5.3 Ex. 1, p.4

(a) $y = 2e^x$

$= 2 \cdot e^x$

$\frac{dy}{dx} = 2 \frac{de^x}{dx}$

$y' = 2e^x$ (use ① in 5.3)

(b) $y = 2e^{3x}$

$\frac{dy}{dx} = 2 \frac{d(e^u)}{dx}$ when $u = 3x$

$= 2e^u \frac{du}{dx}$ (use ② in 5.3)

$= 2e^{3x}(3) = 6e^{3x}$ ($\because u = 3x, \frac{du}{dx} = 3$)

(c) $y = ae^{-bt}$ (Note that both a & b are constants!)

$y' = \frac{dy}{dt} = a \frac{d}{dt}(e^{-bt}) = a \frac{d}{dt}(e^u)$ where $u = -bt$

$= ae^u \frac{du}{dt}$ (use ② in 5.3)

$= ae^u(-b)$ ($\because u = -bt, \frac{du}{dt} = -b$)

$= ae^{-bt}(-b) = -abe^{-bt}$

(d) $y = 5e^{x^4+2x}$

$= 5e^{u}$

$\frac{dy}{dx} = 5\frac{de^{u}}{dx}$ where $u = x^4+2x$

$= 5e^{u}\frac{du}{dx}$ use ② in 5.3

$= 5e^{u}(4x^3+2)$ $(\because u = x^4+2x, \frac{du}{dx} = 4x^3+2)$

$= 5(4x^3+2)e^{x^4+2x}$

(e) $y = \frac{e^{-3x}}{x^2+1}$ ⇒ This is $\frac{\text{exponential function}}{\text{function of } x}$

∴ use quotient rule ⇒ $y = \frac{f(x)}{g(x)}$ $y' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

$y = \frac{e^{-3x}}{x^2+1}$ $e^{-3x} \to f(x)$, $x^2+1 \to g(x)$

$\therefore y' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

$= \frac{-3e^{-3x}\cdot(x^2+1) - (2x)e^{-3x}}{(x^2+1)^2}$

(Note you should understand why $f'(x) = -3e^{-3x}$, this is explained through examples (b) – (d))

$= \frac{-3x^2-3-2x\,(e^{-3x})}{(x^2+1)^2}$

$= e^{-3x}\left[\frac{-3x^2-2x-3}{(x^2+1)^2}\right]$