------------------------------------------
                                                        5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0       2    -8050.4781         .      2116.3764    25.32
    1       6    -7075.2453      0.97993    165.9107    12.25
    2       8    -6992.2899      0.28286
-------------------------------------------------------------------------
                                                        5%
maximum                                       max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0       2    -8050.4781         .      1950.4657    18.96
    1       6    -7075.2453      0.97993    165.9107    12.52
    2       8    -6992.2899      0.28286
-------------------------------------------------------------------------
```

(iii)

```
. vecrank y x,trend(c) lags(1/1) max

                    Johansen tests for cointegration
Trend: constant                                    Number of obs =     499
Sample:  2 - 500                                            Lags =       1
-------------------------------------------------------------------------
                                                        5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0       2    -8050.4781         .      2086.8946    15.41
    1       5    -7083.8611      0.97923    153.6607     3.76
    2       6    -7007.0308      0.26504
-------------------------------------------------------------------------
                                                        5%
maximum                                       max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0       2    -8050.4781         .      1933.2339    14.07
    1       5    -7083.8611      0.97923    153.6607     3.76
    2       6    -7007.0308      0.26504
-------------------------------------------------------------------------
```

(iv)

```
. vecrank y x,trend(rc) lags(1/1) max

                       Johansen tests for cointegration
Trend: rconstant                                        Number of obs =     499
Sample:  2 - 500                                                 Lags =       1
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0      0      -8811.3759           .   3593.1922    19.96
    1      4      -7096.9792     0.99896    164.3988     9.42
    2      6      -7014.7798     0.28069
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0      0      -8811.3759           .   3428.7934    15.67
    1      4      -7096.9792     0.99896    164.3988     9.24
    2      6      -7014.7798     0.28069
-------------------------------------------------------------------------------
```

(v)

```
. vecrank y x,trend(n) lags(1/1) max

                       Johansen tests for cointegration
Trend: none                                             Number of obs =     499
Sample:  2 - 500                                                 Lags =       1
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0      0      -8811.3759           .   3204.1194    12.53
    1      3      -7211.7592     0.99836      4.8860     3.84
    2      4      -7209.3162     0.00974
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0      0      -8811.3759           .   3199.2334    11.44
    1      3      -7211.7592     0.99836      4.8860     3.84
    2      4      -7209.3162     0.00974
-------------------------------------------------------------------------------
```

3.

(i)

```
. vecrank y x,trend(t) lags(1/1) max

                       Johansen tests for cointegration
Trend: trend                                           Number of obs =     499
Sample:  2 - 500                                                Lags =       1
-------------------------------------------------------------------------------
                                                          5%
maximum                                        trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0       4     -7387.4577        .       790.3357    18.17
    1       7     -6992.3441     0.79477      0.1085*    3.74
    2       8     -6992.2899     0.00022
-------------------------------------------------------------------------------
                                                          5%
maximum                                         max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0       4     -7387.4577        .       790.2272    16.87
    1       7     -6992.3441     0.79477      0.1085     3.74
    2       8     -6992.2899     0.00022
-------------------------------------------------------------------------------
```

(ii)

```
. vecrank y x,trend(t) lags(2/2) max

                       Johansen tests for cointegration
Trend: trend                                           Number of obs =     498
Sample:  3 - 500                                                Lags =       2
-------------------------------------------------------------------------------
                                                          5%
maximum                                        trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0       8     -6795.5337        .       187.5771    18.17
    1      11     -6703.3826     0.30932      3.2749*    3.74
    2      12     -6701.7451     0.00655
-------------------------------------------------------------------------------
                                                          5%
maximum                                         max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0       8     -6795.5337        .       184.3022    16.87
    1      11     -6703.3826     0.30932      3.2749     3.74
    2      12     -6701.7451     0.00655
-------------------------------------------------------------------------------
```

(iii)

```
. vecrank y x,trend(t) lags(3/3) max

                       Johansen tests for cointegration
Trend: trend                                           Number of obs =     497
Sample:  4 - 500                                                Lags =       3
-------------------------------------------------------------------------------
                                                          5%
maximum                                        trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0      12      -6756.658        .       140.0434    18.17
    1      15      -6688.113     0.24106      2.9534*    3.74
    2      16     -6686.6363     0.00592
-------------------------------------------------------------------------------
                                                          5%
maximum                                         max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0      12      -6756.658        .       137.0900    16.87
    1      15      -6688.113     0.24106      2.9534     3.74
    2      16     -6686.6363     0.00592
-------------------------------------------------------------------------------
```

4.

From the test, we set the null hypothesis of H0 : r = 1 and H1 : r > 2. One lag term with linear trend is statistically insignificant in 5% critical value (0.1085 < 3.74), therefore, we failed to reject H0, hence one lag term with linear trend is cointegrated at rank = 1 or there is 1 cointegrating equation.

Two lag terms with linear trend is statistically insignificant in 5% critical value (3.2749 < 3.74), therefore, we failed to reject H0, hence one lag term with linear trend is cointegrated at rank = 1 or there is 1 cointegrating equation.

Three lag term with linear trend is statistically insignificant in 5% critical value (2.9534 < 3.84), therefore, we failed to reject H0, hence one lag term with linear trend is cointegrated at rank = 1 or there is 1 cointegrating equation.

5.

(i)

```
. vec y x, lags(1/1)

Vector error-correction model

Sample:  2 - 500                               Number of obs     =        499
                                               AIC               =   28.41227
Log likelihood = -7083.861                     HQIC              =   28.42883
Det(Sigma_ml)  =  7.34e+09                     SBIC              =   28.45448

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_y                  2     331.479   0.9988   399343.6   0.0000
D_x                  2     430.726   0.9953   105050.2   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |  -1.303404   .0095397  -136.63   0.000    -1.322102   -1.284707
             |
       _cons |   -107.843   69.40373    -1.55   0.120    -243.8718    28.18583
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |  -.8364345   .0123959   -67.48   0.000    -.8607301    -.812139
             |
       _cons |   168.0502    90.1839     1.86   0.062    -8.706965    344.8074
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                 1   1.71e+10   0.0000
-------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500508   .0000115 -1.3e+05   0.000    -1.500531   -1.500486
       _cons |  -1678.204          .        .       .            .           .
------------------------------------------------------------------------------
```

(ii)

```
. vec y x, lags(2/2)

Vector error-correction model

Sample:  3 - 500                                Number of obs     =        498
                                                AIC               =   27.18032
Log likelihood = -6758.899                      HQIC              =   27.21018
Det(Sigma_ml)  =  2.11e+09                      SBIC              =   27.25641

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_y                  4     213.415   0.9995   964110.3   0.0000
D_x                  4     263.876   0.9982   280728.2   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |   -.427199   .0483639    -8.83   0.000    -.5219905   -.3324075
             |
           y |
         LD. |   .4022605   .0224005    17.96   0.000     .3583563    .4461646
             |
           x |
         LD. |   .4814843   .0598769     8.04   0.000     .3641276    .5988409
             |
       _cons |   171.4547   44.03061     3.89   0.000     85.15624    257.7531
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |   .2903827   .0597995     4.86   0.000     .1731779    .4075875
             |
           y |
         LD. |   .5734396   .0276971    20.70   0.000     .5191544    .6277249
             |
           x |
         LD. |   .3676105   .0740347     4.97   0.000     .2225051    .5127159
             |
       _cons |    252.237   54.44157     4.63   0.000     145.5335    358.9405
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                 1   8.09e+08   0.0000
-------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500176   .0000527 -2.8e+04   0.000     -1.50028   -1.500073
       _cons |  -577.9622          .        .       .            .           .
------------------------------------------------------------------------------
```

(iii)

```
. vec y x, lags(3/3)

Vector error-correction model

Sample:  4 - 500                                Number of obs     =        497
                                                AIC               =    27.1245
Log likelihood = -6727.439                      HQIC              =   27.16771
Det(Sigma_ml)  =  1.96e+09                      SBIC              =   27.23459
```

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_y	6	211.673	0.9995	980003.4	0.0000
D_x	6	253.098	0.9984	305162.2	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D_y						
_ce1						
L1.	-.3193909	.058155	-5.49	0.000	-.4333726	-.2054092
y						
LD.	.3553209	.0559202	6.35	0.000	.2457194	.4649225
L2D.	.0174858	.0312998	0.56	0.576	-.0438607	.0788323
x						
LD.	.6042668	.0763396	7.92	0.000	.4546439	.7538896
L2D.	.0521968	.0634683	0.82	0.411	-.0721988	.1765925
_cons	181.5838	45.38519	4.00	0.000	92.63049	270.5372
D_x						
_ce1						
L1.	.4434064	.0695359	6.38	0.000	.3071185	.5796943
y						
LD.	.4713427	.0668638	7.05	0.000	.3402922	.6023932
L2D.	.012763	.0374252	0.34	0.733	-.060589	.0861151
x						
LD.	.5220109	.0912792	5.72	0.000	.3431069	.700915
L2D.	.0911602	.0758891	1.20	0.230	-.0575797	.2399001
_cons	130.797	54.26707	2.41	0.016	24.43548	237.1585

Cointegrating equations

Equation	Parms	chi2	P>chi2
_ce1	1	6.64e+08	0.0000

Identification: beta is exactly identified

Johansen normalization restriction imposed

beta	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_ce1						
y	1	.	.	.	.	.
x	-1.500049	.0000582	-2.6e+04	0.000	-1.500163	-1.499935
_cons	-92.13439	.	.	.	.	.

From the estimated VECM model, Cointegrating equation is statistically significant with a p-value of 0.0000 < 0.05., and the optimal lag term is lag term = 3 since it has the lowest SBIC such that there is less error among others with SBIC = 27.23459. Cointegrating equation is

$Yt = \beta_0 + \beta_1 Xt$ or $Yt = 92.13439 + 1.5Xt$ and has the speed of adjustment of Y = -.3193 and for $X = 0.4434$.