Chapter 5

Nonlinear Model and Differential Calculus in Economic Theory

Topics:

- Slope and derivatives of a function
- Rule of differentiation
- Non differentiable functions
- Convexity and concavity
- Inflection point
- Maxima-Minima
- Examples in Economics
    - ➲ Relations among the total, the average and the marginal functions
    - ➲ Derivative and marginality
    - ➲ Elasticity

Comparative Statics and Derivative

VOCAB: "Difference Quotient" / "Derivative"/ "Differentiation"/ "Differential Calculus"

Comparative Statics is concerned with the comparison of different equilibrium states that are associated with different sets of values of parameters and exogenous variables.

For example,

When $G = G_0$

$$\left.\begin{aligned} &(1.)\ Y = C + I + G \\ &(2.)\ C = a + bY_d \\ &(3.)\ Y_d = Y - T \\ &(4.)\ I = I_0 \\ &(5.)\ G = G_0 \end{aligned}\right\} \Rightarrow \boxed{Y_E\big|_{G=G_0} = \frac{a - bT + I_0 + G_0}{1-b}}$$

$$Y_E = f(\text{exo}, \text{parameters})$$

$$\frac{\partial Y_E}{\partial\, \text{exo}}, \quad \frac{\partial Y_E}{\partial\, \text{parameter}}$$

$$\rightarrow \frac{\partial Y_E}{\partial a}, \quad \frac{\partial Y_E}{\partial b}, \quad \frac{\partial Y_E}{\partial T_0}, \quad \frac{\partial Y_E}{\partial I_0}, \quad \frac{\partial Y_E}{\partial G_0}$$

When $G = G_1$

$$\left.\begin{aligned} &(1.)\ Y = C + I + G \\ &(2.)\ C = a + bY_d \\ &(3.)\ Y_d = Y - T \\ &(4.)\ I = I_0 \\ &(5.)\ G = G_1 \end{aligned}\right\} \Rightarrow \boxed{Y_E\big|_{G=G_1} = \frac{a - bT + I_0 + G_1}{1-b}}$$

$$\begin{aligned} &\textit{when} \quad G_0 \quad \rightarrow \quad Y_E\big|_{G_0} \\ &\textit{when} \quad G_1 \quad \rightarrow \quad Y_E\big|_{G_1} \end{aligned}$$

$$\Delta G = G_1 - G_0 \quad \Rightarrow \quad \Delta Y = Y_E\big|_{G_1} - Y_E\big|_{G_0}$$

It should be clear that the problem under consideration is essentially one of finding *a rate of change: the rate of change of the equilibrium value of an endogenous variable with respect to the change in a particular parameter or exogenous variable.*

The notion of rate of change is directly concerned with the mathematical concept of *derivative,* in *differential calculus*.

Looking at a function: $y = f(x)$

$$
\begin{array}{lll}
when & x_1 \rightarrow & y_1 = f(x_1) \\
when & x_2 \rightarrow & y_2 = f(x_2)
\end{array}
$$

$$\Delta x = x_2 - x_1 \quad \Rightarrow \quad \Delta y = y_2 - y_1$$

We are interested in $\frac{\Delta y}{\Delta x}$ which is called " difference quotient"

When $\Delta x \to 0, \frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$.

That is, $\lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{dy}{dx} = f'(x)$

$\frac{dy}{dx}$ $or\ f'(x)$ is called the derivative of f at x.

Notation for derivative of f at x: $f'(x), \frac{df(x)}{dx}, \frac{dy}{dx}, D_x f(x), y'$.

A *derivative* is a function. The word *derivative* means *a derived function*, from the original function $y = f(x)$, which is also called a *primitive function*.

When we have a Primitive function $y = f(x)$ and try to find the derived function $\frac{dy}{dx}$, we call this process "differentiation".

Derivative and differential Calculus will be used in finding maximum, minimum points and in optimization problem.

The Slope of a Curve and The Derivative of a function

The concept of the **slope** of a curve is the geometric counterpart of the concept of the **derivative**. Both concepts deal with the **marginal** notion used in economics.

The slope of a total cost function: $C = F(q)$ measures the change in total cost resulting from a unit increase in output, i.e. the marginal cost(MC). That is,

$$MC(Q) = \frac{\Delta C}{\Delta q}$$

And when $\Delta q \to 0$

$$MC(Q) = \frac{\mathrm{d}C}{\mathrm{d}q}$$

The slope of a utility function: $u = U(x)$ measures the change in utility resulting from a unit increase in consumption, i.e. the marginal utility(MU). That is,

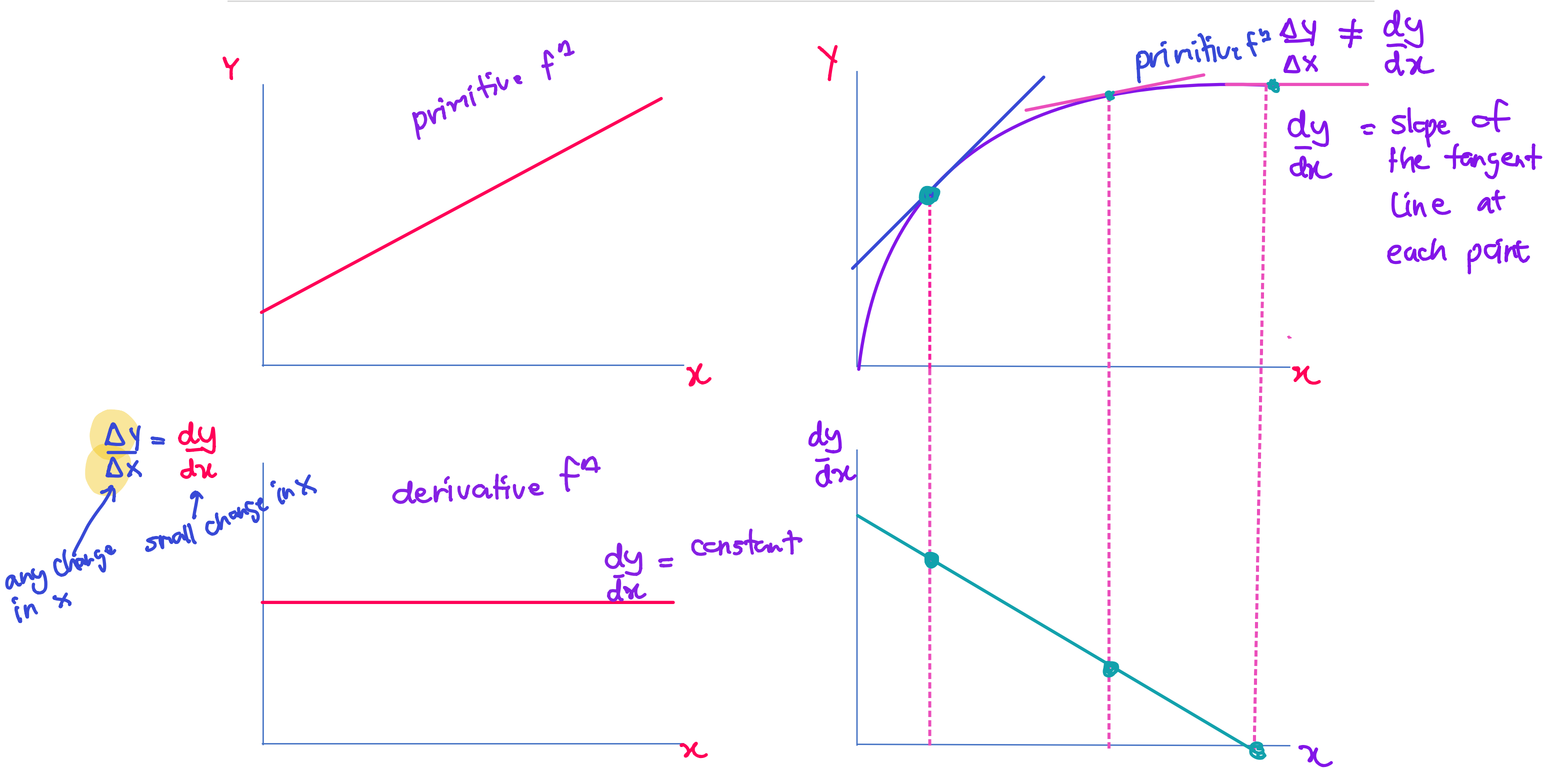
$$MU(x) = \frac{\Delta u}{\Delta x}$$

And when $\Delta x \to 0$

$$MU(x) = \frac{\mathrm{d}u}{\mathrm{d}x}$$

⦿ If a function is linear, the slope is constant and is equal for every points on the linear curve.

⦿ If a function is nonlinear function, the slope is not constant. Slope for each point on the curve might not be equal. Slope at each point on a nonlinear function is the slope of the tangent line at that point.



Differentiation / derivation / process of obtaining the derivative

From definition of derivative, we can find the derivative from a primitive function as the following.

For example, let $f(x)=2x^2+4$ find $f'(x)$:

(1.) $f'(x)=4x$

(2.) Use derivative definition: $f'(x)=\lim\limits_{\Delta x\to 0}\left(\dfrac{\Delta y}{\Delta x}\right)$

Let x increases from x_1 to $x_2=x_1+h$, so $\Delta x=x_2-x_1=h$ with

$$f(x_1)=2x_1^2+4 \text{ และ } f(x_2)=2x_2^2+4=2(x_1+h)^2+4=2x_1^2+4x_1h+2h^2+4$$

Therefore,

$$\begin{aligned}\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}&=\frac{f(x_2)-f(x_1)}{h}\\ &=\frac{(2x_1^2+4x_1h+2h^2+4)-(2x_1^2+4)}{h}\\ &=4x_1+2h\end{aligned}$$

x_1 is actually can be any x $\frac{\Delta y}{\Delta x} = 4x + 2h$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) \\ &= \lim_{h \to 0}(4x + 2h) \\ &= 4x \qquad \# \end{aligned}$$

Differentiability of a function

"A function is differentiable at point x_0 if it is smooth and continuous at point x_0."

✪ Continuity

🔔 Continuity is a necessary condition for a function to be differentiable.

The function f is continuous at a if

1. We can find $f(a)$, i.e. $x = a$ must be in the domain of the function f.
2. We can find $\lim_{x \to a} f(x)$
3. $\lim_{x \to a} f(x) = f(a)$

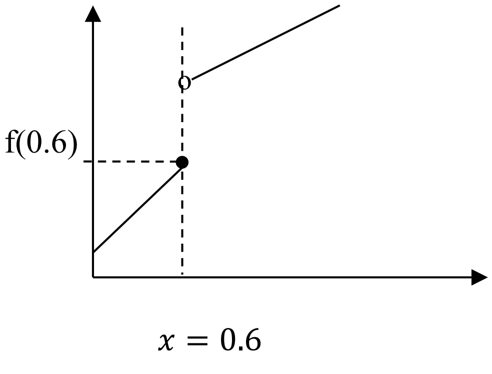
Example of a function that is not continuous at $x = 0.6$

$f(x)$

เนื่องจาก

1. $f(0.6)$ can be found.
2. $\lim_{x \to 0.6^-} f(x) \neq \lim_{x \to 0.6^+} f(x)$

$\therefore f(x)$ is not continuous at $x = 0.6$



H.w.: Is this function continuous $f(x) = \begin{cases} x^2 & when\ x < 2 \\ x + 1 & when\ x \geq 2 \end{cases}$?

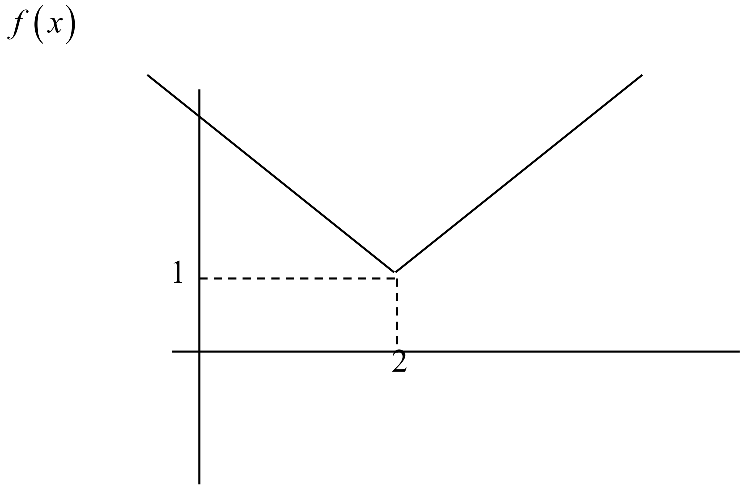
✪ Smooth Function(has no kink)

🔔 Smooth function is a sufficient condition for differentiability.

The differentiability condition is:

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Example of a function that has kink: $f(x) = |x-2| + 1$



Continuity: $f(2) = \lim_{x \to 2} f(x) = 1$, $f(x)$ is continuous at $x = 2$

But if we try to find derivative:

Function f has derivative at a if:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If we let $x = a + h$

when $h \to 0$, $x \to a$ and we can rewrite $f'(a)$ as:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad , \quad x \neq a$$

Derivative of $f(x)$ at $x = 2$ is:

$$f'(2)=\lim_{x\to 2}\frac{|x-2|+1-1}{x-2}=\lim_{x\to 2}\frac{|x-2|}{x-2}$$

$$where\ \lim_{x\to 2^-}\frac{|x-2|}{x-2}=\frac{-(x-2)}{x-2}=-1$$

$$\lim_{x\to 2^+}\frac{|x-2|}{x-2}=\frac{x-2}{x-2}=1$$

$$\therefore \lim_{x\to 2^-}\frac{|x-2|}{x-2}\neq \lim_{x\to 2^+}\frac{|x-2|}{x-2}$$

Therefore, $f'(2)=\lim_{x\to 2}\frac{|x-2|}{x-2}$ cannot be found.

Even $f(x)$ is continuous at $x=2$ but we cannot find $f'(2)$ because $f(x)$ has a kink at $x=2$.

Notation:

$f\in c^{(0)}$ or $f\in c$: means f is continuous.

$f\in c^{(1)}$ or $f\in c'$: means f is continuously differentiable (A function f with a continuous derivative function, i.e. the everywhere-smooth function)

Rule of Differentiation

1. If $f(x)=c$, c is a constant $f'(x)=0$

2. If $f(x)=cg(x)$, c is a constant $f'(x)=cg'(x)$

3. If $f(x)=x^n$, n is any real number $f'(x)=nx^{n-1}$

4. If $f(x)=U(x)\pm V(x)$, $f'(x)=U'(x)\pm V'(x)$

5. If $f(x)=U(x)V(x)$, $f'(x)=U(x)V'(x)+V(x)U'(x)$

6. If $f(x)=\frac{U(x)}{V(x)}$, $f'(x)=\frac{V(x)U'(x)-U(x)V'(x)}{[V(x)]^2}$

7. [chain rule] If we have a differentiable function $z=U(y)$ and another differentiable function $y=V(x)$, then the derivative of z with respect to x is equal to the derivative of z with respect to y, times the derivative of y with respect to x .

$$\frac{dz}{dx}=\frac{dz}{dy}\frac{dy}{dx}=U'(y)V'(x)$$

Change in x determines change in y via function V, and change in y determines change in z via function U.

8. [Derivatives of Inverse Function] Let $y = f(x)$, we have $\frac{dy}{dx}$. The inverse function of $f, x = f^{-1}(y)$, its derivative is $\frac{dx}{dy}$, and

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

9. [Derivatives of log function]

If $y = \log_a x$, $\frac{dy}{dx} = \frac{1}{xlna}$.

If $y = ln\, x$, $\frac{dy}{dx} = \frac{1}{xlne} = \frac{1}{x}$.

If $y = ln\, V(x)$, $\frac{dy}{dx} = \frac{V'(x)}{V(x)}$.

10. [Derivatives of exponential function]

If $y = a^x$, where $a > 0, a \neq 1$, $\frac{dy}{dx} = a^x lna$

If $y = e^x$, $\frac{dy}{dx} = e^x$

If $y = e^{V(x)}$, $\frac{dy}{dx} = e^{V(x)}V'(x)$

H.w.:

(a.) find $f'(x)$ for the following functions:

$f(x) = \sqrt{2}x^{\frac{-2}{3}}$

$f(x) = 2x^3 + 3x^2 - 5x + 1$

$f(x) = (2x+3)(3x^2)$

$f(x) = \frac{ax^2 + b}{cx}$

$f(x) = (x - 4x^2)^3$

$f(x) = (1 - x^2)\sqrt{1 - 2x}$

(b.)

If $y = f(x) = 5x + 25$, find $\dfrac{dx}{dy}$

If $y = f(x) = x^5 + x$, find $\dfrac{dx}{dy}$

(c.)

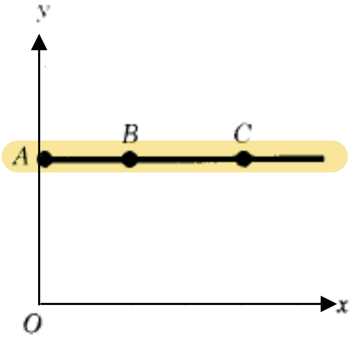
Find $f'(x)$ if $f(x) = e^{3x-1}$
Find $f'(x)$ if $f(x) = \ln(1 + 2x + x^2)$
Find $f'(x)$ if $f(x) = (1+x)(1+e^{x^2})(3-x)^{\frac{1}{2}}$
Find $f'(x)$ if $f(x) = (x^2 - 1)\sqrt{\dfrac{2-3x^2}{1-2x^3}}$

(d.) Let $y = x^4 - x^{\frac{4}{3}} + 6x^{\frac{1}{3}}$, find the second derivative of $f(x)$ with respect to x, $f''(x)$.

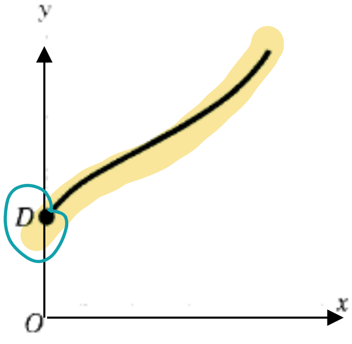
Maxima and Minima, Convexity and Concavity

Global vs. Local Extremum Concept

Consider (a), (b) , (c) and (d)



(a)



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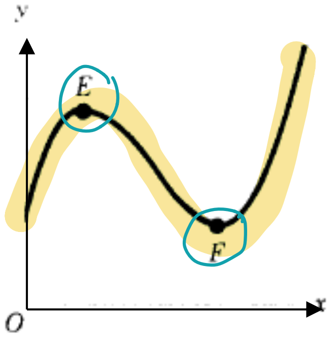


(c)

Graph(a.): Constant Function

No maximum/minimum of y

$\frac{dy}{dx} = 0$

Graph (b.): Strictly Increasing Function

When x increases, y increases. There is no maximum value of y, and the minimum value of y is at point D.

Note:

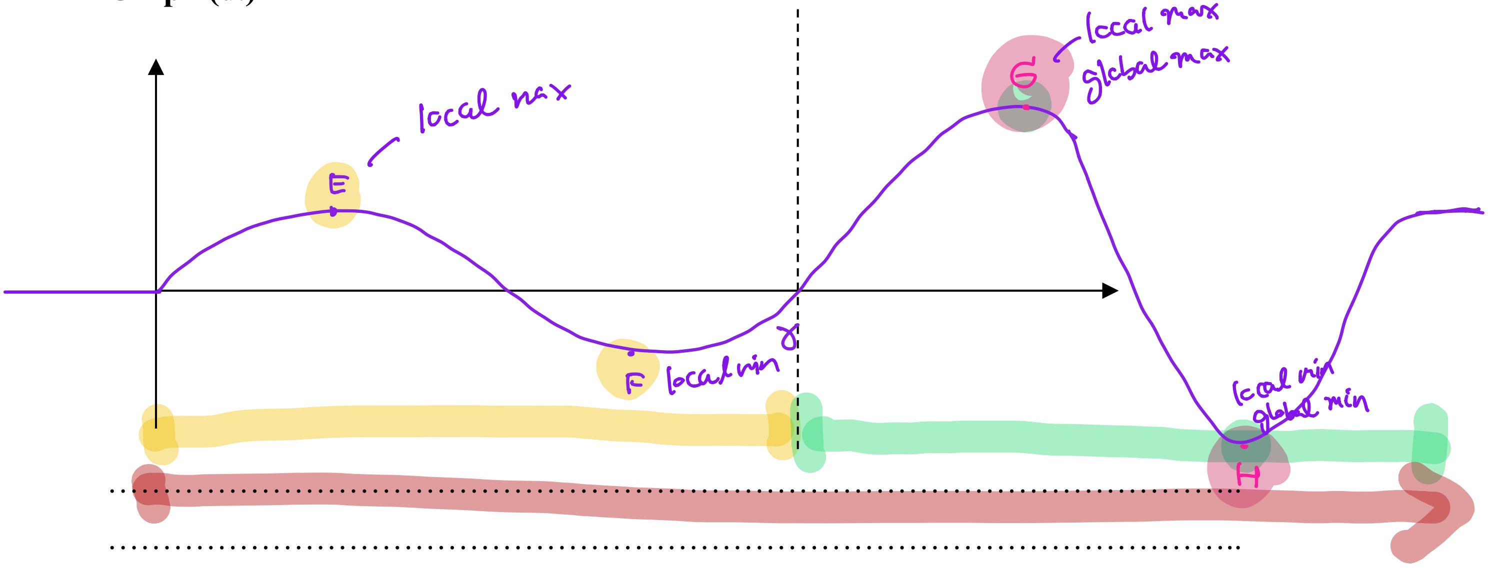
Monotonic increasing function is when $\frac{dy}{dx} \geq 0$.

Strictly monotonic increasing function is when $\frac{dy}{dx} > 0$.

Monotonic decreasing function is when $\frac{dy}{dx} \leq 0$.

Strictly monotonic decreasing function is when $\frac{dy}{dx} < 0$.

Graph (d.)



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How can we test if a critical point is a local max, a local min, or an inflection point?

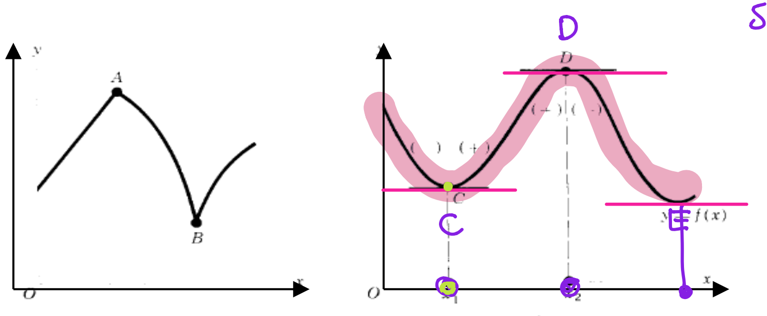
✪ First-Derivative Test ✪

If y = f(x) has Local Max/ Local Min at $x = x_0$, possible cases for the first derivative are:

(a.) $f'(x_0)$ *does not exist*

(b.) $f'(x_0) = 0$

C, D, E are stationary points



critical point / value

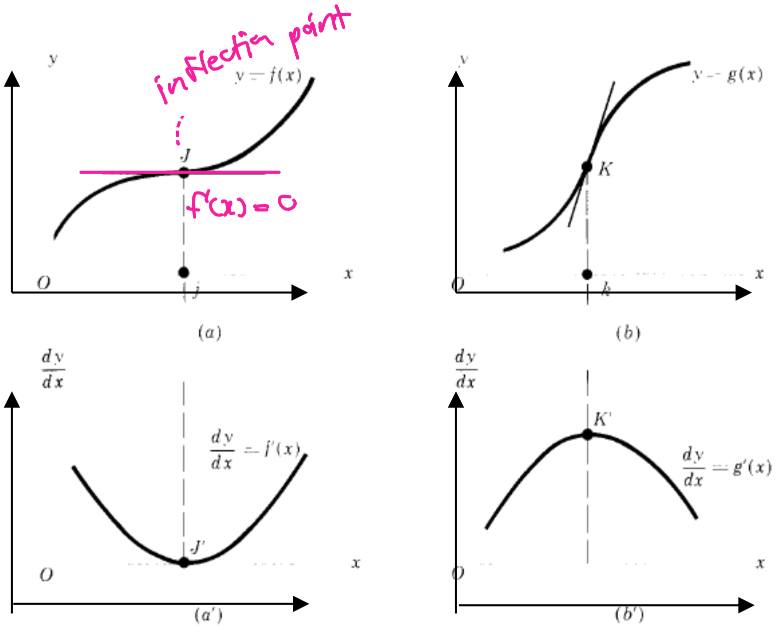
If $f(x)$ **is a smooth function, the necessary condition** for a point to be a local max or local min is:

$$\frac{df(x)}{dx} = \frac{dy}{dx} = f'(x) = 0$$

as x changes in an extremely small unit f(x) doesn't change.

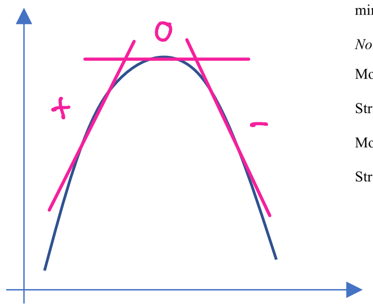
We call the order pair (x, y) that satisfies this necessary condition a stationary point, in which x is "critical point or critical value", and y is "stationary value".

The condition $f'(x) = 0$ is a necessary condition, but not a sufficient condition) for a point to be a local maximum or local minimum.



If the first derivative of f(x) at $x = x_0$ is 0 $(f'(x_0) = 0)$, then $f(x_0)$ will be

1. **Local maximum,** when the first derivative changes from being positive to negative when $x < x_0$ and $x > x_0$



teach

Graph (b.): Strictly Increasing Function

When x increases, y increases. There is no maximum value of y, and the minimum value of y is at point D.

Note:

Monotonic increasing function is when $\frac{dy}{dx} \geq 0$.

Strictly monotonic increasing function is when $\frac{dy}{dx} > 0$.

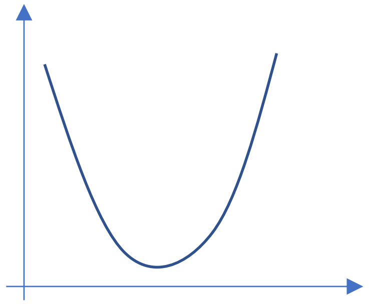
Monotonic decreasing function is when $\frac{dy}{dx} \leq 0$.

Strictly monotonic decreasing function is when $\frac{dy}{dx} < 0$.

when $x < x_0$, ………………………………………………………………………..

when $x > x_0$, ………………………………………………………………………..

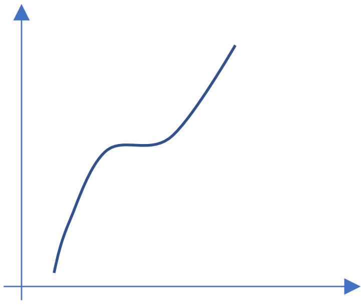
2. **Local minimum,** when the first derivative changes from being negative to positive when $x < x_0$ and $x > x_0$



when $x < x_0$, ………………………………………………………………………..

when $x > x_0$, ………………………………………………………………………..

3. **Inflection point,** when the first derivative doesn't change sign when $x < x_0$ and $x > x_0$



when $x < x_0$, ……………………………………………………………………..

when $x > x_0$, ……………………………………………………………………..

H.w.: Find local maxima or minima of the following functions:

$$y = f(x) = x^3 - 12x^2 + 36x + 8$$

Average-Cost Function $AC = f(Q) = Q^2 - 5Q + 8$

✪ Second-Derivative Test ✪

Interpretation of the Second Derivative

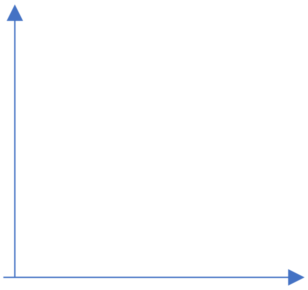
$f'(x)$ ……………………………………………………

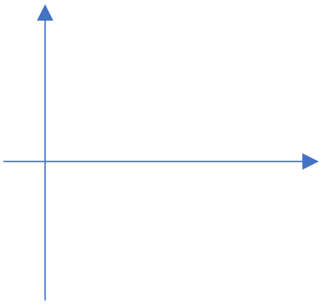
$f''(x)$ ……………………………………………………

$$\left.\begin{array}{l} f'(x) \;\; > 0 \\ f'(x) \;\; < 0 \end{array}\right\} \;\text{f(x) will be}\; \left.\vphantom{\begin{array}{l} a \\ b \end{array}}\right\}$$

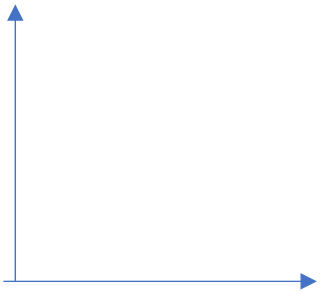
$$\left.\begin{array}{l} f''(x) > 0 \\ f''(x) < 0 \end{array}\right\} \;\ldots\ldots\ldots\ldots\text{f(x) will be}\; \left.\vphantom{\begin{array}{l} a \\ b \end{array}}\right\}$$

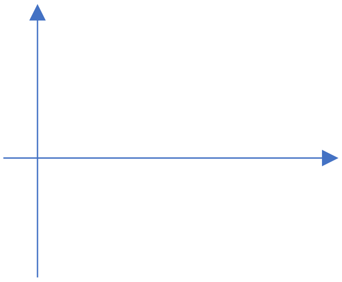
If $f'(x)$ > 0 and $f''(x) > 0$,



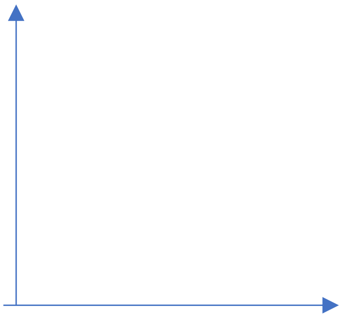


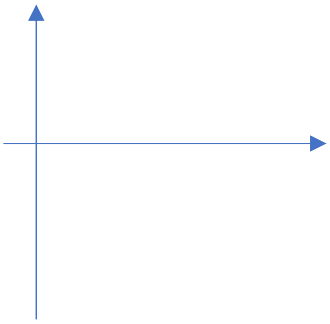
If $f'(x)$ > 0 and $f''(x) < 0$,



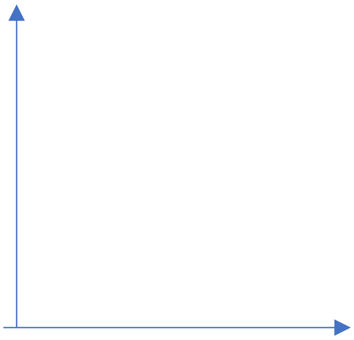


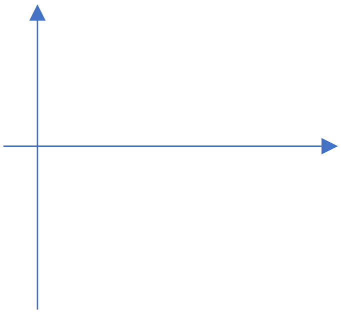
If $f'(x)$ < 0 and $f''(x) > 0$,





If $f'(x)$ < 0 and $f''(x) < 0$,





Curvature of a Graph

A graph can be:

→ Strictly Convex/ Convex

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→ Strictly Concave/Concave

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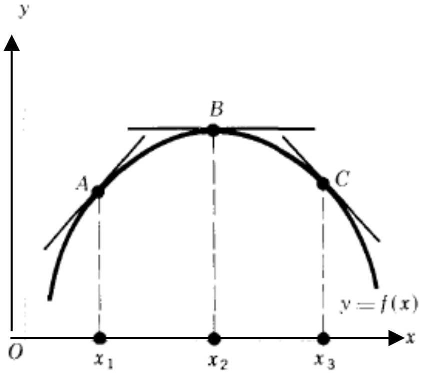
$f(x)$ is strictly convex if a linear line between any two points M and N lies above $f(x)$

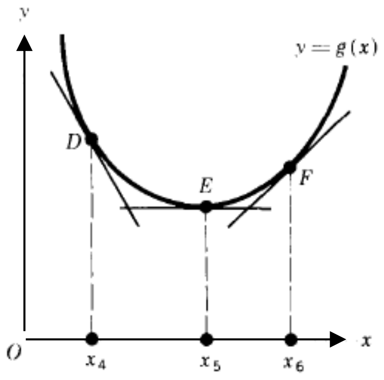
$f''(x)$…………………. for strictly convex function

$f(x)$ is strictly concave if a linear line between any two points M and N lies below $f(x)$

$f''(x)$…………………. for strictly concave function



(a)



(b)

From graph (a.)

$$x_1 < x_2 < x_3$$

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From graph (b.)

$$x_4 < x_5 < x_6$$

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If $f'(x_0) = 0$, $f(x_0)$ will be

→ a local maximum when…………………………………………………………

→ a local minimum when…………………………………………………………

SUMMARY

Condition	Maximum	Minimum
First-order necessary		
Second-order necessary		
Second-order sufficient		

H.W.:

(1.) $f(x)=\frac{1}{8}\left(x^4-8x^2\right)$

(2.) $f(x)=\frac{x^4}{4}-\frac{3}{2}x^2$

(3.) $f(x)=x^4$

Application of differential calculus in economics

Total Cost, Average Cost, and Marginal Cost

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Relationship between MC and AC

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How much does total cost increase if labor in production increases?

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TR, AR, MR

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The relation between MR, AR, and Price elasticity of demand

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The elasticity of total revenue with respect to output

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How much does total revenue increase if labor in production increases?

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H.W. Let $P = 25 - 0.1Q$, $Q = 5L$, find MRP

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TP, AP$_L$, MP$_L$

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Proof:

1.) At the maximum of AP_L, $AP_L = MP_L$

2.) When $MP_L > AP_L$, AP_L increases.

3.) When $MP_L < AP_L$, AP_L decreases.

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MC and MP_L

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AVC and AP_L

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Demand **(1.)** $Q^d = a - bP;\ a, b > 0$

I. $\frac{dQ^d}{dP}$

II. $\frac{d^2Q^d}{dP^2}$

The graphs are:

(2.) $Q^d = \frac{a}{P}$

$\frac{dy}{dx}$ and marginality

- Utility

$\frac{\mathrm{dU}}{\mathrm{dx}}$

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$\frac{\mathrm{d^2U}}{\mathrm{dx^2}}$

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H.W.
(a.) Find marginal utility when $U = 5x^{1/2} + 100$
(b.) Find marginal cost when $TC=5Q^2+3Q+200$
(c.) Find marginal revenue when $TR = 25Q$

✍ $\frac{dy}{dx}$ **and Elasticity**

- $\frac{dy}{dx}$ and price elasticity of demand: law of demand

Demand **(1.)** $Q^d = 250 - 10P$ **(2.)** $Q^d = 50P^{-1/4}$

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- $\frac{dy}{dx}$ and income elasticity of demand: inferior vs. normal good

$Q^d = f(I)$

$$E_I = \frac{dQ^d}{dI} \cdot \frac{I}{Q^d}$$

$E_I < 0$ ……………………………………………………………………..……………..

$E_I > 0$ ……………………………………………………………………..……………..

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- $\frac{dy}{dx}$ and cross price elasticity of demand: substitute vs. complementary goods

$Q_a^d = f(P_b)$

$$E_C = \frac{dQ_a^d}{dP_b} \cdot \frac{P_b}{Q_a^d}$$

$E_C < 0$ ……………………………………………………………………..……………..

$E_C > 0$ ……………………………………………………………………..……………..

- $\frac{dy}{dx}$ and law of supply

$$E_S = \frac{dQ^s}{dP} \cdot \frac{P}{Q^s}$$

- $\frac{dy}{dx}$ and output elasticity of labor

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