

OLS vs WLS vs GLS**Simulated Data**

```
. set obs 10
number of observations (_N) was 0, now 10
. set seed 123
. g x1=rnormal(1,100)
. g x2=rnormal(2,200)
. g x3=abs(rnormal(0,10))
. g s2=2*x3^2+rnormal(0,1)
. g u=rnormal(0,s2)
. g y=1+0.99*x1+2*x2+u
```

OLS

```
. reg y x1 x2
```

Source	SS	df	MS	Number of obs	=	10
Model	3027361.28	2	1513680.64	F(2, 7)	=	57.98
Residual	182748.905	7	26106.9864	Prob > F	=	0.0000
				R-squared	=	0.9431
				Adj R-squared	=	0.9268
Total	3210110.19	9	356678.91	Root MSE	=	161.58

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x1	1.649781	.6064788	2.72	0.030	.215687 3.083876
x2	2.501466	.3331438	7.51	0.000	1.713706 3.289226
_cons	36.78376	51.3774	0.72	0.497	-84.70449 158.272

```
. predict uhat, r
. g uhat2=uhat^2
. g x32=x3^2
. reg uhat2 x32, noconst
```

Source	SS	df	MS	Number of obs	=	10
Model	5.8206e+09	1	5.8206e+09	F(1, 9)	=	29.80
Residual	1.7578e+09	9	195306892	Prob > F	=	0.0004
				R-squared	=	0.7681
				Adj R-squared	=	0.7423
Total	7.5783e+09	10	757833039	Root MSE	=	13975

uhat2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x32	209.4194	38.36127	5.46	0.000	122.6402 296.1986

```
. predict vhat2, xb
. g vhat=sqrt(vhat2)
```

WLS

```
. wls y x1 x2, sd(vhat)
```

Variance-weighted least-squares regression				Number of obs	=	10
Goodness-of-fit chi2(7)	=	7.80	Model chi2(2)	=	189.82	
Prob > chi2	=	0.3509	Prob > chi2	=	0.0000	

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	.9657276	.4069476	2.37	0.018	.168125 1.76333
x2	2.142351	.1594087	13.44	0.000	1.829916 2.454786
_cons	-6.174503	27.56531	-0.22	0.823	-60.20151 47.85251

Matrix Approach

```
. g one=1
. mkmat one x1 x2, matrix(x)
. mat list x
x[10,3]
      one      x1      x2
r1      1  50.076767 -63.368835
r2      1 -139.50819 -240.76537
r3      1 -19.405062  .23585288
r4      1  211.97264  327.33344
r5      1  45.607742 -257.28363
r6      1 -31.318707  212.41083
r7      1  76.942497  200.37794
r8      1 -124.71904  40.628117
r9      1 -70.446632 -4.2360992
r10     1  12.753014 -43.672867
```

```
. mkmat y, mat(y)
. mat list y
y[10,1]
      y
r1 -235.75397
r2 -621.93738
r3 -2.7740371
r4  1458.6753
r5 -543.53473
r6  391.29703
r7  498.34872
r8  9.9493847
r9 -80.996529
r10 -56.312798
```

OLS

```
. matrix b=inv(x'*x)*(x'*y)
. mat list b
b[3,1]
      y
one  36.783764
x1  1.6497815
x2  2.5014664
. mat u=Y-X*b
. mat V=u*u'
. mat list v
symmetric v[10,10]
      r1      r2      r3      r4      r5      r6      r7      r8      r9      r10
r1  38666.676
r2 -34156.761  30172.862
r3  1599.3916 -1412.8454  66.156539
r4 -49822.162  44011.118 -2060.8222  64196.048
r5  2354.7331 -2080.0871  97.400155 -3034.0827  143.39913
r6  24610.533 -21740.066  1017.9794 -31710.767  1498.7385  15664.091
r7  32762.333 -28941.075  1355.1669 -42214.393  1995.1689  20852.542  27759.575
r8 -15199.152  13426.388 -628.6911  19584.166 -925.60179 -9673.9431 -12878.264  5974.5039
r9 -1777.1628  1569.8822 -73.509791  2289.8812 -108.22611 -1131.1271 -1505.7928  698.56967  81.680352
r10 961.56918 -849.41587  39.77393 -1238.9856  58.557885  612.01874  814.73903 -377.97497 -44.194774  23.912459
. mat u2=vecdiag(v)
. mat list u2
u2[1,10]
      r1      r2      r3      r4      r5      r6      r7      r8      r9      r10
r1  38666.676  30172.862  66.156539  64196.048  143.39913  15664.091  27759.575  5974.5039  81.680352  23.912459
. mat u2t=u2'
. mat list u2t
u2t[10,1]
      r1
r1  38666.676
r2  30172.862
r3  66.156539
r4  64196.048
r5  143.39913
r6  15664.091
r7  27759.575
r8  5974.5039
r9  81.680352
r10 23.912459
. mkmat x32, mat(x32)
. mat b2=inv(x32'*x32)*x32'*u2t
. mat u2hat=x32*b2
```

```

. mat u2hatT=u2hat'
. mat list u2hatT
u2hatT[1,10]
r1 13685.593 24773.965 1897.4132 65346.331 9157.7929 1329.8996 10753.107 4565.7394 3985.646 22534.11
. mat vhat=diag(u2hatT)
. mat list vhat
symmetric vhat[10,10]
r1 13685.593
r2 0 24773.965
r3 0 0 1897.4132
r4 0 0 0 65346.331
r5 0 0 0 0 9157.7929
r6 0 0 0 0 0 1329.8996
r7 0 0 0 0 0 0 10753.107
r8 0 0 0 0 0 0 0 4565.7394
r9 0 0 0 0 0 0 0 0 3985.646
r10 0 0 0 0 0 0 0 0 0 22534.11

```

GLS – WLS

```

. mat bgls=inv(X'*inv(vhat)*X)*(X'*inv(vhat)*Y)
. mat list bgls
bgls[3,1]
y
one -6.1745036
x1 .96572763
x2 2.1423509
. vwls y x1 x2, sd(vhat)

```

```

Variance-weighted least-squares regression      Number of obs      =      10
Goodness-of-fit chi2(7)      =      7.80      Model chi2(2)      =      189.82
Prob > chi2      =      0.3509      Prob > chi2      =      0.0000

```

```

-----+-----
      y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----+-----
      x1 |   .9657276   .4069476     2.37   0.018     .168125   1.76333
      x2 |   2.142351   .1594087    13.44   0.000     1.829916   2.454786
     _cons |  -6.174503   27.56531    -0.22   0.823    -60.20151   47.85251
-----+-----

```