Scaling and Units of Measurement

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$$
$$Y_i^* = w_1 Y_i$$
$$X_i^* = w_2 X_i$$

w_1 & w_2 are called "scale factors"

$$Y_i^* = \hat{\beta}_1^* + \hat{\beta}_2^* X_i^* + \hat{u}_i^*$$
$$\hat{\beta}_1^* = w_1 \hat{\beta}_1$$
$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2$$
$$\hat{\sigma}^{*2} = w_1^2 \hat{\sigma}^2$$
$$\text{var}(\hat{\beta}_1^*) = w_1^2 \text{ var}(\hat{\beta}_1)$$
$$\text{var}(\hat{\beta}_2^*) = \left(\frac{w_1}{w_2}\right)^2 \text{var}(\hat{\beta}_2)$$

TABLE 6.2
Gross Private Domestic Investment and GDP, United States, 1990–2005 (Billions of chained [2000] dollars, except as noted; quarterly data at seasonally adjusted annual rates)

Source: *Economic Report of the President,* 2007, Table B-2, p. 328.

Year	GPDIBL	GPDIM	GDPB	GDPM
1990	886.6	886,600.0	7,112.5	7,112,500.0
1991	829.1	829,100.0	7,100.5	7,100,500.0
1992	878.3	878,300.0	7,336.6	7,336,600.0
1993	953.5	953,500.0	7,532.7	7,532,700.0
1994	1,042.3	1,042,300.0	7,835.5	7,835,500.0
1995	1,109.6	1,109,600.0	8,031.7	8,031,700.0
1996	1,209.2	1,209,200.0	8,328.9	8,328,900.0
1997	1,320.6	1,320,600.0	8,703.5	8,703,500.0
1998	1,455.0	1,455,000.0	9,066.9	9,066,900.0
1999	1,576.3	1,576,300.0	9,470.3	9,470,300.0
2000	1,679.0	1,679,000.0	9,817.0	9,817,000.0
2001	1,629.4	1,629,400.0	9,890.7	9,890,700.0
2002	1,544.6	1,544,600.0	10,048.8	10,048,800.0
2003	1,596.9	1,596,900.0	10,301.0	10,301,000.0
2004	1,713.9	1,713,900.0	10,703.5	10,703,500.0
2005	1,842.0	1,842,000.0	11,048.6	11,048,600.0

Note: GPDIBL = gross private domestic investment, billions of 2000 dollars.
GPDIM = gross private domestic investments, millions of 2000 dollars.
GDPB = gross domestic product, billions of 2000 dollars.
GDPM = gross domestic product, millions of 2000 dollars.

GPDIBL = Gross private domestic investment, billions of 2000 dollars
GPDIM = Gross private domestic investment, millions of 2000 dollars
GDPB = Gross domestic product, billions of 2000 dollars
GDPM= Gross domestic product, millions of 2000 dollars

Example 1

Both GDPI and GDP in billions of dollars:

$$\widehat{GPDI}_t = -926.090 + 0.2535GDP_t$$

$$se = (116.358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in billions of dollars $\longrightarrow$ millions of dollars

GDP in billions of dollars $\longrightarrow$ millions of dollars

$$w_1 = 1000$$

$$w_2 = 1000$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = 1000 * -926.090 = -926,090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{1000}{1000}\right)0.2535 = 0.2535$$

$$\widehat{GPDI}_t = -926,090 + 0.2535GDP_t$$

$$se = (116,358) \quad (0.0129)$$

$$r^2 = 0.9648$$

Example 2

Both GDPI and GDP in millions of dollars:

$$\widehat{GPDI}_t = -926,090 + 0.2535GDP_t$$

$$se = (116,358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in millions of dollars $\longrightarrow$ billions of dollars

GDP in millions of dollars $\longrightarrow$ billions of dollars

$$w_1 = \frac{1}{1000}$$

$$w_2 = \frac{1}{1000}$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = \frac{1}{1000} * -926,090 = -926.090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{\frac{1}{1000}}{\frac{1}{1000}}\right)0.2535 = 0.2535$$

$$\widehat{GPDI}_t = -926.090 + 0.2535GDP_t$$

$$se = (116.358) \quad (0.0129)$$

$$r^2 = 0.9648$$

Example 3

Both GDPI and GDP in billions of dollars:

$$\widehat{GPDI}_t = -926.090 + 0.2535GDP_t$$

$$se = (116.358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in billions of dollars $\longrightarrow$ billions of dollars

GDP in billions of dollars $\longrightarrow$ millions of dollars

$$w_1 = 1$$

$$w_2 = 1000$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = 1 * -926.090 = -926.090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{1}{1000}\right)0.2535 = 0.0002535$$

$$\widehat{GPDI}_t = -926.090 + 0.0002535GDP_t$$

$$se = (116.358) \quad (0.0000129)$$

$$r^2 = 0.9648$$

Example 4

Both GDPI and GDP in millions of dollars:

$$\widehat{GPDI}_t = -926,090 + 0.2535GDP_t$$

$$se = (116,358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in millions of dollars $\longrightarrow$ billions of dollars

GDP in millions of dollars $\longrightarrow$ millions of dollars

$$w_1 = \frac{1}{1000}$$

$$w_2 = 1$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = \frac{1}{1000} * -926,090 = -926.090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{\frac{1}{1000}}{1}\right)0.2535 = 0.0002535$$

$$\widehat{GPDI}_t = -926.090 + 0.0002535GDP_t$$

$$se = (116.358) \quad (0.0000129)$$

$$r^2 = 0.9648$$

Example 5

Both GDPI and GDP in billions of dollars:

$$\widehat{GPDI}_t = -926.090 + 0.2535GDP_t$$

$$se = (116.358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in billions of dollars $\longrightarrow$ millions of dollars

GDP in billions of dollars $\longrightarrow$ billions of dollars

$$w_1 = 1000$$

$$w_2 = 1$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = 1000 * -926.090 = -926,090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{1000}{1}\right)0.2535 = 253.524$$

$$\widehat{GPDI}_t = -926,090 + 253.524GDP_t$$

$$se = (116,358) \quad (12.9465)$$

$$r^2 = 0.9648$$

Example 6

Both GDPI and GDP in millions of dollars:

$$\widehat{GPDI}_t = -926,090 + 0.2535GDP_t$$

$$se = (116,358) \quad (0.0129)$$

$$r^2 = 0.9648$$

GDPI in millions of dollars $\longrightarrow$ millions of dollars

GDP in millions of dollars $\longrightarrow$ billions of dollars

$$w_1 = 1$$

$$w_2 = \frac{1}{1000}$$

$$\hat{\beta}_1^* = w_1\hat{\beta}_1 = 1 * -926,090 = -926,090$$

$$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\hat{\beta}_2 = \left(\frac{1}{\frac{1}{1000}}\right)0.2535 = 253.524$$

$$\widehat{GPDI}_t = -926,090 + 253.524GDP_t$$

$$se = (116,358) \quad (12.9465)$$

$$r^2 = 0.9648$$