Labor Demand

Introduction

Firms hire workers because consumers want to purchase a variety of goods and services.

Demand for workers **is derived from the wants and desires of consumers**.

Central questions: **how many workers are hired and what are they paid?**

The Firm's Production Function

Describes the technology that the firm uses to produce goods and services.

The firm's output can be produced by a variety of capital–labor combinations.

The marginal product of labor is the change in output resulting from hiring an additional worker, holding constant the quantities of other inputs.

The marginal product of capital is the change in output resulting from employing one additional unit of capital, holding constant the quantities of other inputs.

The Production Function

- The production function describes the technology that the firm uses to produce goods and services

- Assume that there are only two factors of production
  - The number of employee-hours hired by the firm (E)
  - Capital (K)

$$q = f(E, K)$$

Marginal Product and Average Product

Marginal product of labor

- The change in output resulting from hiring an additional worker, holding constant the quantities of all other inputs

Marginal product of capital

- The change in output resulting from a one-unit increase in the capital stock, holding constant the quantities of all other inputs

Average product of labor

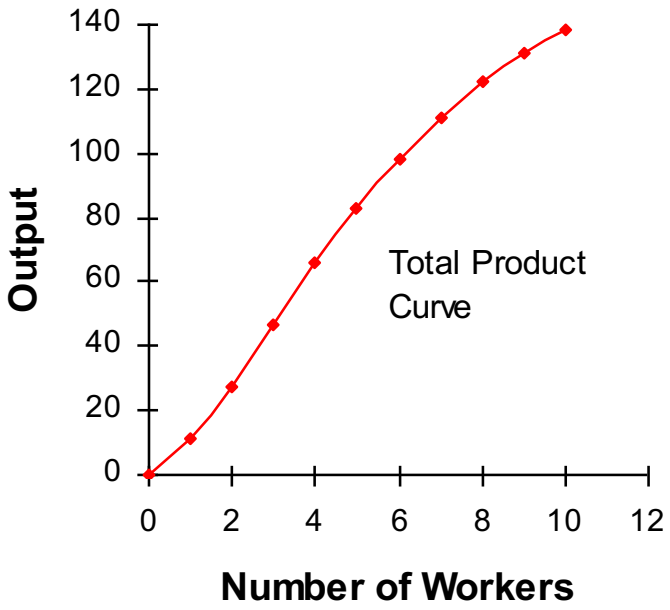
- The amount of output produced by the typical worker

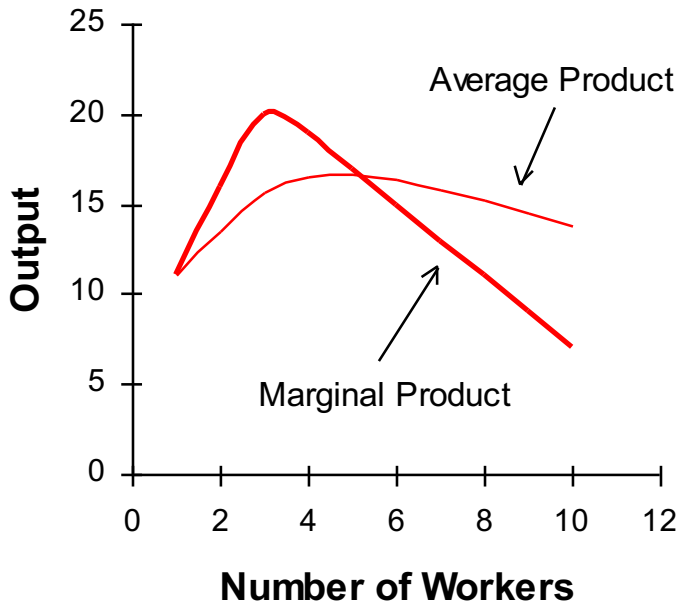
Assumption of Law of diminishing returns implies that the average product of labor curve will eventually decline

Number of Workers Employed	Output (Units)	Marginal Product (Units)	Average Product (Units)	Value of Marginal Product ($)	Value of Average Product ($)
0	0	—	—	—	—
1	11	11	11.0	22	22.0
2	27	16	13.5	32	27.0
3	47	20	15.7	40	31.3
4	66	19	16.5	38	33.0
5	83	17	16.6	34	33.2
6	98	15	16.3	30	32.7
7	111	13	15.9	26	31.7
8	122	11	15.3	22	30.5
9	131	9	14.6	18	29.1
10	138	7	13.8	14	27.6

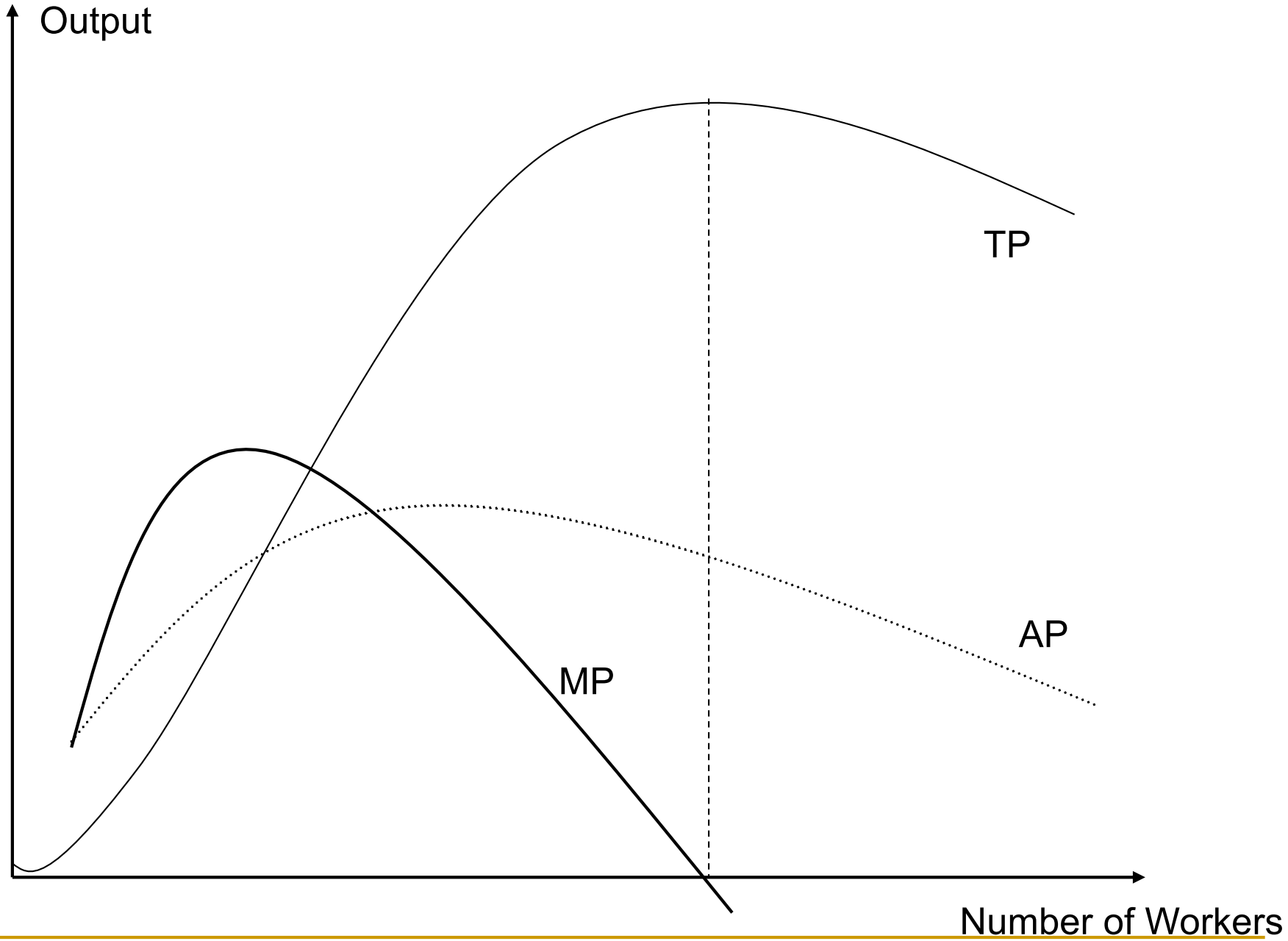
Note: The calculations for the value of marginal product and the value of average product assume that the price of the output is $2.

The total Product, the Marginal Product, and the Average Product Curves





The total product curve gives the relationship between output and the number of workers hired by the firm (holding capital fixed). The marginal product curve shows the output produced by each additional worker, and the average product curve shows output per worker.


Output
TP
AP
MP
Number of Workers

Example

- Cobb-Douglas

$$q = E^{\alpha} K^{1-\alpha}$$

Marginal Product

Marginal Product of Labor

$$MP_E = \frac{\partial q}{\partial E} = \frac{\partial E^{\alpha} K^{1-\alpha}}{\partial E} = \alpha E^{\alpha-1} K^{1-\alpha} = \alpha \frac{K^{1-\alpha}}{E^{1-\alpha}}$$

Marginal Product of capital

$$MP_K = \frac{\partial q}{\partial K} = \frac{\partial E^{\alpha} K^{1-\alpha}}{\partial K} = (1-\alpha) E^{\alpha} K^{-\alpha} = (1-\alpha) \frac{E^{\alpha}}{K^{\alpha}}$$

Profit Maximization

Objective of the firm is to maximize profits.

The profit function is:

- **Profits = $pq - wE - rK$**
  - Total Revenue = pq
  - Total Costs = $(wE + rk)$

Perfectly competitive firms cannot influence prices of output or inputs.

The Employment Decision in the Short Run

- Value of Marginal Product

$$VMP_E = p \times MP_E$$

- Value of Average Product

$$VAP_E = p \times AP_E$$

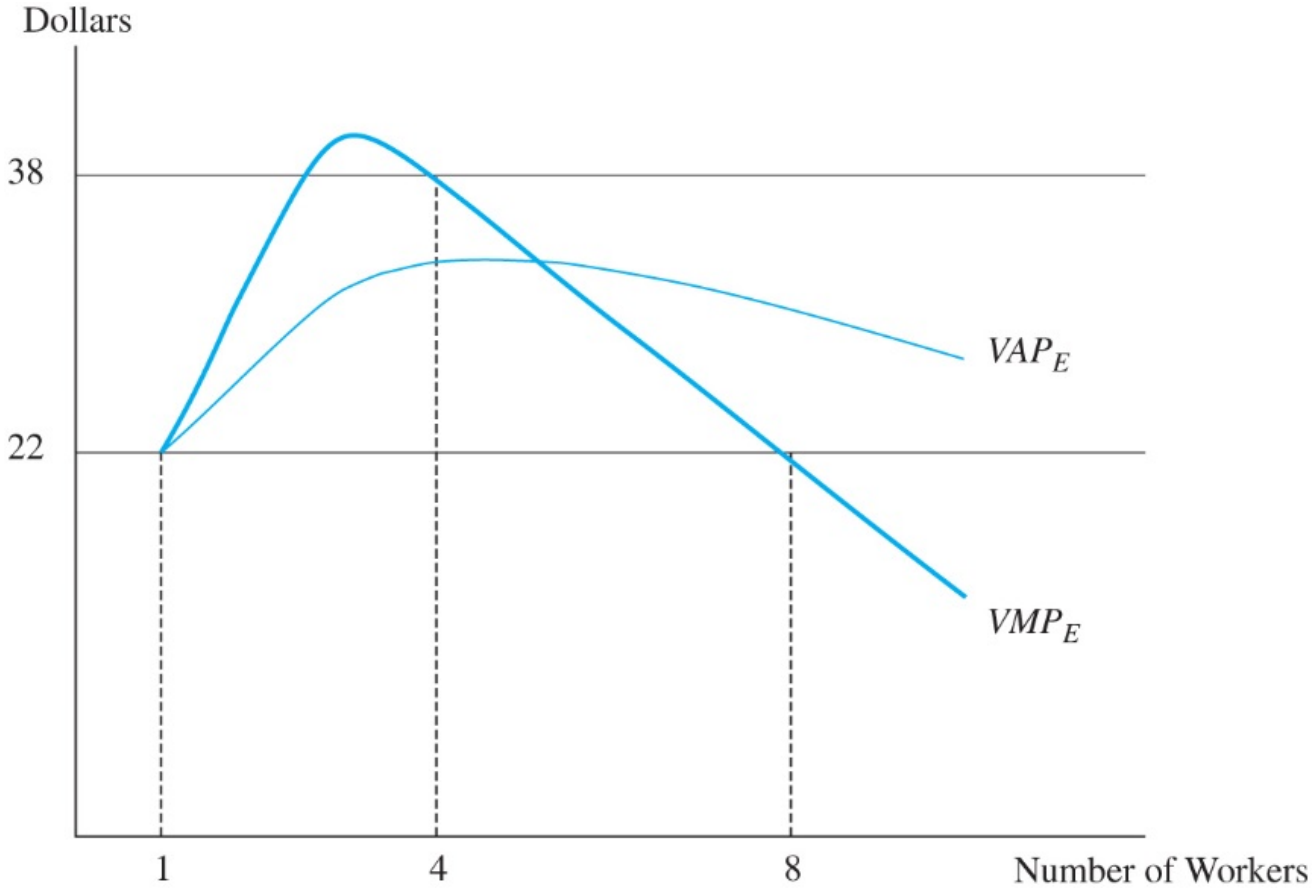
- Optimal at condition

$$w = p \times MP_E = VMP_E$$

Short Run Hiring Decision

- Value of Marginal Product (VMP) is the marginal product of labor times the dollar value of the output.
- VMP indicates the dollar benefit derived from hiring an additional worker, holding capital constant.
- Value of Average Product is the dollar value of output per worker.

The Firm's Hiring Decision in the Short-Run



A profit-maximizing firm hires workers up to the point where the wage rate equals the value of marginal product of labor. If the wage is $22, the firm hires eight workers.

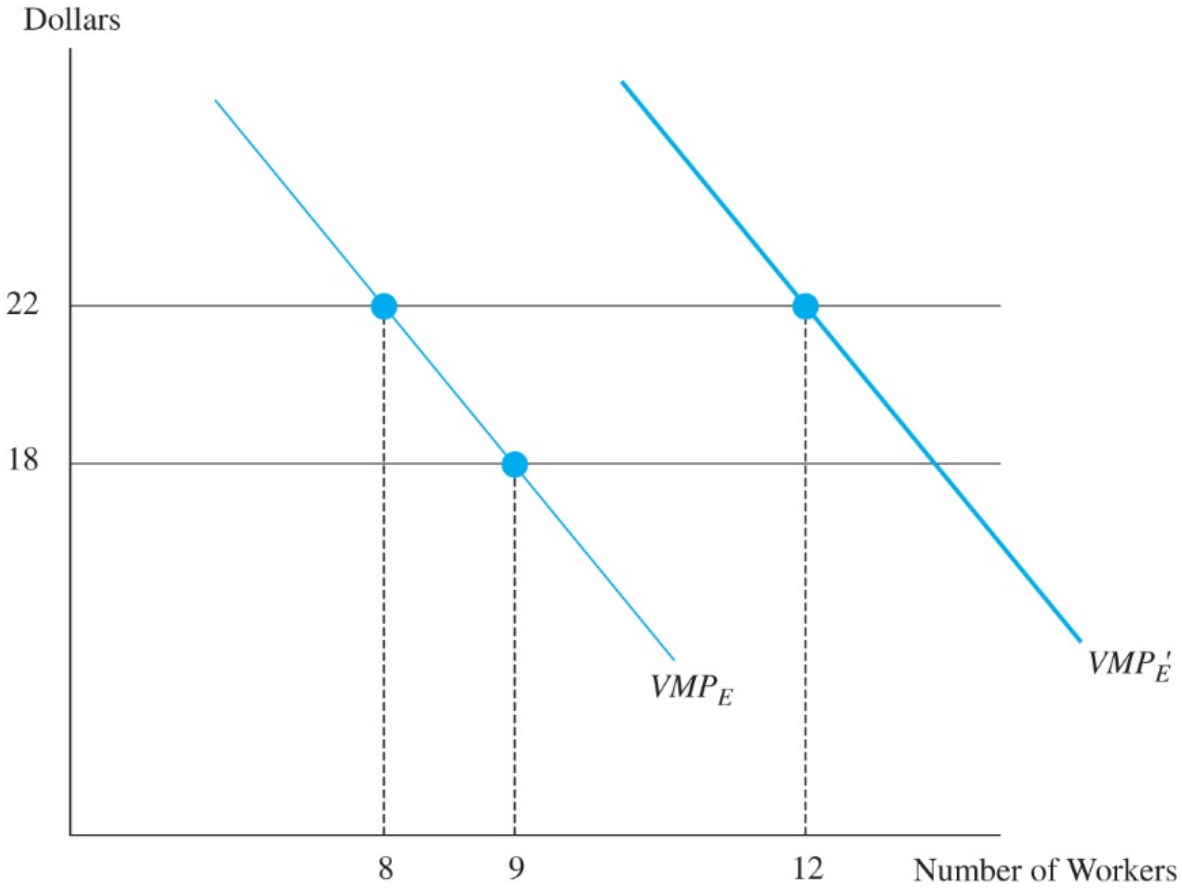
How many workers should the firm hire?

- $VMP_E = w$ and VMP_E is declining

Labor Demand Curve

- The demand curve for labor indicates how many workers the firm hires for each possible wage, holding capital constant.
- The labor demand curve is downward sloping. This reflects the fact that additional workers are costly and alter average production due to the Law of Diminishing Returns.

The Short-Run Demand Curve for Labor

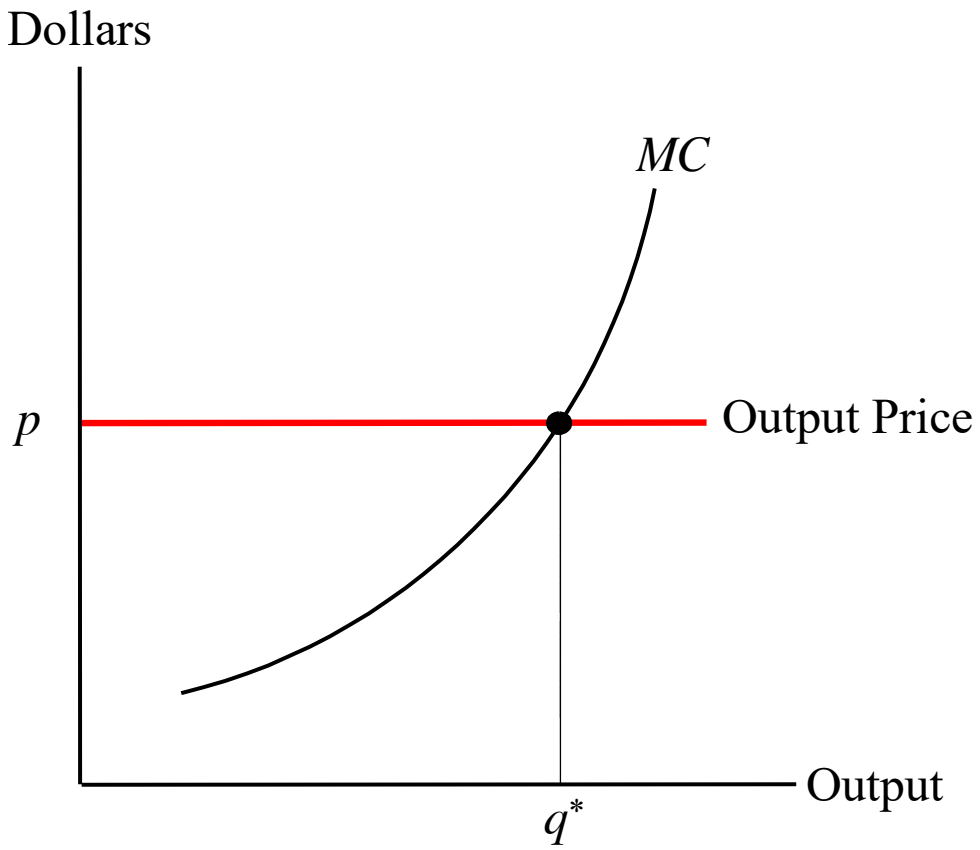


Because marginal product eventually declines, the short-run demand curve for labor is downward sloping. A drop in the wage from $22 to $18 increases the firm's employment. An increase in the price of the output shifts the value of marginal product curve upward (to the right), and increases employment.

Maximizing Profits: Two Rules

- The profit maximizing firm should produce up to the point where the cost of producing an additional unit of output (marginal cost) is equal to the revenue obtained from selling that output (marginal revenue).
- Marginal Productivity Condition: hire labor up to the point where the value of marginal product equals the added cost of hiring the worker (i.e., the wage).

The Firm's Output Decision



A profit-maximizing firm produces up to the point where the output price equals the marginal cost of production.

Hire labor up to the point where the value of marginal product equals to the wage

$$VMP_E = p \times MP_E$$

$$VMP_E = w$$

$$w = p \times MP_E$$

The Mathematics of Marginal Productivity Theory

- The cost of producing an extra unit of output:

$$MC = w \times (1 \times MP_E)$$

- The condition: produce to the point where MC = P (for the competitive firm, P = MR)

$$w \times (1 \times MP_E) = P$$

The optimal level of output

The profit maximizing condition equating price and marginal cost is identical to the profit-maximizing condition equating the wage and the value of marginal product of labor

$$MC = w \times \frac{1}{MP_E}$$

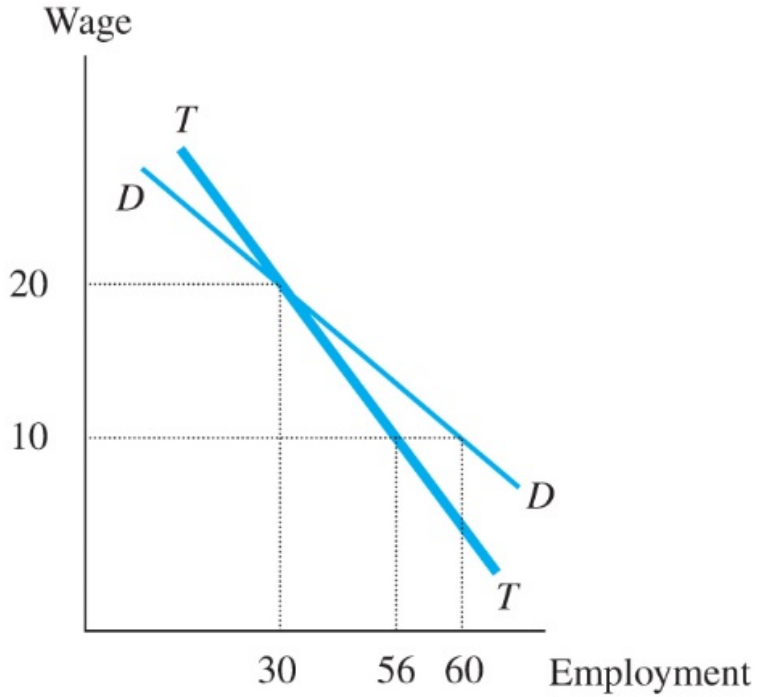
$$w \times \frac{1}{MP_E} = P$$

Critiques of Marginal Productivity Theory

- A common criticism is that the theory bears little relation to the way that employers make hiring decisions.
- Another criticism is that the assumptions of the theory are not very realistic.
- However, employers act as if they know the implications of marginal productivity theory (hence, they try to make profits and remain in business).

The short-run demand curve for the industry

(*a*) Individual Firms



(*b*) Industry

The Short-Run elasticity of labor demand

- Measure the responsiveness of employment in the industry to changes in the wage rate
- The percentage change in short-run employment resulting from a 1 percentage change in the wage:

$$\delta_{SR} = \frac{Percent\ \ change\ in\ employment}{Percent\ \ change\ in\ the\ wage} = \frac{\Delta E_{SR} / E_{SR}}{\Delta w / w} = \frac{\Delta E_{SR}}{\Delta w} \cdot \frac{w}{E_{SR}}$$

Example

$$\delta_{SR} = \frac{Percent\ \ change\ in\ employment}{Percent\ \ change\ in\ the\ wage} = \frac{(56-30)/30}{(10-20)/20} = -1.733$$

- Labor demand is said to be elastic if the absolute value of the elasticity is greater than one
- Labor demand is said to be inelastic if the absolute value of the elasticity is less than one

Class exercise

- Suppose the hourly wage is \$10 and the price of each unit of capital is \$25. The price of output is constant at \$50 per unit. The production function is

$$f(E,K) = E^{1/2}K^{1/2}$$

so that the marginal product of labor is

$$MP_E = \left(1/2\right)\left(K/E\right)^{1/2}$$

If the current capital stock is fixed at 1,600 units, how much labor should the firm employ in the short run? How much profit will the firm earn?

The Employment Decision in the Long Run

- The firm maximizes profits by choosing both how many workers to hire and how much plant and equipment to invest in

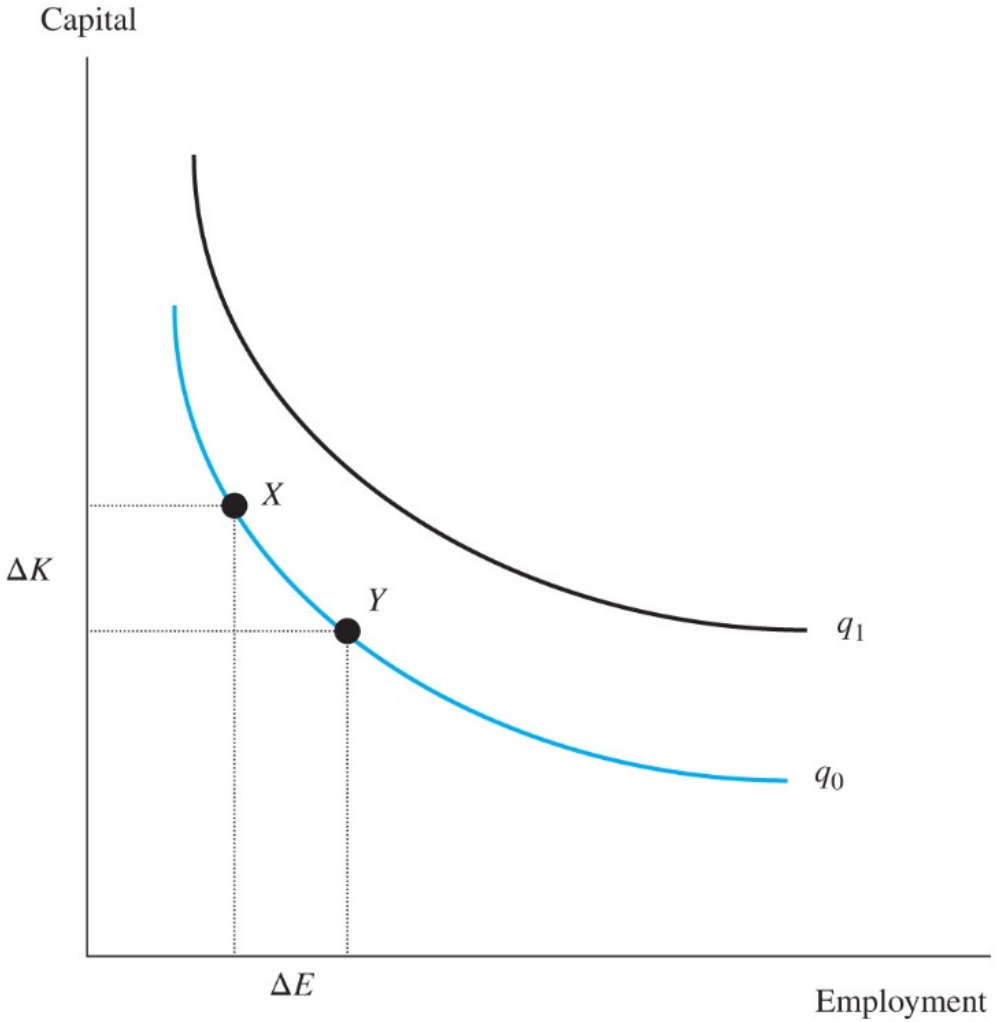
- Isoquant curves describe the possible combinations of labor and capital that produce the same level of output.

Isoquant

- The possible combinations of labor and capital that produce the same level of output

- Properties of these constant-output curves
  - Must be downward sloping
  - Do not intersect
  - Higher isoquants are associated with higher levels of output
  - Convex to the origin

Isoquant Curves



All capital-labor combinations that lie on a single isoquant produce the same level of output. The input combinations at points X and Y produce q_0 units of output. Combinations of input bundles that lie on higher isoquants must produce more output.

Isocost Lines

- The isocost line indicates all labor–capital bundles that exhaust a specified budget for the firm.
- Isocost lines indicate equally costly combinations of inputs.
- Higher isocost lines indicate higher costs.

Marginal Rate of Technical Substitution (MRTS)

$$\frac{\Delta K}{\Delta E} = -\frac{MP_E}{MP_K}$$

The assumption that isoquants are convex to the origin is an assumption about how the MRTS changes as the firm switches from capital to labor

The convexity assumption implies *diminishing marginal rate of technical substitution* as the firm substitutes more labor for capital

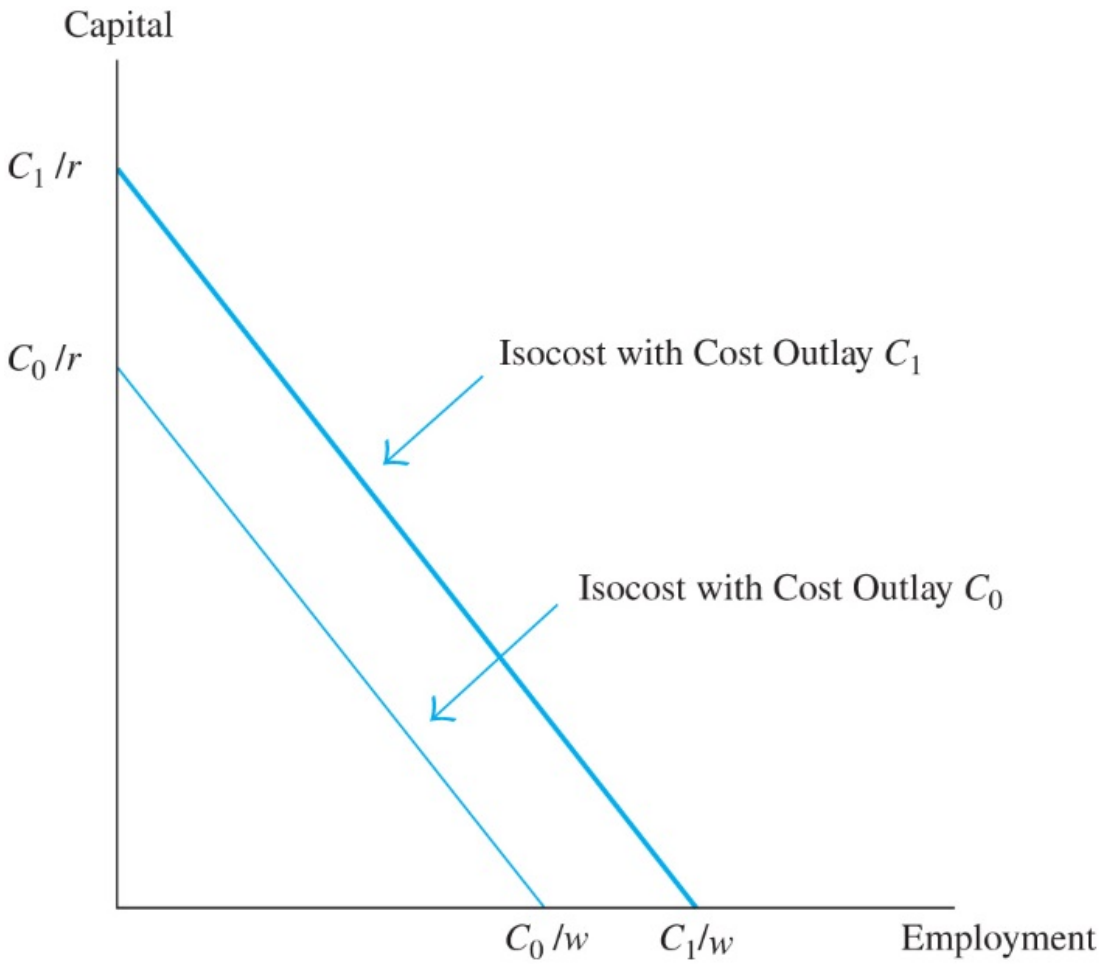
Isocosts

The firm's costs of production (C)

$$C = wE + rK$$

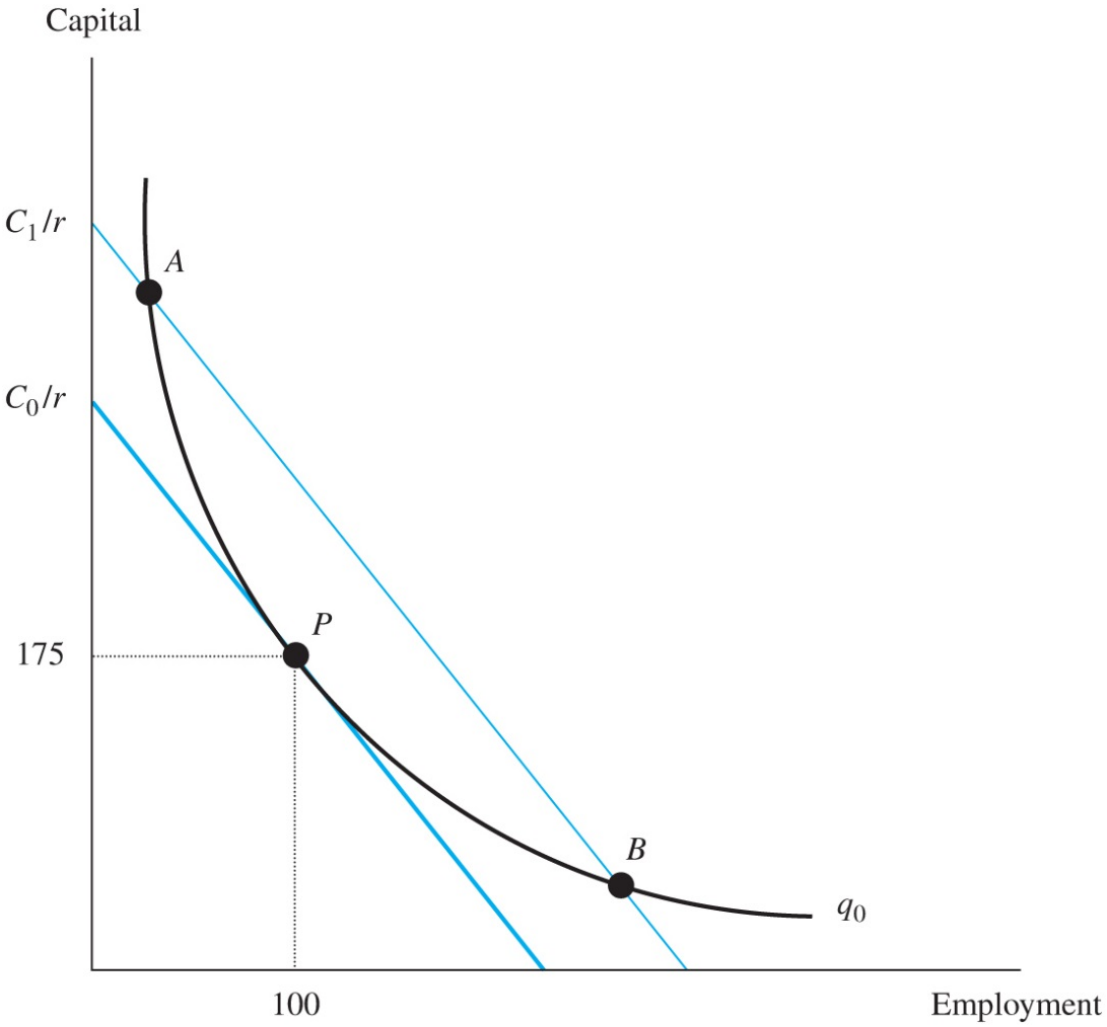
$$K = \frac{C}{r} - \frac{w}{r}E$$

Isocost Lines



All capital-labor combinations that lie on a single isocost curve are equally costly. Capital-labor combinations that lie on a higher isocost curve are more costly. The slope of an isoquant equals the ratio of input prices (-*w/r*).

The Firm's Optimal Combination of Inputs



A firm minimizes the cost of producing q_0 units of output by using the capital-labor combination at point *P*, where the isoquant is tangent to the isocost. All other capital-labor combinations (such as those given by points *A* and *B*) lie on a higher isocost curve.

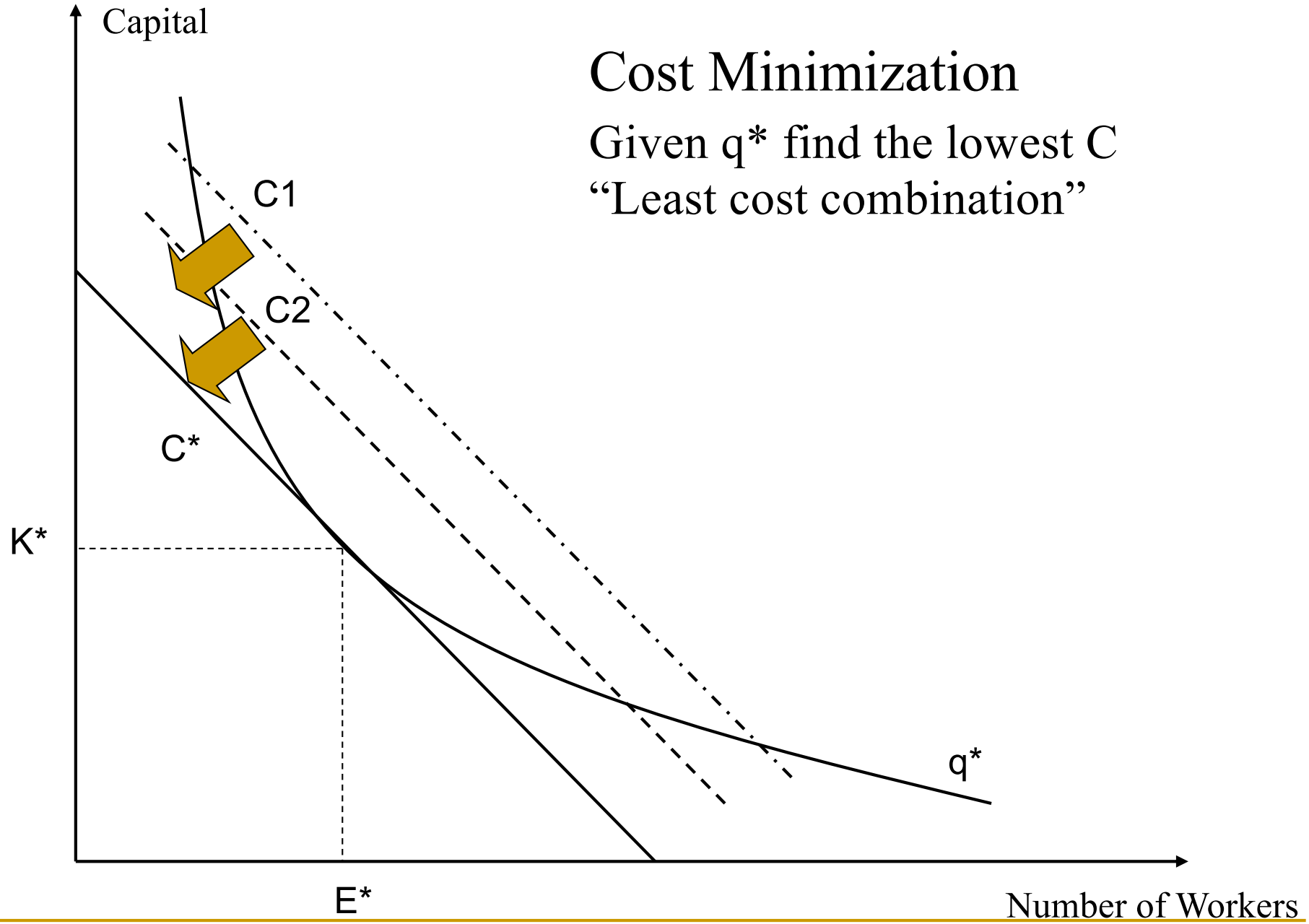
Cost Minimization

- Profit maximization implies cost minimization.
- The firm chooses the least-cost combination of capital and labor.
- This least-cost choice is where the isocost line is tangent to the isoquant.
- Marginal rate of substitution equals the ratio of input prices, w / r, at the least-cost choice.

Cost Minimization

Given q* find the lowest C

"Least cost combination"



At the cost-minimizing solution P, the slope of the isocost equals the slope of the isoquant

$$\frac{MP_E}{MP_K} = \frac{w}{r}$$

Cost minimization, therefore requires that MRTS = the ratio of input prices

$$\frac{MP_E}{w} = \frac{MP_K}{r}$$

Cost minimization requires that the last dollar spent on labor yield as much output as the last dollar spent on capital

The profit-maximizing condition that tells that firm how much capital to hire is obtained by equating the price of capital (r) and VMP_K

Therefore, long-run profit maximization requires that labor and capital be hired up to the point where

$$w = p \times MP_E$$

$$r = p \times MP_K$$

The Long Run Demand Curve for Labor

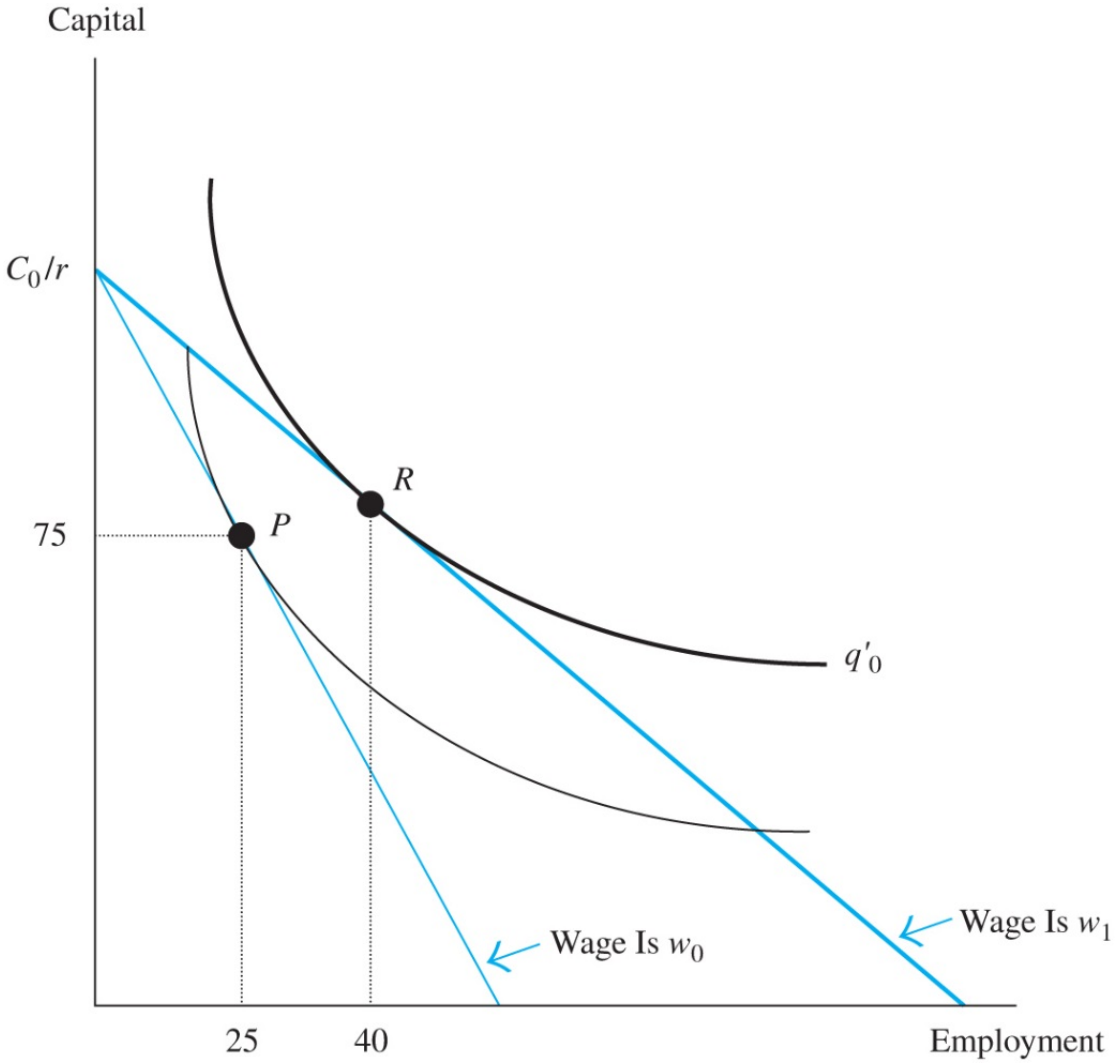
- We assume that this output is the profit-maximizing level of output, at the level of production, output price equals marginal cost

- A profit maximization firm will produce this output at the lowest cost possible

Long Run Demand for Labor

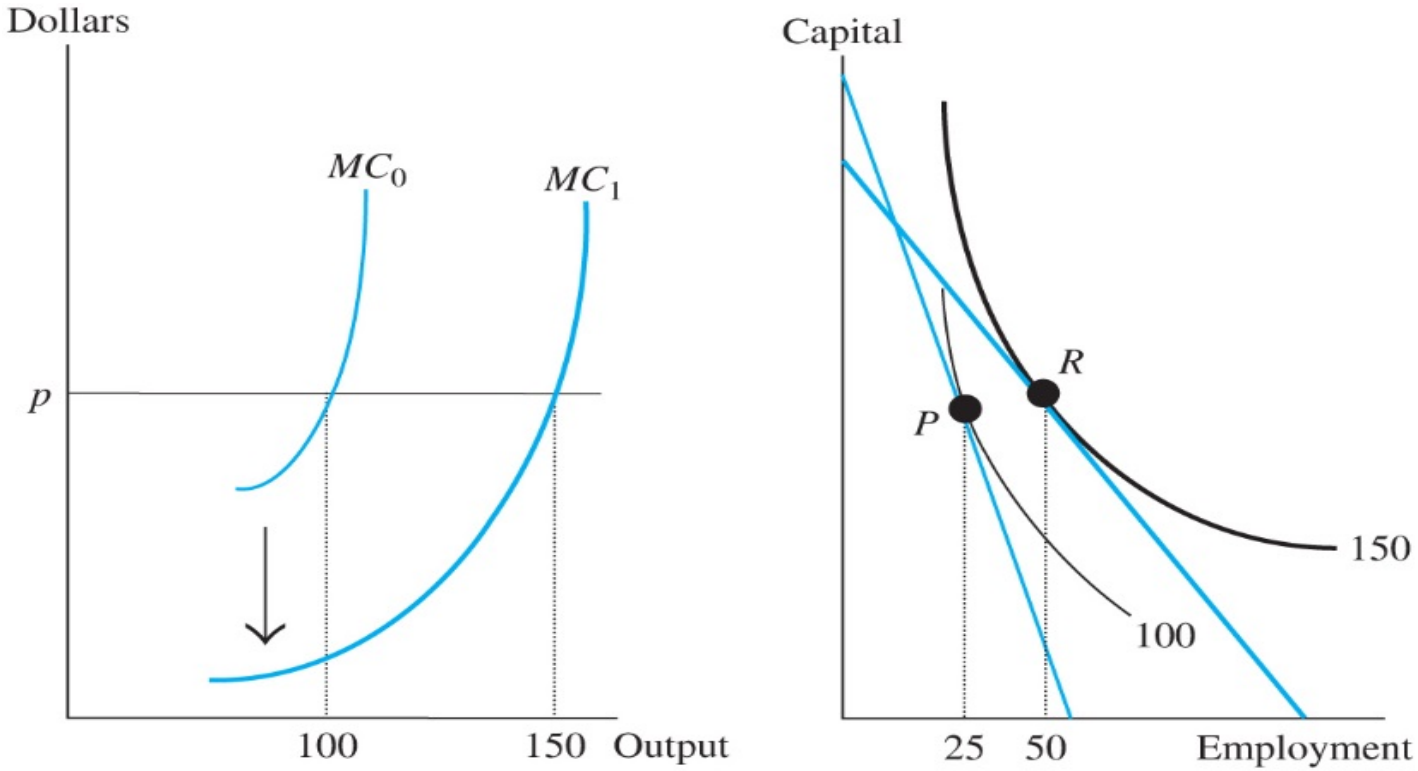
- When the wage drops, two effects arise.
  - The firm takes advantage of the lower price of labor by expanding production (the scale effect).
  - The firm takes advantage of the wage change by rearranging its mix of inputs even if holding output constant (the substitution effect)

The Impact of a Wage Reduction Holding Costs Constant



A wage reduction flattens the isocost curve. If the firm were to hold the initial cost outlay constant at C_0 dollars, the isocost would rotate around C_0 and the firm would move from point P to point R. A profit-maximizing firm, however, will not generally want to hold the cost outlay constant when the wage changes.

The Impact of a Wage Reduction on the Output and Employment of a Profit-Maximizing Firm



(*a*) Firm's Output Decision (*b*) Firm's Hiring Decision

•A wage cut reduces the marginal cost of production and encourages the firm to expand (from producing 100 to 150 units).

•The firm moves from point *P* to point *R*, increasing the number of workers hired from 25 to 50.

The impact of a wage reduction on the output and employment of a profit-maximizing firm

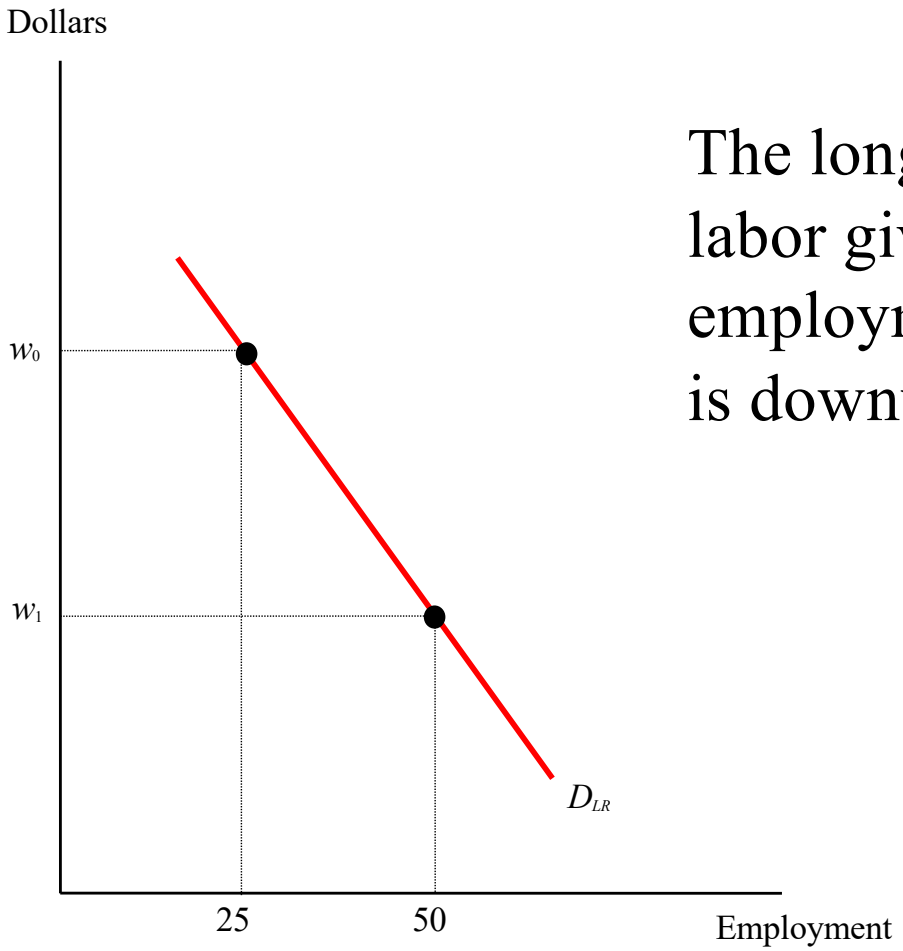
Firm's Output Decision

- A wage cut reduces the MC of production and encourages the firm to expand

Firm's Hiring Decision

- The firm increases the number of workers hired

Long Run Demand Curve for Labor



The long-run demand curve for labor gives the firm's employment at a given wage and is downward sloping.

Substitution and Scale Effects

A wage cut generates substitution and scale effects

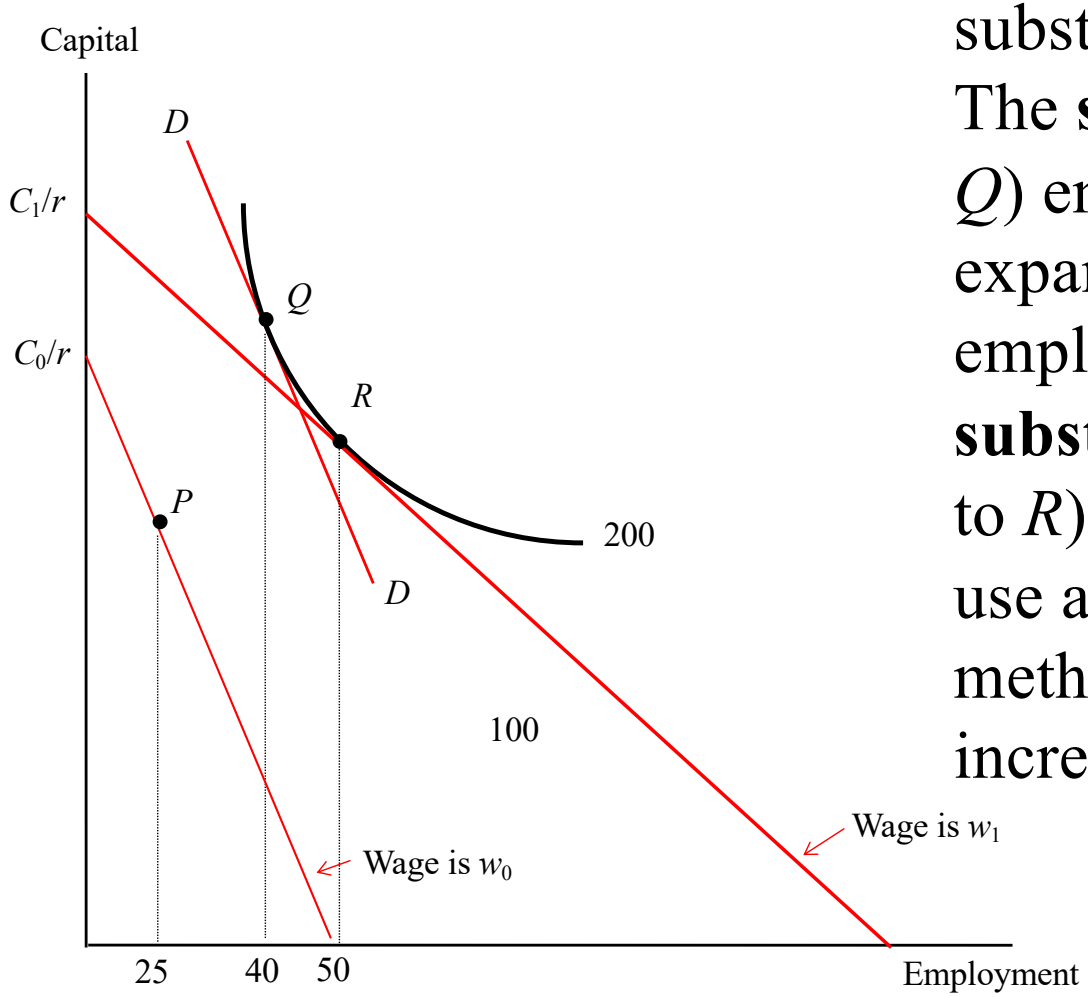
The scale effect

- The move from P to Q
- Encourages the firm to expand, increasing the firm's employment

The substitution effect

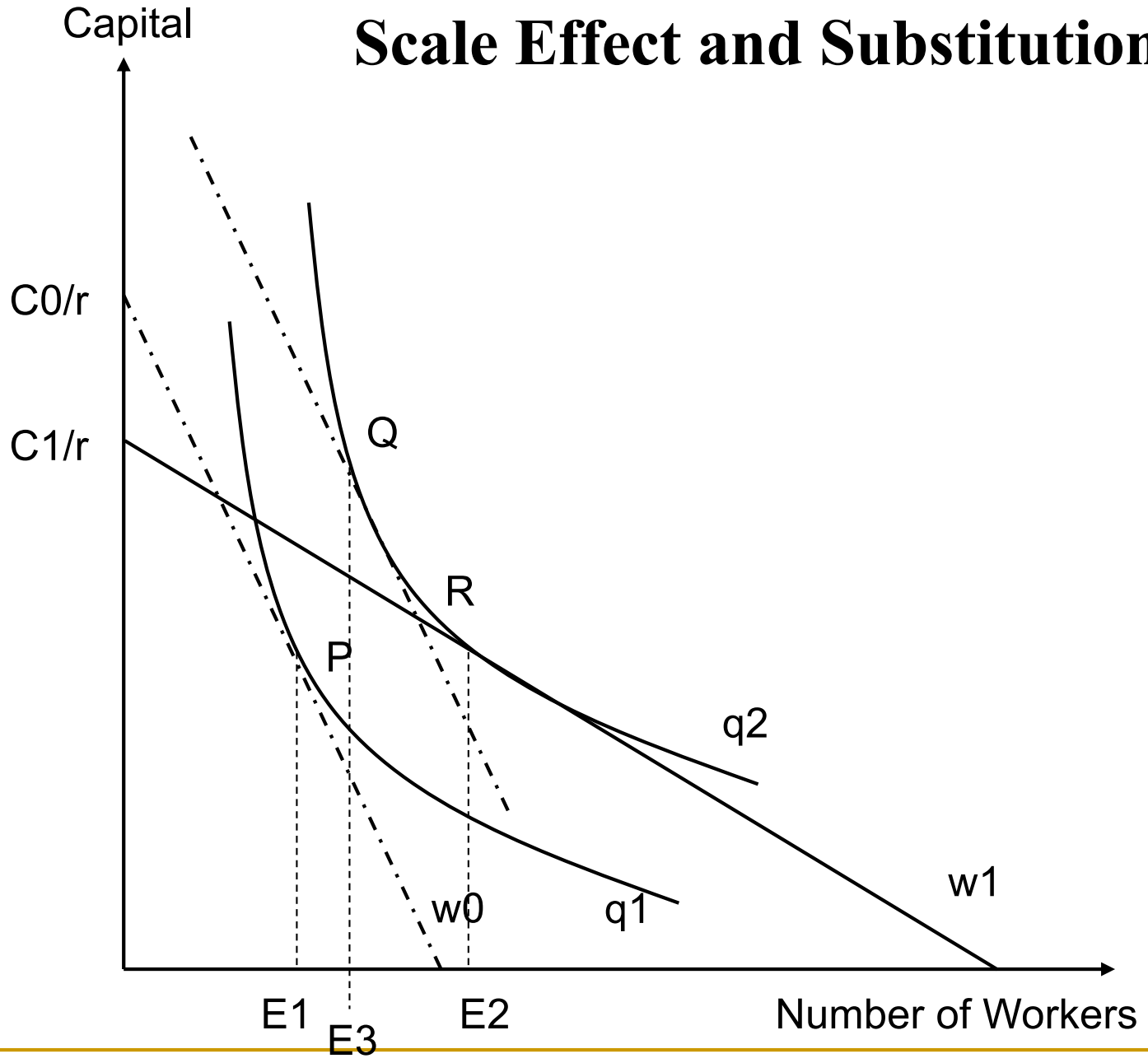
- The move from Q to R
- Encourages the firm to use a more labor-intensive method of production, further increasing employment

Substitution and Scale Effects



A wage cut generates substitution and scale effects. The **scale effect** (from *P* to *Q*) encourages the firm to expand, increasing the firm's employment. The **substitution effect** (from *Q* to *R*) encourages the firm to use a more labor-intensive method of production, further increasing employment.

Scale Effect and Substitution Effect

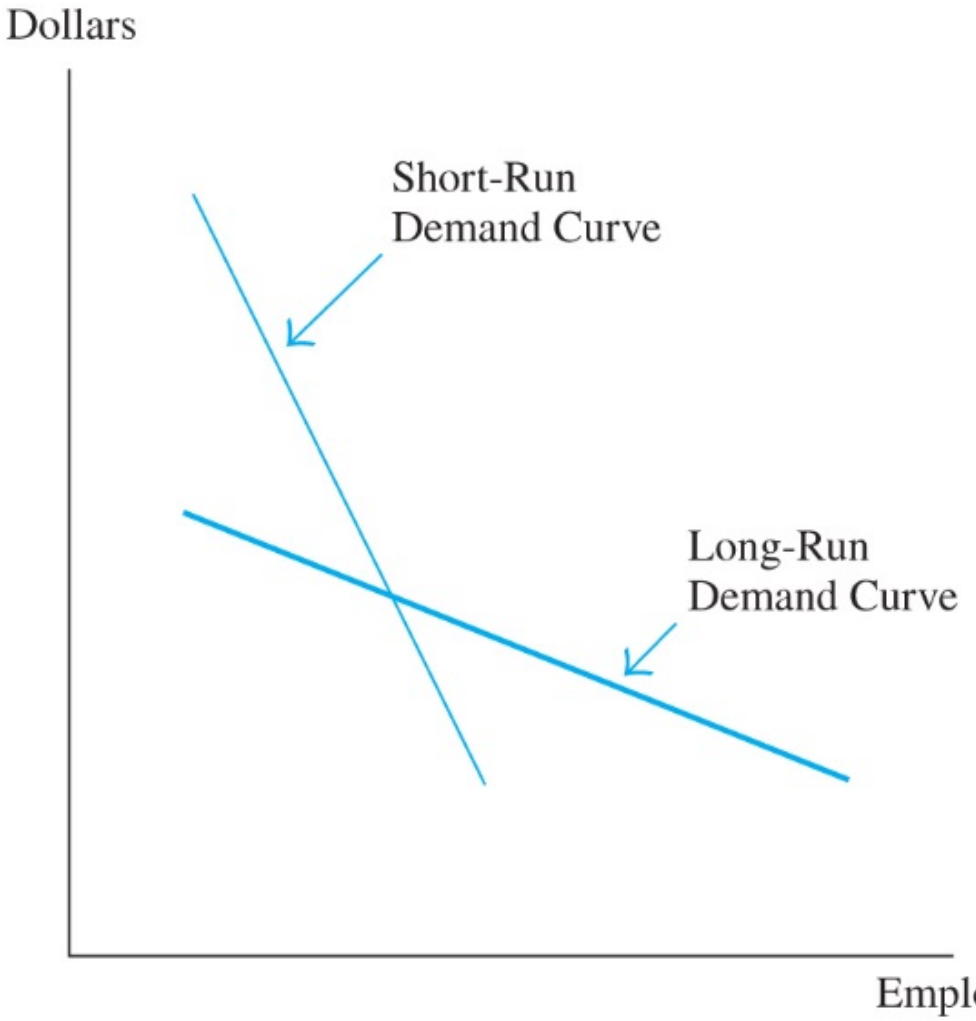


The Long-Run elasticity of labor demand

- Measure the responsiveness of changes in long-run employment to changes in the wage

$$\delta_{LR} = \frac{Percent\ \ change\ in\ employment}{Percent\ \ change\ in\ the\ wage} = \frac{\Delta E_{LR} / E_{LR}}{\Delta w / w} = \frac{\Delta E_{LR}}{\Delta w} \cdot \frac{w}{E_{LR}}$$

The Short- and Long-Run Demand Curves for Labor



In the long run, the firm can take full advantage of the economic opportunities introduced by a change in the wage. As a result, the long-run demand curve is **more elastic** than the short-run demand curve.

The Elasticity of Substitution

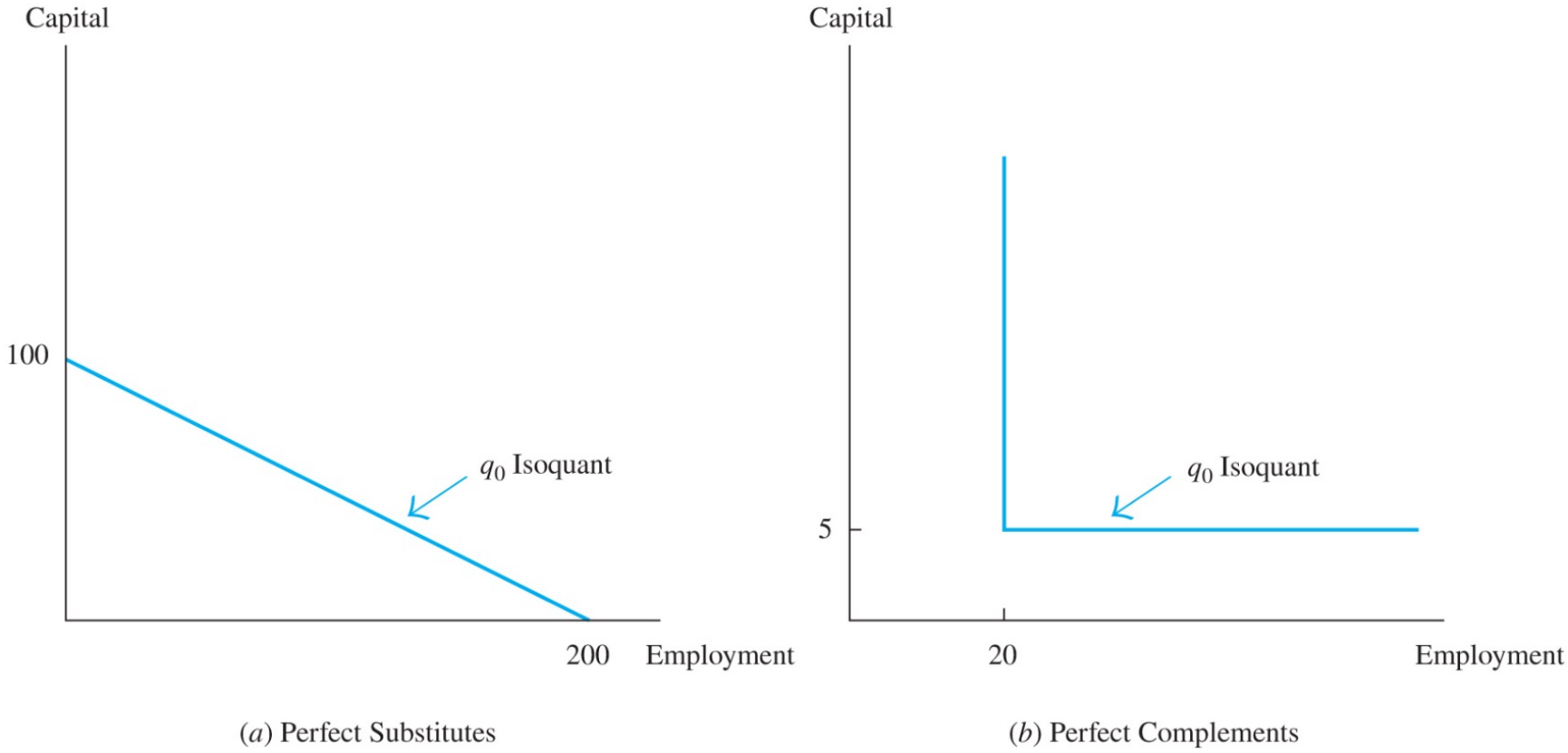
Perfect substitutes

- Whenever any two inputs in production can be substituted at a constant rate

Perfect complements

- When the isoquant between any two inputs is right-angled

Two Special Cases of Isoquants



Capital and labor are perfect substitutes if the isoquant is linear (so that two workers can always be substituted for one machine). The two inputs are perfect complements if the isoquant is right-angled. The firm then gets the same output when it hires 5 machines and 20 workers as when it hires 5 machines and 25 workers.

The elasticity of substitution

- To measure the curvature of the isoquant
- The elasticity of substitution between capital and labor (holding output constant) is defined by

$$Elasticity\ of\ substitution = \frac{Percent\ change\ in\ (K/E)}{Percent\ change\ in\ (w/r)}$$

- The elasticity of substitution gives the percentage change in the capital/labor ratio resulting from a 1 percent change in the relative price of labor. As the relative price of labor increases, the substitution effect tells us that the capital/labor ratio increases

Elasticity of Substitution

- Example:

  If the elasticity of substitution is 5, then a 10% increase in the ratio of wages to the price of capital would result in the firm increasing its capital-to-labor ratio by 50%.

About labor demand

- Always be **downward sloping**
- Decreasing in wage rate
  - Scale effect → Increase Production → increase employment
  - Substitution Effect → labor intensive → increase employment
- Factors will be reallocated
- More elastic of long run labor demand than short run labor demand

- Determination factors of elasticity of demand
  - Elasticity of substitution
  - Elasticity of demand for product
    - Increasing price → lower demand → decreasing labor (if labor intensive production)
  - Elasticity of supply for other inputs
    - Depend on substitutable between inputs

Marshall's Rule of Derived Demand

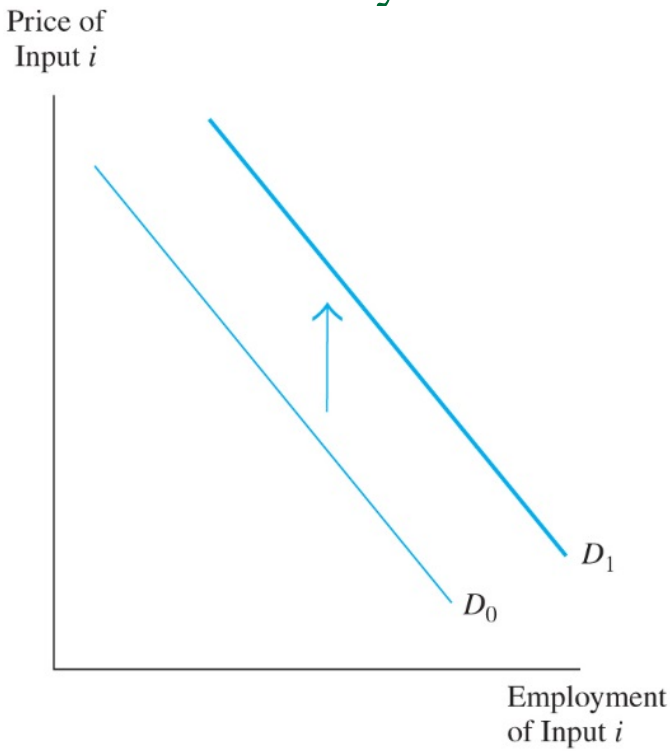
Labor Demand is more elastic when:

- The elasticity of substitution is greater.
- The elasticity of demand for the firm's output is greater.
- Labor's share in total costs of production is greater.
- The elasticity of supply of other factors of production such as capital is greater.

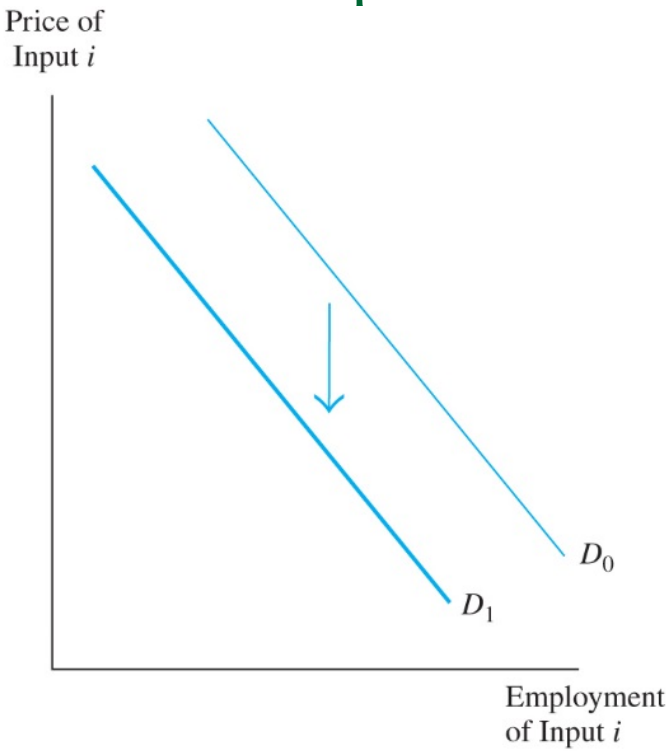
Factor Demands when there are Several Inputs

- There are many different inputs.
  - Skilled and unskilled labor
  - Old and young
  - Old and new machines
- Cross-elasticity of factor demand.
  - $\%\Delta D_i \div \%\Delta w_j$
  - If cross-elasticity is positive, the two inputs are said to be substitutes in production.

The Demand Curve for a Factor of Production is Affected by the Prices of Other Inputs



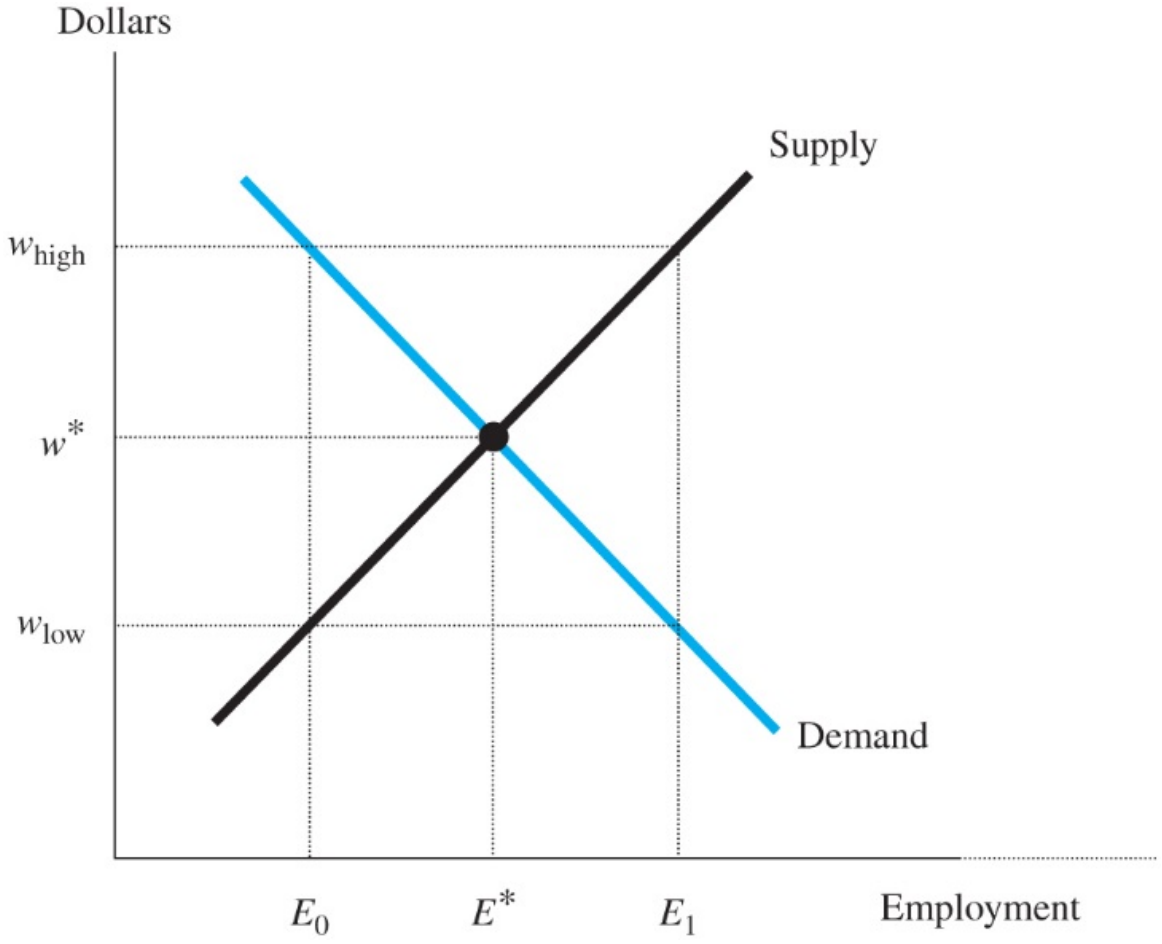
(a) inputs are i and j are substitutes



(b) inputs i and j are complements

The labor demand curve for input i shifts when the price of another input changes. (a) If the price of a substitutable input rises, the demand curve for input i shifts up. (b) If the price of a complement rises, the demand curve for input i shifts down.

Labor Market Equilibrium

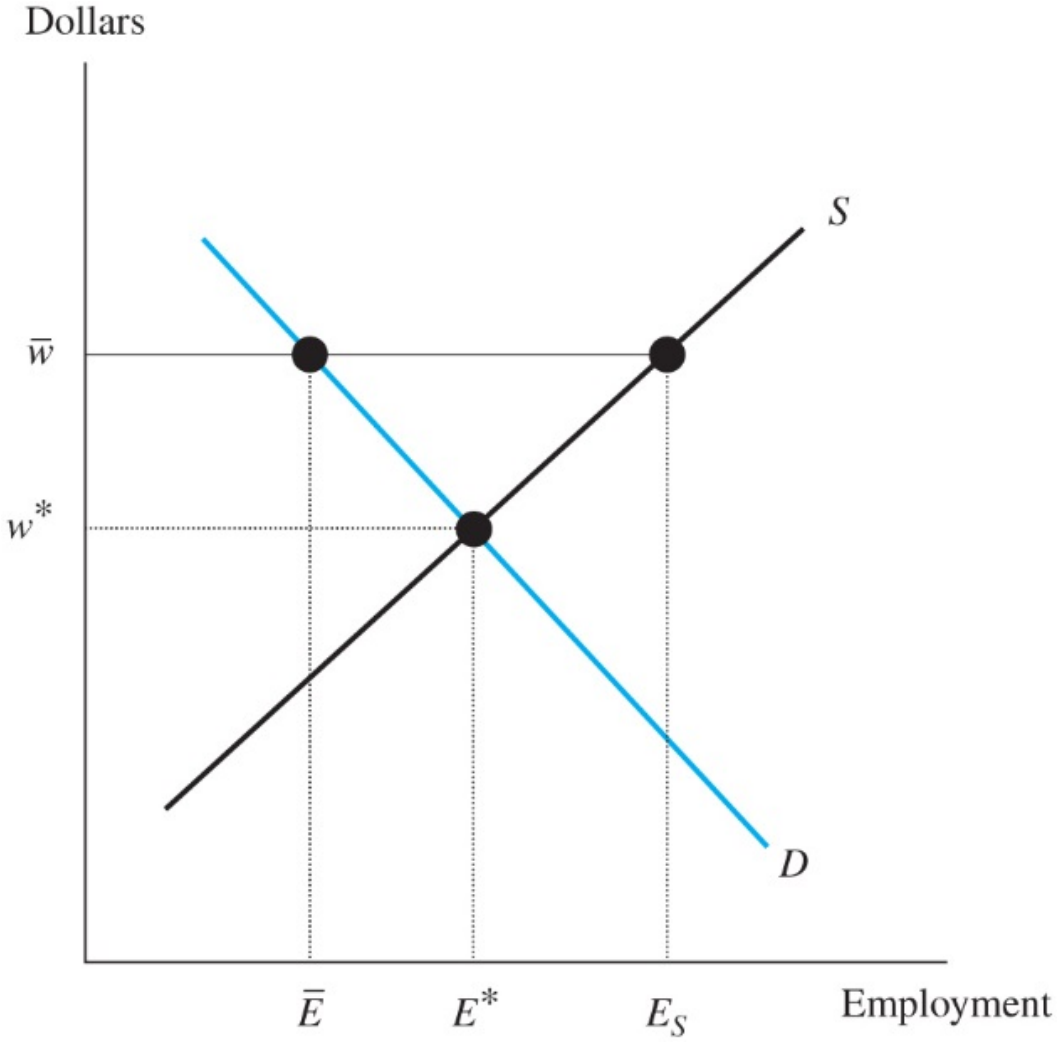


In a competitive labor market, equilibrium is attained at the point where supply equals demand. The market-clearing wage is w^* at which E^* workers are employed.

Application: The Employment Effects of Minimum Wages

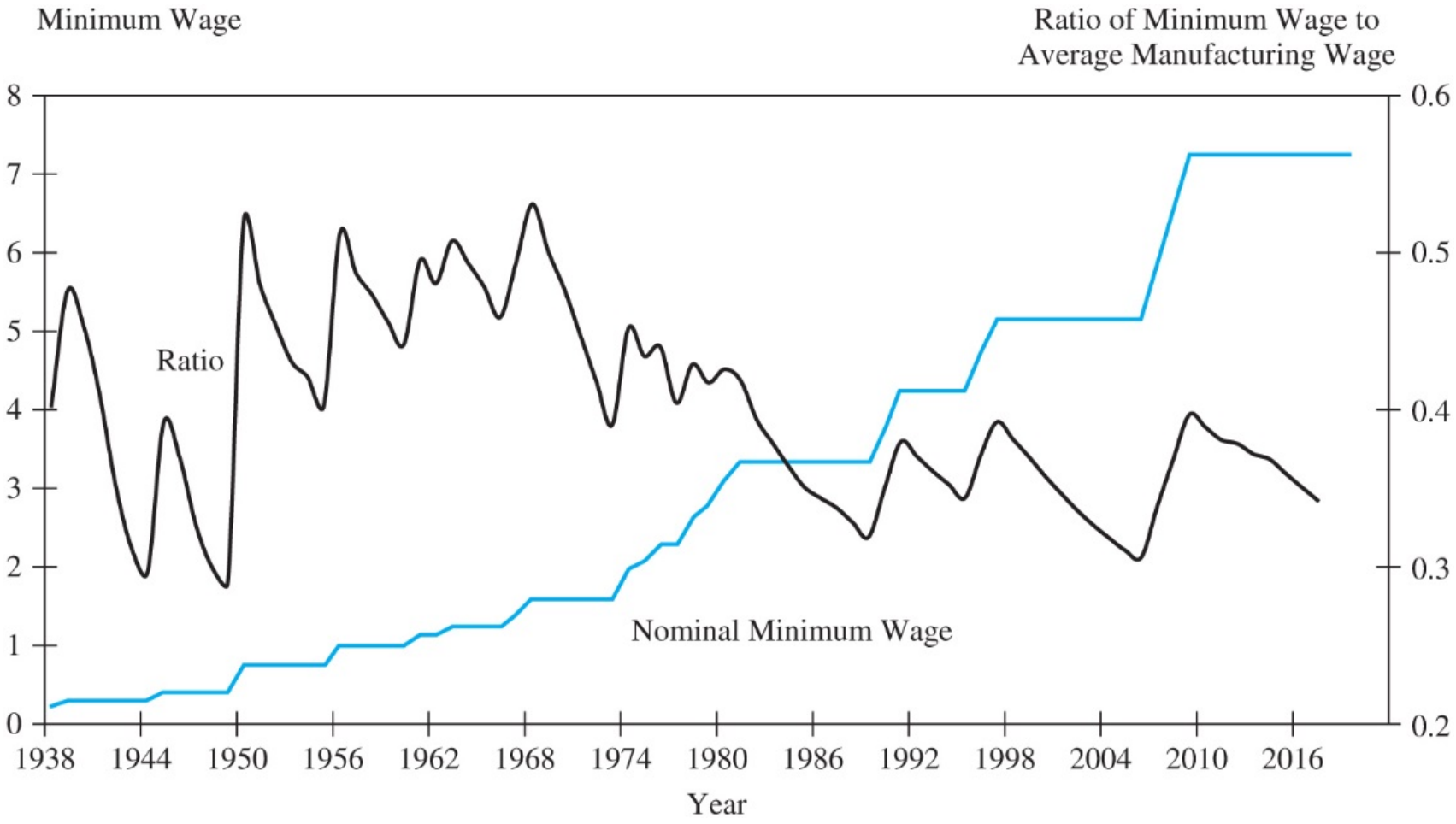
- The unemployment rate is higher the higher the minimum wage and the more elastic are the labor supply and demand curves.
- The benefits of the minimum wage accrue mostly to workers who are not at the bottom of the distribution of permanent income.

The Impact of the Minimum Wage on Employment

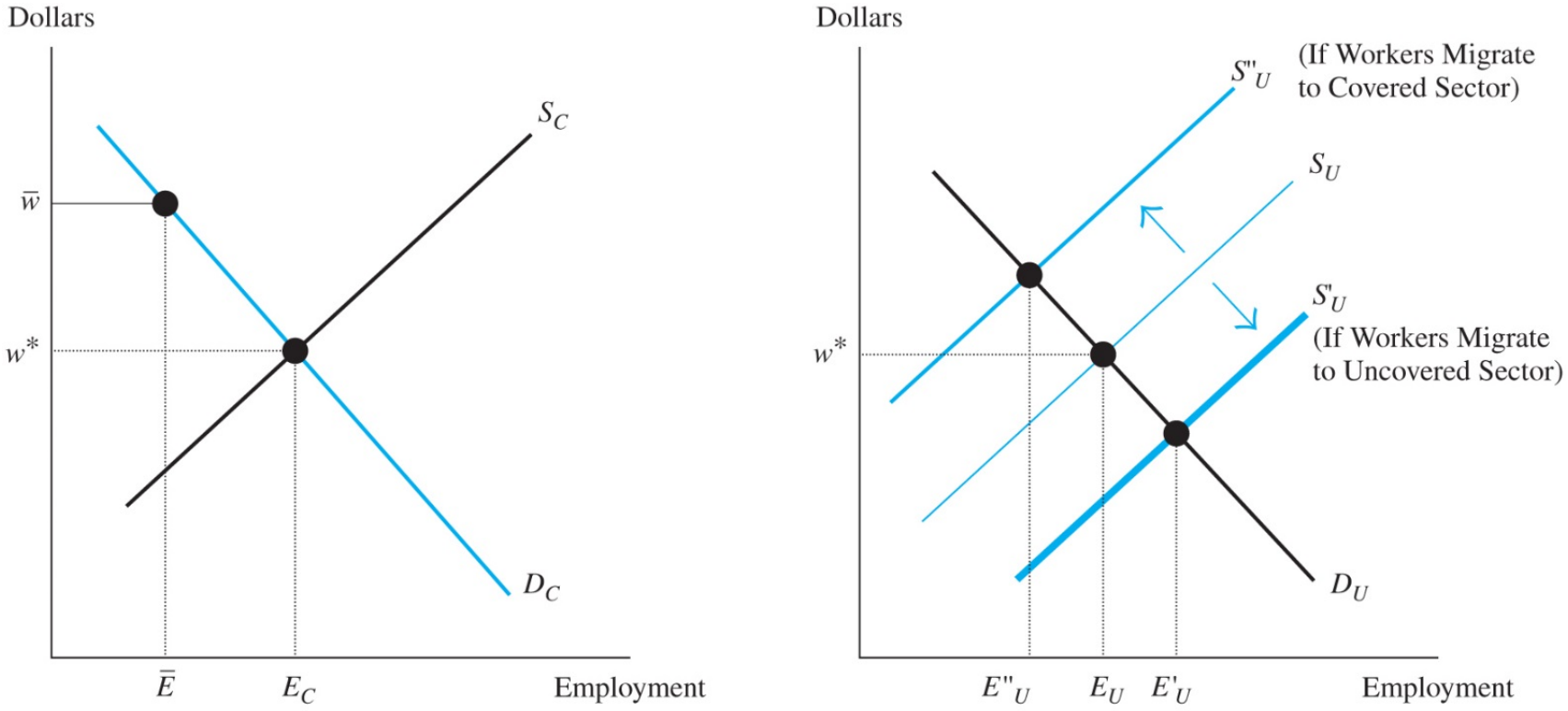


A minimum wage set at w^- results in employers cutting employment from E^* to E^-. The higher wage also encourages $E_S - E^*$ workers to enter the market. Thus, under a minimum wage, $E_S - E^-$ workers are unemployed.

Minimum Wages in the United States, 1938-2011



The Impact of Minimum Wages on the Covered and Uncovered Sectors



(*a*) Covered Sector (*b*) Uncovered Sector

If the minimum wage applies only to jobs in the covered sector, the displaced workers might move to the uncovered sector, shifting the supply curve to the right and reducing the uncovered sector's wage. If it is easy to get a minimum wage job, workers in the uncovered sector might quit their jobs and wait in the covered sector until a job opens up, shifting the supply curve in the uncovered sector to the left and raising the uncovered sector's wage.

The employment effect of minimum wages in New Jersey and Pennsylvania

Source: David Card and Alan B. Krueger, "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," *American Economic Review* 84 (September 1994), Table 3.

	Employment in Typical Fast-Food Restaurant (in full-time equivalents)	
	New Jersey	Pennsylvania
Before New Jersey increased the minimum wage	20.4	23.3
After New Jersey increased the minimum wage	21.0	21.2
Difference	0.6	−2.1
Difference-in-differences	**2.7**	

Discussion ☺☺

- Should the government increase the minimum wage? Why?
- Is the minimum wage an effective antipoverty program? Why?