Solution to Homework2

CHAPTER 6

6. Points on the curve are derived by solving for $E(r)$ in the following equation:

$$U = 0.05 = E(r) - 0.5A\sigma^2 = E(r) - 1.5\sigma^2$$

The values of $E(r)$, given the values of σ^2, are therefore:

σ	σ^2	$E(r)$
0.00	0.0000	0.05000
0.05	0.0025	0.05375
0.10	0.0100	0.06500
0.15	0.0225	0.08375
0.20	0.0400	0.11000
0.25	0.0625	0.14375

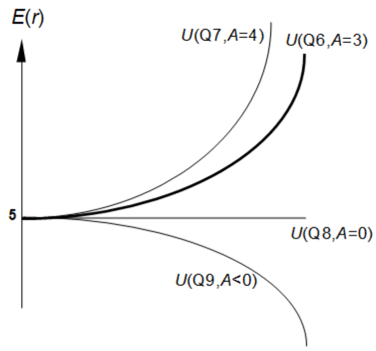
The bold line in the graph on the next page (labeled Q6, for Question 6) depicts the indifference curve.

7. Repeating the analysis in Problem 6, utility is now:

$$U = E(r) - 0.5A\sigma^2 = E(r) - 2.0\sigma^2 = 0.05$$

The equal-utility combinations of expected return and standard deviation are presented in the table below. The indifference curve is the upward sloping line in the graph on the next page, labeled Q7 (for Question 7).

σ	σ^2	$E(r)$
0.00	0.0000	0.0500
0.05	0.0025	0.0550
0.10	0.0100	0.0700
0.15	0.0225	0.0950
0.20	0.0400	0.1300
0.25	0.0625	0.1750



The indifference curve in Problem 7 differs from that in Problem 6 in slope. When A increases from 3 to 4, the increased risk aversion results in a greater slope for the indifference curve since more expected return is needed in order to compensate for additional σ.

8. The coefficient of risk aversion for a risk neutral investor is zero. Therefore, the corresponding utility is equal to the portfolio's expected return. The corresponding indifference curve in the expected return-standard deviation plane is a horizontal line, labeled Q8 in the graph above (see Problem 6).

9. A risk lover, rather than penalizing portfolio utility to account for risk, derives greater utility as variance increases. This amounts to a negative coefficient of risk aversion. The corresponding indifference curve is downward sloping in the graph above (see Problem 6), and is labeled Q9.

13. Expected return = (0.7 × 18%) + (0.3 × 8%) = 15%

Standard deviation = 0.7 × 28% = 19.6%

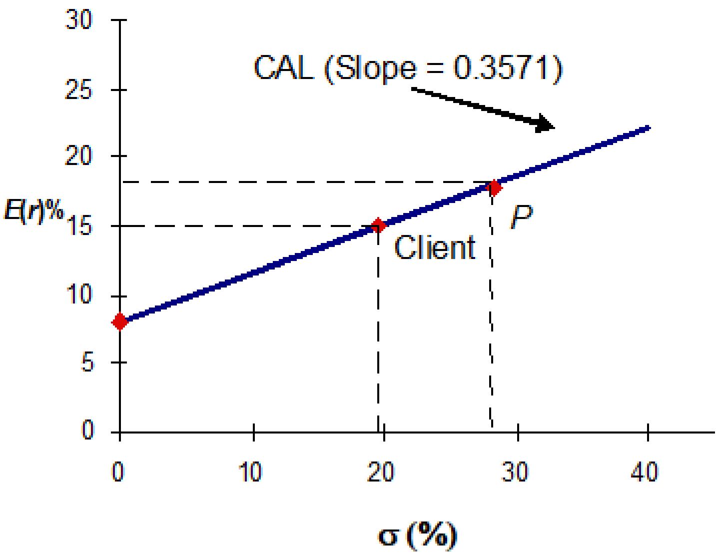
14. Investment proportions:

	30.0% in T-bills
0.7 × 25% =	17.5% in Stock A
0.7 × 32% =	22.4% in Stock B
0.7 × 43% =	30.1% in Stock C

15. Your reward-to-volatility (Sharpe) ratio: $S = \frac{.15 - .08}{.196} = 0.3571$

Client's reward-to-volatility (Sharpe) ratio: $S = \frac{.15 - .08}{.196} = 0.3571$

16.



17. a. $E(r_C) = r_f + y \times [E(r_P) - r_f] = .08 + y \times (.18 - .08)$

If the expected return for the portfolio is 16%, then:

$$16\% = 8\% + 10\% \times y \Rightarrow \quad y = \frac{.16 - .08}{.10} = 0.8$$

Therefore, in order to have a portfolio with expected rate of return equal to 16%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b.

Client's investment proportions:		20.0% in T-bills
	$0.8 \times 25\% =$	20.0% in Stock A
	$0.8 \times 32\% =$	25.6% in Stock B
	$0.8 \times 43\% =$	34.4% in Stock C

c. $\sigma_C = 0.8 \times \sigma_P = 0.8 \times 28\% = 22.4\%$

18. a. $\sigma_C = y \times 28\%$

If your client prefers a standard deviation of at most 18%, then:

$y = 18/28 = 0.6429 = 64.29\%$ invested in the risky portfolio.

b. $E(r_C) = .08 + .1 \times y = .08 + (0.6429 \times .1) = 14.429\%$

19. a. $$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.18 - 0.08}{3.5 \times 0.28^2} = \frac{0.10}{0.2744} = 0.3644$$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

b. $E(r_C) = 0.08 + 0.10 \times y^* = 0.08 + (0.3644 \times 0.1) = 0.1164$ or 11.644%

$\sigma_C = 0.3644 \times 28 = 10.203\%$

CHAPTER 7

4. The parameters of the opportunity set are:

$E(r_S) = 20\%$, $E(r_B) = 12\%$, $\sigma_S = 30\%$, $\sigma_B = 15\%$, $\rho = 0.10$

From the standard deviations and the correlation coefficient we generate the covariance matrix [note that] $Cov(r_S, r_B) = \rho \times \sigma_S \times \sigma_B$

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

The minimum-variance portfolio is computed as follows:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$w_{Min}(B) = 1 – 0.1739 = 0.8261$

The minimum variance portfolio mean and standard deviation are:

$E(r_{Min}) = (0.1739 \times .20) + (0.8261 \times .12) = .1339 = 13.39\%$

$$\sigma_{Min} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)]^{1/2}$$

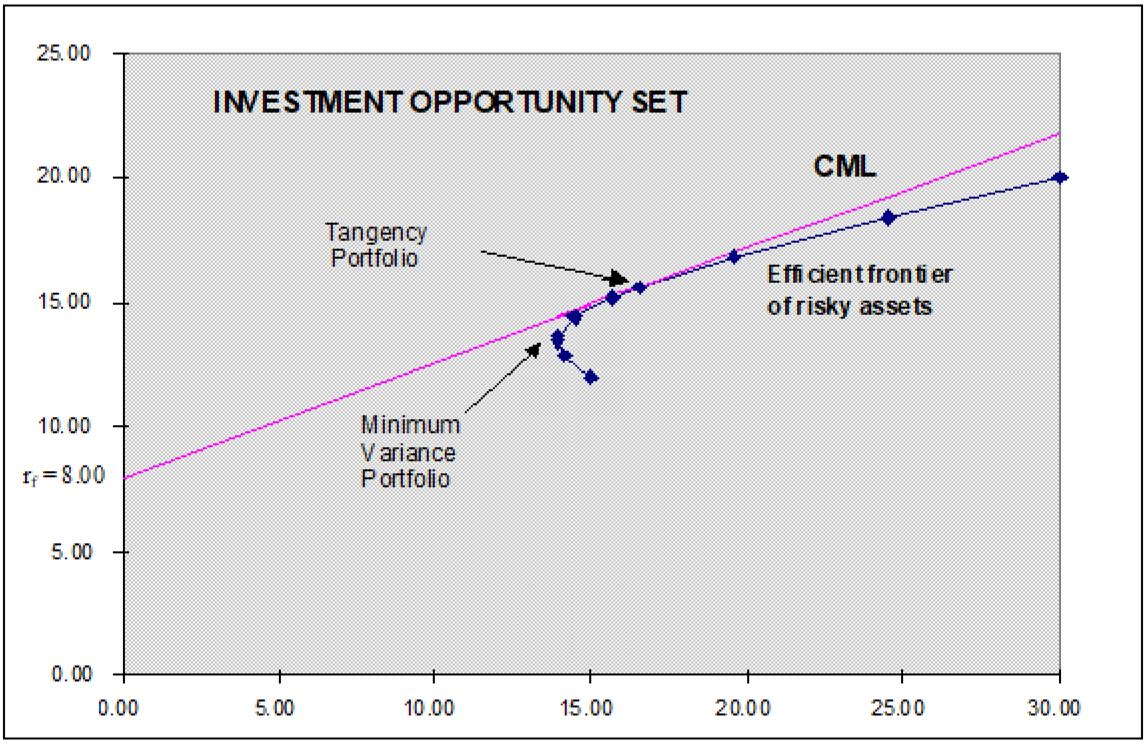
$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2}$$

$= 13.92\%$

5.

Proportion in stock fund	Proportion in bond fund	Expected return	Standard Deviation	
0.00%	100.00%	12.00%	15.00%	
17.39%	82.61%	13.39%	13.92%	minimum variance
20.00%	80.00%	13.60%	13.94%	
40.00%	60.00%	15.20%	15.70%	
45.16%	54.84%	15.61%	16.54%	tangency portfolio
60.00%	40.00%	16.80%	19.53%	
80.00%	20.00%	18.40%	24.48%	
100.00%	0.00%	20.00%	30.00%	

Graph shown below.



6. The above graph indicates that the optimal portfolio is the tangency portfolio with expected return approximately 15.6% and standard deviation approximately 16.5%.

7. The proportion of the optimal risky portfolio invested in the stock fund is given by:

$$w_S = \frac{[E(r_S)-r_f]\times\sigma_B^2-[E(r_B)-r_f]\times Cov(r_S,r_B)}{[E(r_S)-r_f]\times\sigma_B^2+[E(r_B)-r_f]\times\sigma_S^2-[E(r_S)-r_f+E(r_B)-r_f]\times Cov(r_S,r_B)}$$

$$=\frac{[(.20-.08)\times 225]-[(.12-.08)\times 45]}{[(.20-.08)\times 225]+[(.12-.08)\times 900]-[(.20-.08+.12-.08)\times 45]}=0.4516$$

$$w_B = 1-0.4516 = 0.5484$$

The mean and standard deviation of the optimal risky portfolio are:

$E(r_P) = (0.4516 \times .20) + (0.5484 \times .12) = .1561$

$= 15.61\%$

$\sigma_P = [(0.4516^2 \times 900) + (0.5484^2 \times 225) + (2 \times 0.4516 \times 0.5484 \times 45)]^{1/2}$

$= 16.54\%$

8. The reward-to-volatility ratio of the optimal CAL is:

$$\frac{E(r_p)-r_f}{\sigma_p}=\frac{.1561-.08}{.1654}=0.4601$$

or should be .4603 (rounding)

9. a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL. The equation for this CAL is:

$$E(r_C)=r_f+\frac{E(r_p)-r_f}{\sigma_P}\sigma_C = .08+0.4601\sigma_C$$

or should be .4603 (rounding)

If $E(r_C)$ is equal to 14%, then the standard deviation of the portfolio is 13.03%.

b. To find the proportion invested in the T-bill fund, remember that the mean of the complete portfolio (i.e., 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$$E(r_C) = (1-y)\times r_f + y\times E(r_P) = r_f + y\times[E(r_P)-r_f] = .08 + y\times(.1561-.08)$$

Setting $E(r_C) = 14\%$ we find: y = 0.7881 and (1 − y) = 0.2119 (the proportion invested in the T-bill fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = 0.7881 × 0.4516 = 0.3559

Proportion of bonds in complete portfolio = $0.7881 \times 0.5484 = 0.4322$

10. Using only the stock and bond funds to achieve a portfolio expected return of 14%, we must find the appropriate proportion in the stock fund (w_S) and the appropriate proportion in the bond fund ($w_B = 1 - w_S$) as follows:

$$.14 = .20 \times w_S + .12 \times (1 - w_S) = .12 + .08 \times w_S \Rightarrow w_S = 0.25$$

So, the proportions are 25% invested in the stock fund and 75% in the bond fund. The standard deviation of this portfolio will be:

$$\sigma_P = [(0.25^2 \times 900) + (0.75^2 \times 225) + (2 \times 0.25 \times 0.75 \times 45)]^{1/2} = 14.13\%$$

This is considerably greater than the standard deviation of 13.04% achieved using T-bills and the optimal portfolio.