

Time Series Models

Simulated AR Time Series

```
. set obs 500
obs was 0, now 500

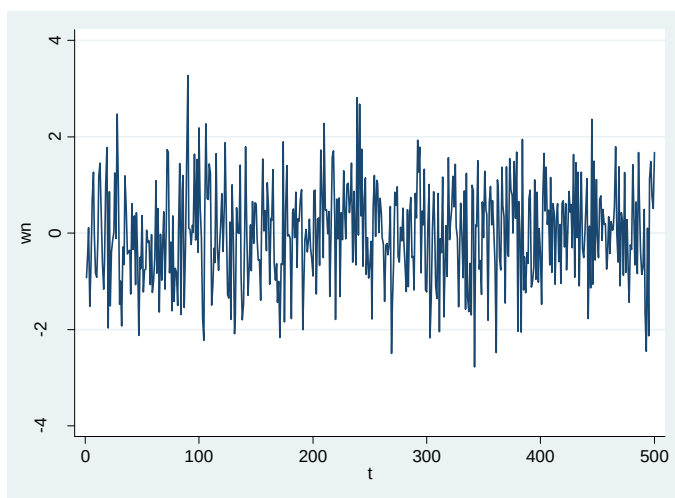
. g t=_n

. tsset t
    time variable: t, 1 to 500
        delta: 1 unit

. set seed 1234

. g wn=rnormal(0,1)

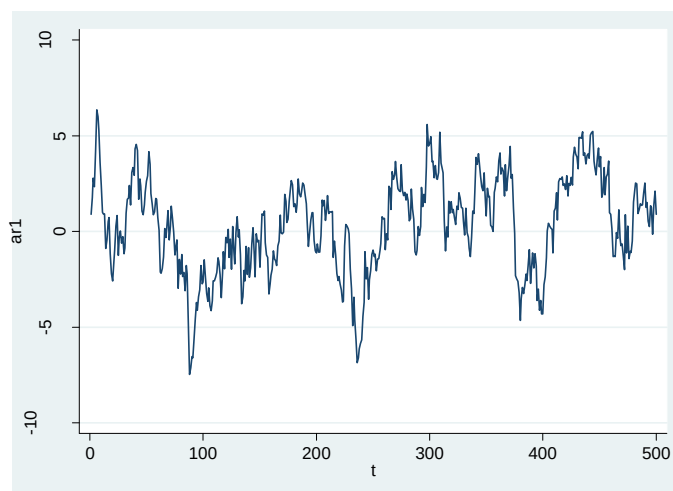
. line wn t
```



```
. g ar1=0.9 in 1
(499 missing values generated)

. replace ar1=0.9*1.ar1+rnormal(0,1) if t>1
(499 real changes made)

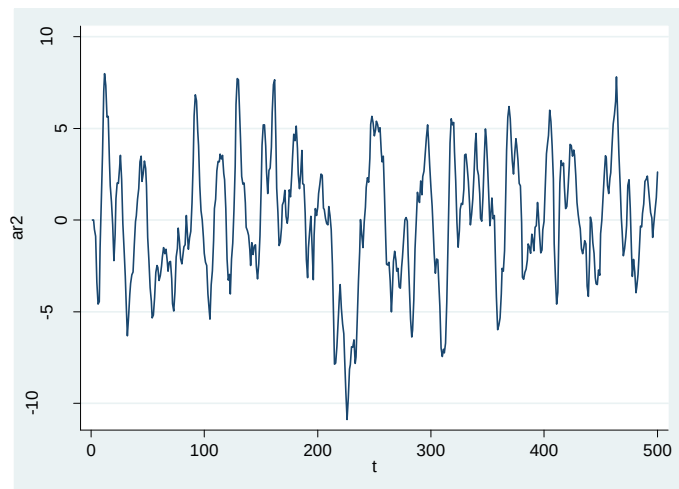
. line ar1 t
```



```
. g ar2=0 if t<3
(498 missing values generated)
```

```
. replace ar2=1.5*1.ar2-0.6*12.ar2+rnormal(0,1)      if      t>2
(498 real changes made)

. line ar2 t
```



Random Walk without Drift:

$$Y_{1t} = Y_{1t-1} + u_t$$

where: u_t is white noise error term with mean = 0 and variance = σ^2

Random Walk with Drift:

$$Y_{2t} = \delta + Y_{2t-1} + u_t$$

```
. set obs 200
obs was 0, now 200

. gener t=_n

. tsset t
    time variable:  t, 1 to 200

. gener u= rnormal(0,1)

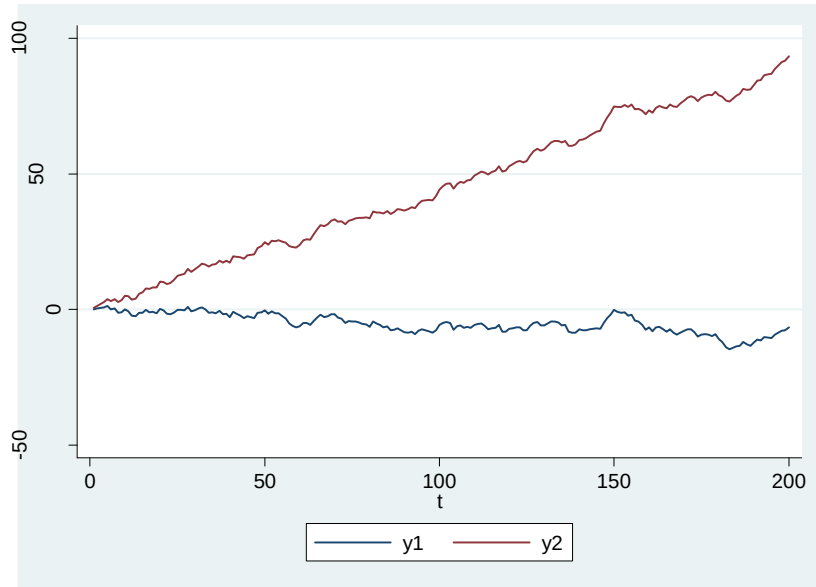
. gener y1=0 in 1
(199 missing values generated)

. gener y2=0.5 in 1
(199 missing values generated)

. replace y1=1.y1 +u if t>1
(199 real changes made)

. replace y2=0.5+1.y2 + u if t>1
(199 real changes made)

. line y1 y2 t
```



Trend Stationary (TS) and Difference Stationary (DS)**Pure random walk**

$$\beta_1 = 0, \beta_2 = 0, \beta_3 = 1$$

$$Y_{1t} = Y_{1t-1} + u_t$$

Random walk with drift

$$\beta_1 \neq 0, \beta_2 = 0, \beta_3 = 1$$

$$Y_{2t} = \beta_1 + Y_{2t-1} + u_t$$

Deterministic trend

$$\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 = 0$$

$$Y_{3t} = \beta_1 + \beta_2 t + u_t$$

Random walk with drift and deterministic trend

$$\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 = 1$$

$$Y_{4t} = \beta_1 + \beta_2 t + Y_{4t-1} + u_t$$

Deterministic trend with stationary AR(1) component

$$\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 < 1$$

$$Y_{5t} = \beta_1 + \beta_2 t + \beta_3 Y_{5t-1} + u_t$$

```
. set obs 200
obs was 0, now 200

. gener time=_n

. tsset time
    time variable:  time, 1 to 200

. gener y1=0 in 1
(199 missing values generated)

. gener y2=0 in 1
(199 missing values generated)

. gener y3=0 in 1
(199 missing values generated)

. gener y4=0 in 1
(199 missing values generated)

. gener y5=0 in 1
(199 missing values generated)

. gener u = rnormal(0,1)

. gener t=_n

. replace y1=l.y1+u if time>1
(199 real changes made)

. replace y2=0.5+l.y2+u if time>1
(199 real changes made)

. replace y3=0.5+0.5*t+u if time>1
(199 real changes made)

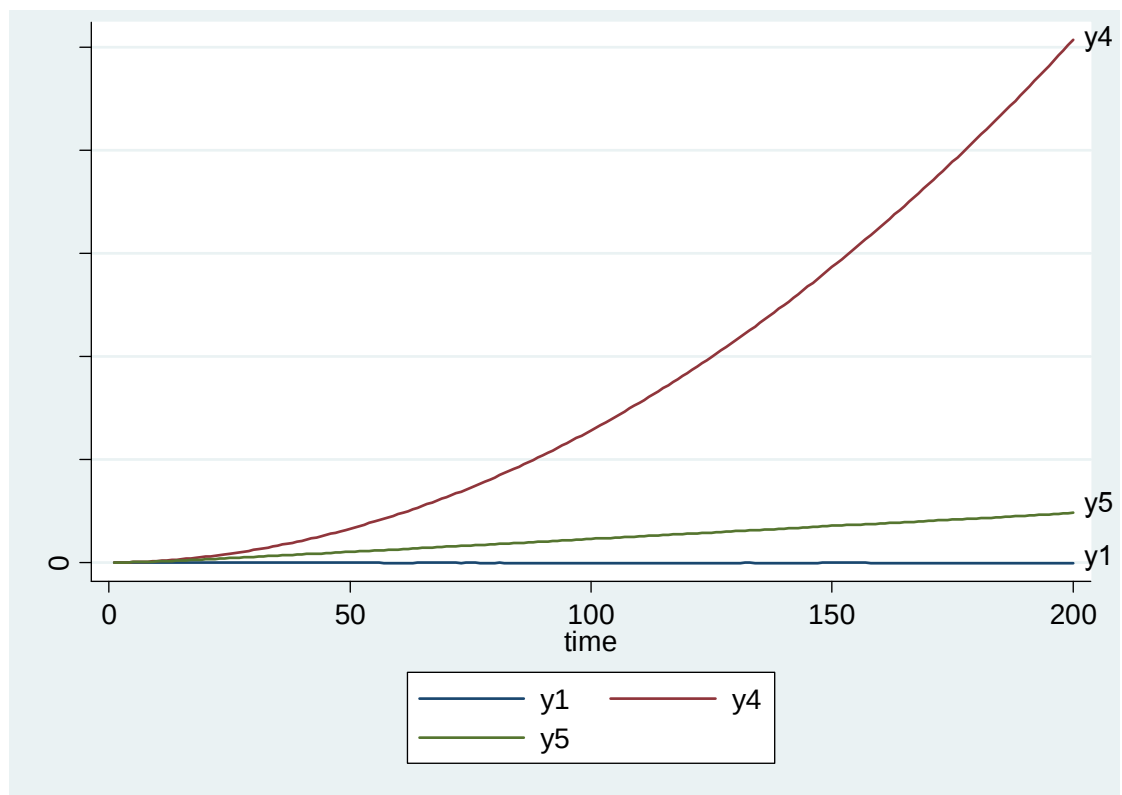
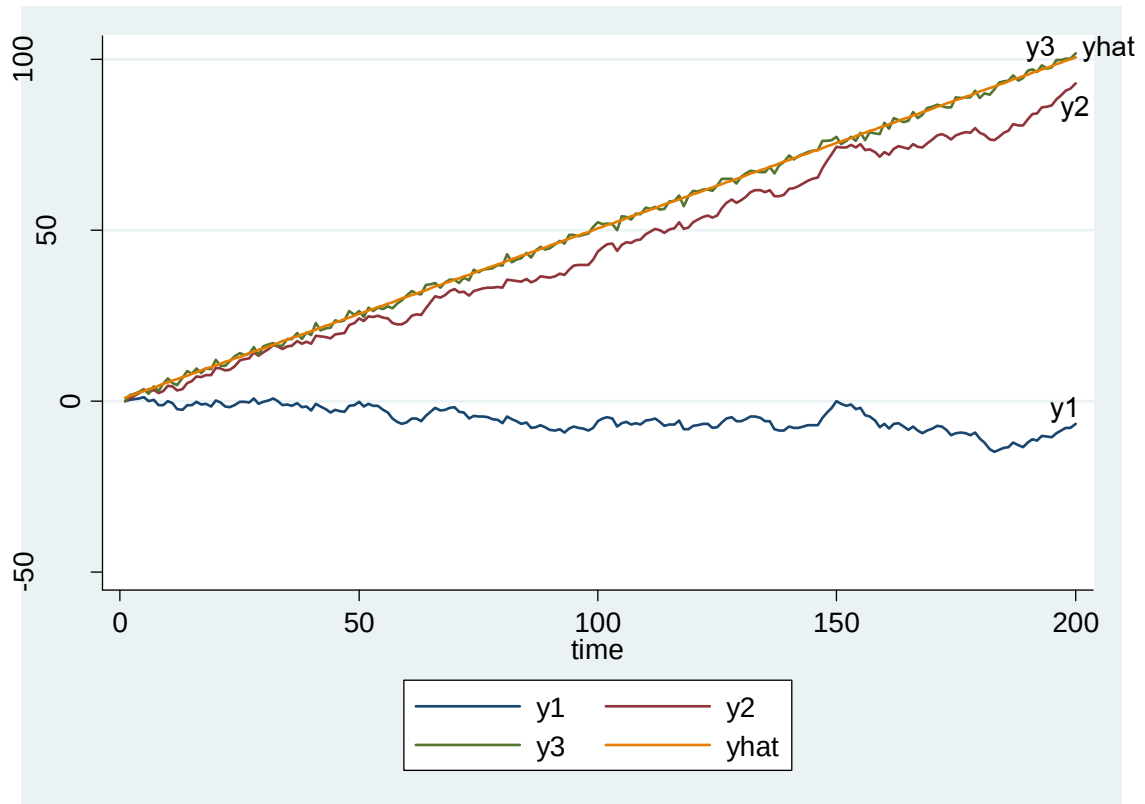
. replace y4=0.5+0.5*t+l.y4+u if time>1
(199 real changes made)

. replace y5=0.5+0.5*t+0.9*l.y5+u if time>1
(199 real changes made)

. gener yhat=0.5+0.5*t

. line y1 y2 y3 yhat time

. line y1 y4 y5 time
```



Unit Root Problem**Autoregressive (Stationary) Series**

$$x_{1t} = 0.05 + 0.95 x_{1t-1} + u_{1t}$$

Random Walk with Drift

$$x_{2t} = 0.05 + x_{2t-1} + u_{2t}$$

Explosive Series

$$x_{3t} = 0.05 + 1.05 x_{3t-1} + u_{3t}$$

```
. set obs 500
obs was 0, now 500

. gener time=_n

. tsset time
    time variable:  time, 1 to 500

. gener x1=0 in 1
(499 missing values generated)

. gener x2=0 in 1
(499 missing values generated)

. gener x3=0 in 1
(499 missing values generated)

. gener u1= rnormal(0,1)

. gener u2= rnormal(0,1)

. gener u3= rnormal(0,1)

. replace x1=0.05+0.95*_l.x1+u1 if time>1
(499 real changes made)

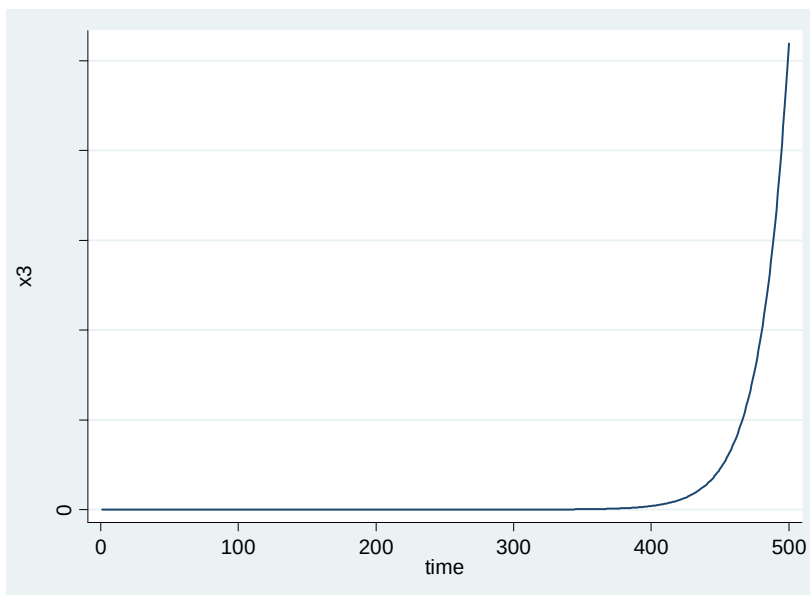
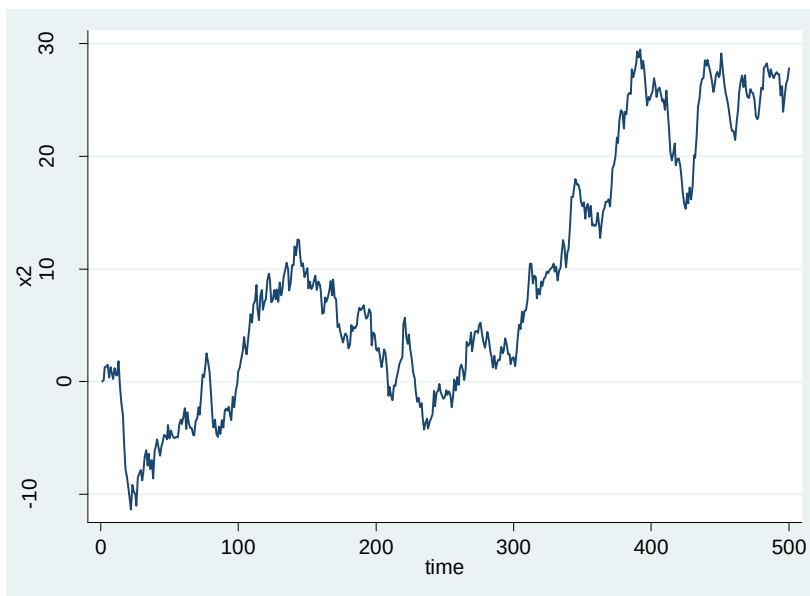
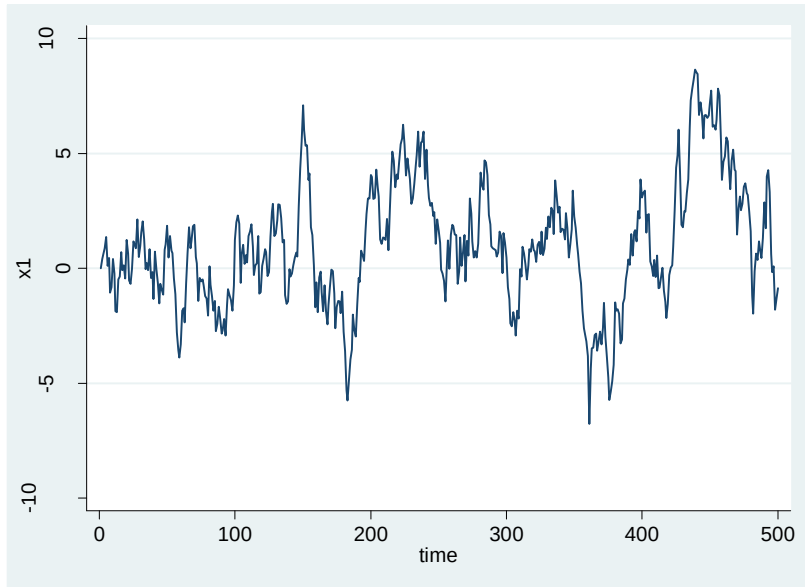
. replace x2=0.05+_l.x2+u2 if time>1
(499 real changes made)

. replace x3=0.05+1.05*_l.x3+u3 if time>1
(499 real changes made)

. line x1 time

. line x2 time

. line x3 time
```



Spurious Regression

$$Y_t = Y_{t-1} + u_t$$

$$X_t = X_{t-1} + v_t$$

If X_t and Y_t are uncorrelated $I(1)$ processes, regression $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t$ can lead to spurious problem.

```
. set obs 500
obs was 0, now 500

. gener time=_n

. tsset time
    time variable:  time, 1 to 500
```

```
. gener y=0 in 1
(499 missing values generated)
```

```
. gener x=0 in 1
(499 missing values generated)
```

```
. gener u= rnormal(0,1)
```

```
. gener v= rnormal(0,1)
```

```
. replace y=l.y+u if time>1
(499 real changes made)
```

```
. replace x=l.x+v if time>1
(499 real changes made)
```

```
. regress y x
```

source	SS	df	MS	Number of obs	=	500
Model	1563.54402	1	1563.54402	F(1, 498)	=	75.33
Residual	10336.0273	498	20.7550748	Prob > F	=	0.0000
				R-squared	=	0.1314
				Adj R-squared	=	0.1297
Total	11899.5713	499	23.8468362	Root MSE	=	4.5558

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x	-.1385236	.0159599	-8.68	0.000	-.1698807 -.1071665
_cons	-10.1792	.24019	-42.38	0.000	-10.65111 -9.707289

```
. dwstat
```

```
Durbin-watson d-statistic( 2, 500) = .0573669
```


Unit Root Test

1. Test with all terms (intercept, trend, and lags):

```
. dfuller y, trend lags(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 498

	Test Statistic	----- 1% Critical Value	Interpolated Dickey-Fuller 5% Critical Value	----- 10% Critical Value
Z(t)	-2.049	-3.980	-3.420	-3.130

Mackinnon approximate p-value for Z(t) = 0.5747

D.y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y						
L1.	-.0187253	.0091402	-2.05	0.041	-.0366837	-.0007668
LD.	-.1694873	.0445717	-3.80	0.000	-.2570608	-.0819138
_trend	.0299962	.0292846	1.02	0.306	-.0275415	.0875339
_cons	-14.66152	9.461164	-1.55	0.122	-33.25061	3.927562

The result shows p-value=0.5747 (fail to reject null hypothesis of unit root). Then, check whether trend is significant and find out that p-value=0.306. Thus, there must be no trend. Go to next step:

2. Test with intercept and lags:

```
. dfuller y, lags(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 498

	Test Statistic	----- 1% Critical Value	Interpolated Dickey-Fuller 5% Critical Value	----- 10% Critical Value
Z(t)	-1.935	-3.440	-2.870	-2.570

Mackinnon approximate p-value for Z(t) = 0.3156

D.y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y						
L1.	-.0175501	.0090684	-1.94	0.054	-.0353673	.0002672
LD.	-.1685012	.0445635	-3.78	0.000	-.2560582	-.0809442
_cons	-6.704068	5.400393	-1.24	0.215	-17.31459	3.906451

The result shows p-value=0.3156 (fail to reject null hypothesis of unit root). Then, check whether constant term is significant and find out that p-value=0.215. Thus, there must be no constant term. Go to next step:

3. Test with only lags:

```
. dfuller y, nocon lags(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 498

	Test Statistic	----- 1% Critical Value	Interpolated Dickey-Fuller 5% Critical Value	----- 10% Critical Value
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z(t)	-1.484	-2.580	-1.950	-1.620		

D.y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	

y						
L1.	-.0104078	.0070134	-1.48	0.138	-.0241875	.0033718
LD.	-.1723889	.0444776	-3.88	0.000	-.2597766	-.0850012

Dickey-Fuller GLS Test

```
. dfglm y, maxlag(10)
```

DF-GLS for y Number of obs = 489

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
10	-2.206	-3.480	-2.849	-2.563
9	-2.229	-3.480	-2.852	-2.567
8	-2.166	-3.480	-2.856	-2.570
7	-2.508	-3.480	-2.859	-2.572
6	-2.452	-3.480	-2.862	-2.575
5	-2.062	-3.480	-2.865	-2.578
4	-2.044	-3.480	-2.868	-2.581
3	-1.894	-3.480	-2.871	-2.583
2	-1.831	-3.480	-2.874	-2.586
1	-1.698	-3.480	-2.876	-2.588

Opt Lag (Ng-Perron seq t) = 8 with RMSE 90.41123
 Min SC = 9.089339 at lag 1 with RMSE 92.94486
 Min MAIC = 9.062736 at lag 8 with RMSE 90.41123

```
. dfglm d.y, maxlag(10)
```

DF-GLS for D.y Number of obs = 488

[lags]	DF-GLS tau Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
10	-4.642	-3.480	-2.849	-2.564
9	-4.526	-3.480	-2.852	-2.567
8	-4.790	-3.480	-2.856	-2.570
7	-5.308	-3.480	-2.859	-2.572
6	-4.967	-3.480	-2.862	-2.575
5	-5.404	-3.480	-2.865	-2.578
4	-6.980	-3.480	-2.868	-2.581
3	-7.989	-3.480	-2.871	-2.583
2	-10.145	-3.480	-2.874	-2.586
1	-13.424	-3.480	-2.876	-2.588

Opt Lag (Ng-Perron seq t) = 7 with RMSE 92.82793
 Min SC = 9.149196 at lag 5 with RMSE 93.36743
 Min MAIC = 9.550801 at lag 9 with RMSE 92.51999

Phillips-Perron Test

```
. pperron y, trend regress
```

Phillips-Perron test for unit root Number of obs = 499
Newey-west lags = 2

	Test Statistic	1% Critical Value	Interpolated Dickey-Fuller 5% Critical Value	10% Critical Value

Z(rho)	-9.437	-28.898	-21.499	-18.100
Z(t)	-2.228	-3.980	-3.420	-3.130

Mackinnon approximate p-value for Z(t) = 0.4742

y		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y							
L1.		.9779577	.0091986	106.32	0.000	.9598848	.9960306
_trend		.0262301	.0295507	0.89	0.375	-.03183	.0842901
_cons		-14.93069	9.53107	-1.57	0.118	-33.65693	3.795564

. pperron y, regress

Phillips-Perron test for unit root

Number of obs = 499
Newey-West lags = 5

	Test Statistic	----- 1% Critical Value	Interpolated Dickey-Fuller 5% Critical Value	----- 10% Critical Value
z(rho)	-9.754	-20.499	-14.000	-11.200
z(t)	-2.223	-3.440	-2.870	-2.570

Mackinnon approximate p-value for z(t) = 0.1981

y		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y							
L1.		.9789795	.0091243	107.29	0.000	.9610525	.9969064
_cons		-7.988098	5.445442	-1.47	0.143	-18.68702	2.710825

. pperron y, nocon regress

Phillips-Perron test for unit root

Number of obs = 499
Newey-West lags = 5

	Test Statistic	----- 1% Critical Value	Interpolated Dickey-Fuller 5% Critical Value	----- 10% Critical Value
z(rho)	-5.695	-13.700	-8.000	-5.700
z(t)	-1.691	-2.580	-1.950	-1.620

y		Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y							
L1.		.9874316	.0070831	139.41	0.000	.9735151	1.001348