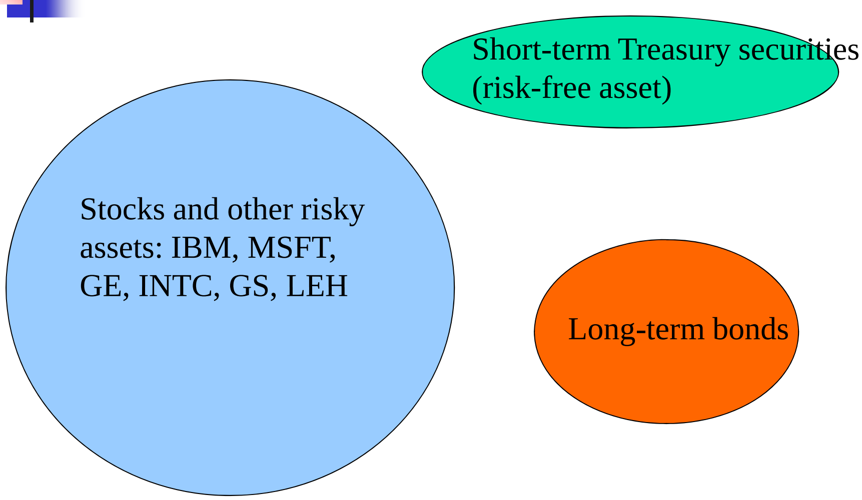
BACHELOR of ECONOMICS
International Program

Thammasat University | Thailand

Challenge: How to invest optimally?



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Outline

- The interest rate
- Return
- Variance risk
- Investors preference
- Risk-return trade off

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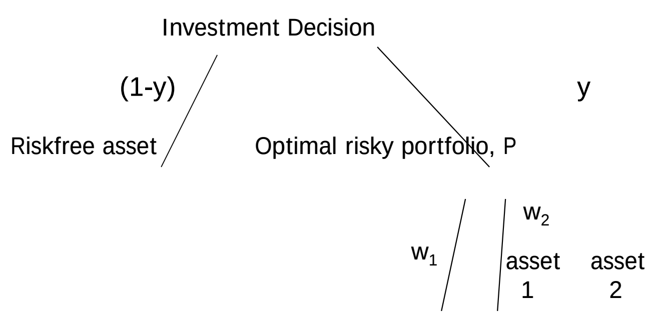
Investments

What is the optimal risky portfolio?



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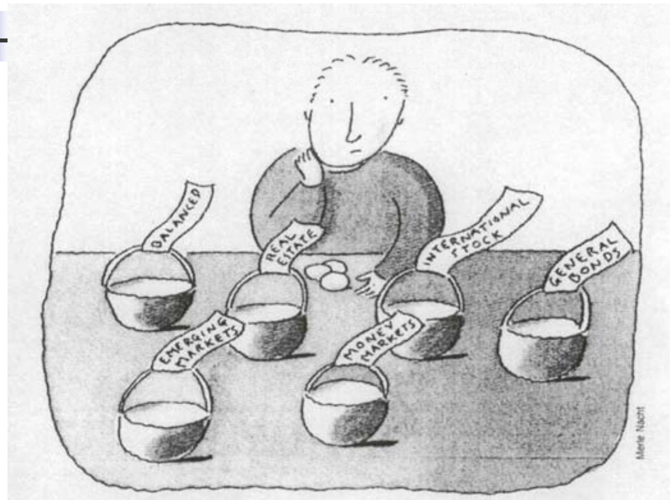
What to do?



Road map/Key ideas

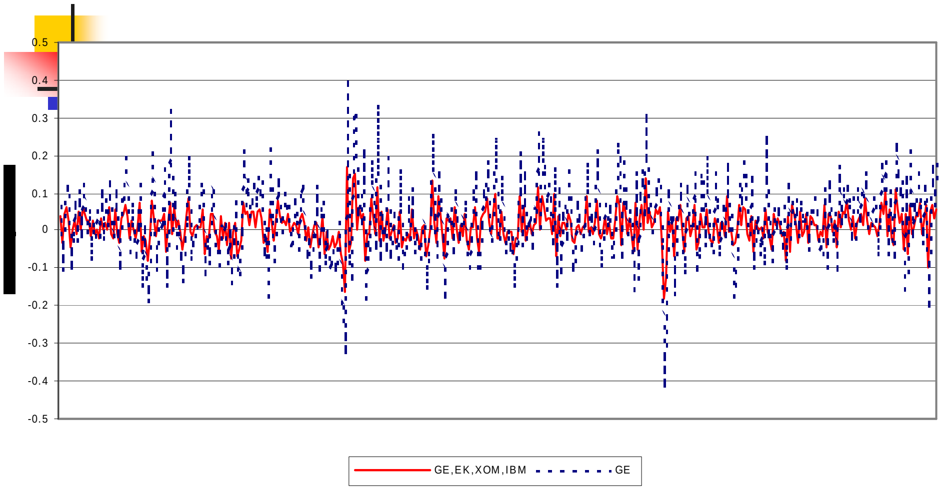
- Simple diversification
- The effect of correlation on diversification
- Optimal diversification
- Limits to diversification

Over the period 1962 – 1998 (Annualized):

Portfolio	Average Rate of Return	Standard Deviation
MKT (Equal-weighted)	12.7%	15.1%
IBM	13.4%	22.0%
GE	16.1%	22.0%
EK	11.3%	21.8%
XOM	15.3%	16.8%

- The STD of the market portfolio is lower than most individual stocks

Simple Diversification (1962-1998)



Simple diversification (1962-1998)

Simple diversification: Equal-weighted portfolio

	IBM	GE	EK	XOM	MKT	IBM,GE	IBM,GE,EK	All 4 stocks
MEAN (per year)	0.134	0.161	0.113	0.153	0.127	0.147	0.136	0.140
STD (per year)	0.227	0.220	0.218	0.168	0.151	0.191	0.175	0.153

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Simple diversification

How diversification works

- Prices of different stocks do not move exactly together
- Variation of a stock price = market-wide variation + firm specific variation.
- Firm-specific variations of different stocks cancel each other out
- STD of a well-diversified portfolio <= STD of an individual stock that make up the portfolio

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Simple diversification

A Well-diversified portfolio should

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Simple diversification

A Well-diversified portfolio should

- Include large number of assets (>40)
- Include assets in different industry/market segment
- Include assets in different countries
- Consider the net current wealth including the present value of future income

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The effect of correlation on diversification

You have $100,000 invested in the S&P, which will return

0.5 → 35%
0.5 → 5%

$E[\tilde{r}]=20\%$

$\sigma=15\%$

Change composition: 70% is invest in the S&P and 30% in either corporate bonds or gold. Which one to choose?

R) Bonds: 0.5 → 16%; 0.5 → 2%

G) Gold: 0.5 → -2%; 0.5 → 20%

$E[\tilde{r}]=9\%$ $\sigma=0.07$

$E[\tilde{r}]=9\%$ $\sigma=0.11$

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The effect of correlation on diversification

1. 70% S&P + 30% Bonds

0.5 → 35%x0.7+16%x0.3=29.3%
0.5 → 5%x0.7+2%x0.3=4.1%

$E[\tilde{r}]=16.7\%$

$\sigma=12.6\%$

2. 70% S&P + 30% Gold

0.5 → 35%x0.7+(-0.2%)x0.3=23.9%
0.5 → 5%x0.7+20%x0.3=9.5%

$E[\tilde{r}]=16.7\%$

$\sigma=7.2\%\%$

The correlation between two risky securities matters in obtaining a portfolio with lower risk

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The effect of correlation on diversification

Consider a portfolio P that invest w1 in stock 1 and w2 in stock 2. The correlation between the two stocks is ρ

- Expected return

$$E[\tilde{r}_P] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

- Variance

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2$$

- The bordered variance-covariance matrix

	w1	w2
w1	σ_1^2	$\rho\sigma_1\sigma_2$
w2	$\rho\sigma_1\sigma_2$	σ_2^2

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The effect of correlation on diversification

The bordered variance-covariance matrix for 3 assets

	w1	w2	w3
w1	σ_1^2	$\rho_{12}\sigma_1\sigma_2$	$\rho_{13}\sigma_1\sigma_3$
w2	$\rho_{12}\sigma_1\sigma_2$	σ_2^2	$\rho_{23}\sigma_2\sigma_3$
w3	$\rho_{13}\sigma_1\sigma_3$	$\rho_{23}\sigma_2\sigma_3$	σ_3^2

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The effect of correlation on diversification

Consider two risky assets: stock 1 and stock 2

- Stock 1: $E[\tilde{r}_1] = 8\% = \mu_1, \sigma[\tilde{r}_1] = \sigma_1 = 12\%$
- Stock 2: $E[\tilde{r}_2] = 13\% = \mu_2, \sigma[\tilde{r}_2] = \sigma_2 = 20\%$
- Riskfree rate is 0.5%

$$E[\tilde{r}_P] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

$$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho \sigma_1 \sigma_2$$

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The effect of correlation on diversification

When $\rho = 1$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_1 \sigma_2,$$

$$= (w_1 \sigma_1 + w_2 \sigma_2)^2$$

$$\sigma_p = w_1 \sigma_1 + w_2 \sigma_2$$

When the correlation equals 1 the STD of the portfolio equals the weighted average of the STD of the stocks that make up the portfolio. There is no diversification benefit.

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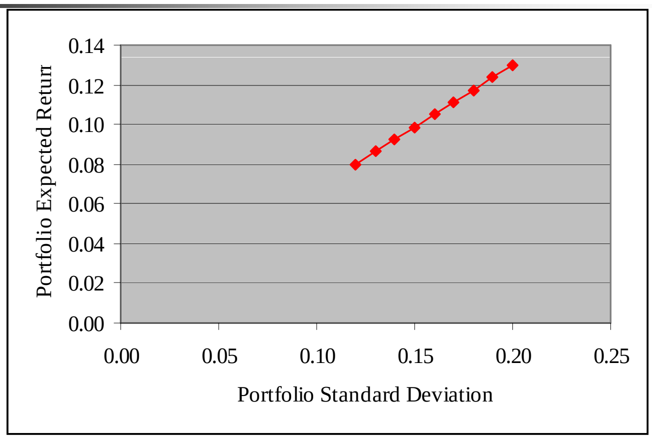
The effect of correlation on diversification

Case: $\rho = 1$ there is no diversification benefit



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The effect of correlation on diversification

- When $\rho < 1$ there is always potential for diversification benefit

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2} < w_1\sigma_1 + w_2\sigma_2$$

- Assets with $\rho < 0$ are often referred to as hedged assets.

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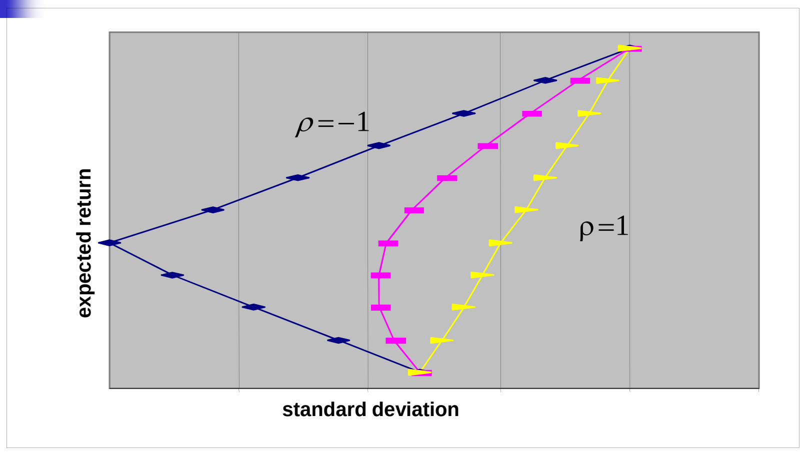
The effect of correlation on diversification



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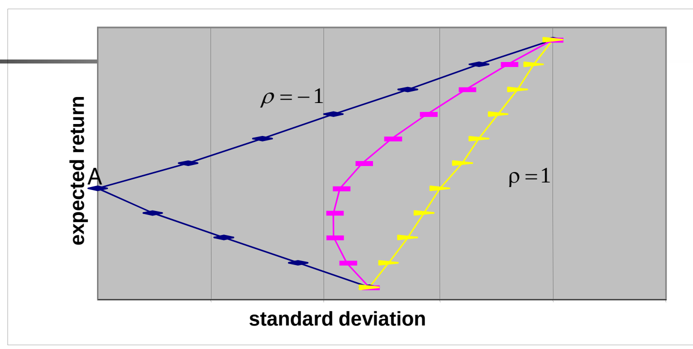
Concept check



What should the expected return at point A be?

G) Zero

Y) Riskfree rate

R) The return on the market portfolio

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The effect of correlation on diversification

Case: $\rho = -1$

$$E[\tilde{r}_p] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 - 2 w_1 w_2 \sigma_1 \sigma_2$$

$$= (w_1 \sigma_1 - w_2 \sigma_2)^2$$

$$\sigma_p = w_1 \sigma_1 - w_2 \sigma_2$$

To find the point where volatility is zero

$$0 = w_1 \sigma_1 - (1 - w_1) \sigma_2$$

$$w_1 = \frac{\sigma_2}{\sigma_1 + \sigma_2}$$

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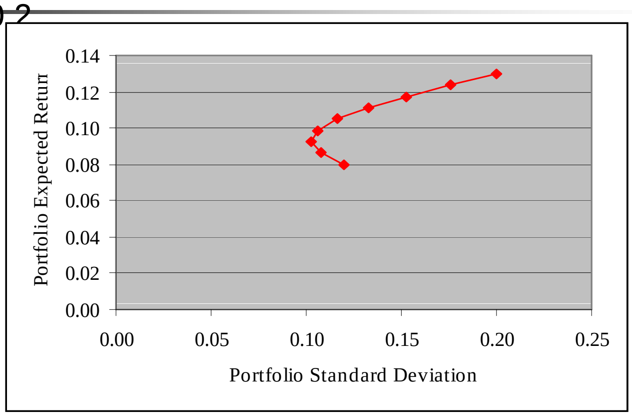
Optimal risky portfolio

Investment opportunity set of two risky assets when $\rho = 0.2$



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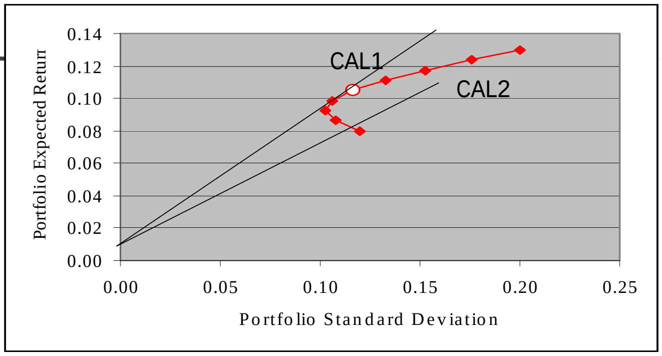
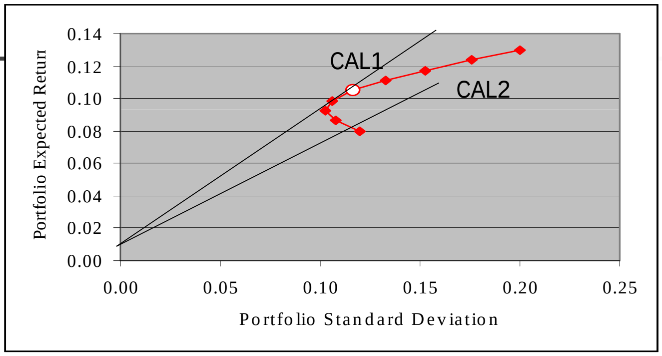
Optimal risky portfolio (visual)



- Adding the riskfree asset generates a number of CAL choices.
- Which CAL is better?

Optimal risky portfolio (visual)



- Adding the riskfree asset generates a number of CAL choices.
- Which CAL is better?
- Choose the CAL with the highest slope (Sharpe ratio)

Optimal risky portfolio (algebra)

Maximize $\quad (E[\tilde{r}_p] - r_f)/\sigma_p$

w_1, w_2

Subject to: $w_1 = (1 - w_2)$

$$E[\tilde{r}_P] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2]$$

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \rho\sigma_1\sigma_2$$

We obtain equation 8.7 in the book

$$w_1 = \frac{[E(r_1) - r_f]\sigma_2^2 - [E(r_2) - r_f]Cov(r_1, r_2)}{[E(r_1) - r_f]\sigma_2^2 + [E(r_2) - r_f]\sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f]Cov(r_1, r_2)}$$

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Optimal risky portfolio

Example: Investing $100,000, riskfree=0.5%

- Stock s 1 and 2: $E[\tilde{r}_1] = 8\%$, $\sigma_1 = 12\%$ $E[\tilde{r}_2] = 13\%$, $= \sigma_2 = 20\%$
- The correlation between the two stock returns is 0

2. Solve for P

$$w_1 = \frac{[E(r_1) - r_f]\sigma_2^2 - [E(r_2) - r_f]Cov(r_1, r_2)}{[E(r_1) - r_f]\sigma_2^2 + [E(r_2) - r_f]\sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f]Cov(r_1, r_2)}$$

$$w_1 = \frac{[0.08 - 0.005]0.2^2}{[0.08 - 0.005]0.2^2 + [0.13 - 0.005]0.12^2} = 0.625$$

$$w_2 = 0.375$$

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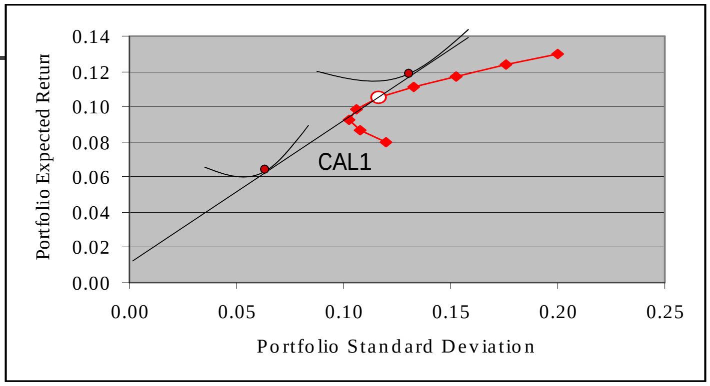
Optimal risky portfolio



- Allocate investment between riskfree asset and P using each investor's utility function

$$y^* = (E[\tilde{r}_p] - r_f)/(A \bullet \sigma_p^2)$$

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Optimal risky portfolio

Investing $100,000, A=8, riskfree=0.5%

1. Stock s 1 and 2: $E[\tilde{r}_1] = 8\%$, $\sigma_1 = 12\%$ $E[\tilde{r}_2] = 13\%$, $= \sigma_2 = 20\%$

- The correlation between the two stock returns is 0

2. We have $w_1 = 0.625$ and $w_2 = 0.375$

$$E[\tilde{r}_P] = 0.625x0.08 + 0.375x0.13 = 0.09875$$

$$\sigma_P = \sqrt{0.625^2 0.12^2 + 0.375^2 0.20^2 + 2x0.625x0.375x0x0.12x0.20} = 0.106$$

3. y*=(0.09875-0.005)/(8x0.106^2) = 1.04

Borrow 4,000; Invest 0.625*1.04*100,000= 65,000 in stock 1; and 0.375*1.04*100,000=39,000 in stock 2

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Concept check

Because diversification is so important, there is never a time when it is optimal to invest more than 90% of a portfolio in one specific asset. Is this true or false?

G) True
R) False

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Example: Investing $100,000, riskfree=0.5%

- Stock 1: $E[\tilde{r}_1] = 8\%\,,\ \sigma_1 = 12\%$
- Stock 2: $E[\tilde{r}_2] = 13\%\,,\ \sigma_2 = 3\%$
- The correlation between the two stock returns is 0

2. Solve for P

$$w_1 = \frac{[E(r_1) - r_f]\sigma_2^2 - [E(r_2) - r_f]Cov(r_1, r_2)}{[E(r_1) - r_f]\sigma_2^2 + [E(r_2) - r_f]\sigma_1^2 - [E(r_1) - r_f + E(r_2) - r_f]Cov(r_1, r_2)}$$

$$w_1 = \frac{[0.08 - 0.005]0.03^2}{[0.08 - 0.005]0.03^2 + [0.13 - 0.005]0.12^2} = 0.036$$

$$w_2 = 0.964$$

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Limits to Diversification

Date 09/21/2001

Diversification Helps... but It Has Its Limits

Performance of various mutual-fund categories since last week's disasters and over longer periods. All returns through Wednesday.

Fund type	From Sept.10	Year-to-date	Past 3 years, annualized
Gold-oriented	+3.5%	+16.8%	+1.8%
Interm. U.S.-govt. bond	+0.9	+6.9	+5.6
Money market	+0.03	+2.9	+4.9
Real-estate	−3.8	+1.6	+9.0
Balanced*	−4.2	−11.6	+3.1
International-stock	−5.9	−28.5	−1.1
International small-cap	−7	−28.4	+4.6
Diversified U.S.-stock	−7.9	−23.3	+4.1
Emerging markets	−9.4	−20.2	+3.4
Natural resources	−9.7	−20.1	+8.9

*Holds a mix of stocks and bonds.

Source: Lipper

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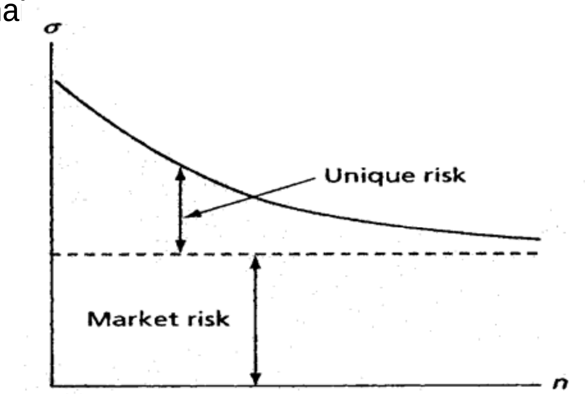
Limits to Diversification

Total variation of a stock = market risk + unique risk

- Unique risk is diversified away, but market risk rema



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Summary

- Simple diversification
- The effect of correlation on diversification
  - Portfolio of less that perfectly correlated assets always offer better risk-return opportunities than the individual component securities on their own
- Optimal diversification with two assets
  - Find the investment opportunity set. Pick the best CAL line.
- Limits to diversification
  - Firm specific risk is diversified away, but market risk remains