

GMM using Moment Equations

In the study of interest rate structure, the continuous time model can be specified as:

$$r_{t+\Delta t} - r_t = (\alpha + \beta r_t)\Delta t + \varepsilon_{t+\Delta t} \quad (1)$$

where: $E[\varepsilon_{t+\Delta t}] = 0$ and $E[\varepsilon_{t+\Delta t}^2] = \sigma^2 r_t^{2\gamma} \Delta t$

Then, the model can be transformed to be discrete time model by setting $\Delta t = 1$. The discrete time model can be stated as:

$$r_{t+1} - r_t = \Delta r_t = \alpha + \beta r_t + \varepsilon_{t+1} \quad (2)$$

where: $E[\varepsilon_{t+1}] = 0$ and $E[\varepsilon_{t+1}^2] = \sigma^2 r_t^{2\gamma}$

The above model consists of four parameters including α , β , σ^2 , γ . The model can be estimate using GMM. Four moment condition equations can be stated as:

$$\begin{aligned} \text{Zero mean condition:} & \quad E(\varepsilon_{t+1}) = 0 \\ \text{Orthogonality condition:} & \quad E(\varepsilon_{t+1} r_t) = 0 \\ \text{Variance condition:} & \quad E(\varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma}) = 0 \\ \text{Zero covariance condition:} & \quad E\left(\left(\varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma}\right) r_t\right) = 0 \end{aligned}$$

The above model can be claimed as unrestricted model for other eight interest rate structure models which can be indicated as follows:

Model	α	β	σ^2	γ
Unrestricted				
Merton		0		0
Vasicek				0
CIR SR				0.5
Dothan	0	0		1
GBM	0			1
Brennan & Schwartz				1
CIR VR	0	0		1.5
CEV	0			

Unrestricted model has four unknown parameters to be estimated and four moment condition equations, thus, method of estimate will be method of moment since it is exact identify equations model. Other eight restricted models will be estimated using GMM.

GMM can be performed by using the following command:

Unrestricted model:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r^(2*{gamma})) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r^(2*{gamma}))*r)
winitial(identity)
```

```
warning: 1 missing value returned for equation 1 at initial values
warning: 1 missing value returned for equation 2 at initial values
warning: 1 missing value returned for equation 3 at initial values
warning: 1 missing value returned for equation 4 at initial values
```

```
Step 1
numerical derivatives are approximate
flat or discontinuous region encountered
Iteration 0:   GMM criterion Q(b) = .00005673
Iteration 1:   GMM criterion Q(b) = .00005192   (backed up)
Iteration 2:   GMM criterion Q(b) = .00004097
Iteration 3:   GMM criterion Q(b) = 3.562e-06
Iteration 4:   GMM criterion Q(b) = 2.174e-07
Iteration 5:   GMM criterion Q(b) = 1.644e-07
```

```
Step 2
Iteration 0:   GMM criterion Q(b) = .01229538
Iteration 1:   GMM criterion Q(b) = .0055188
Iteration 2:   GMM criterion Q(b) = .00045234
Iteration 3:   GMM criterion Q(b) = 8.141e-07
Iteration 4:   GMM criterion Q(b) = 2.461e-12
Iteration 5:   GMM criterion Q(b) = 1.451e-23
```

GMM estimation

```
Number of parameters = 4
Number of moments = 4
Initial weight matrix: Identity
GMM weight matrix: Robust
Number of obs = 1429
```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/alpha	-.0009679	.0012837	-0.75	0.451	-.0034839	.0015481
/beta	.0014035	.0005181	2.71	0.007	.0003882	.0024189
/sigma2	.0006419	.0001156	5.55	0.000	.0004153	.0008686
/gamma	.1273995	.0720773	1.77	0.077	-.0138695	.2686685

```
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons
```

```
. estat overid
```

```
Test of overidentifying restriction:
```

```
Hansen's J chi2(0) = 2.1e-20 (p = .)
```

```
. est store Unrestricted
```

Merton model:

```
. gmm (dr-{alpha}) ((dr-{alpha})*r) ((dr-{alpha})^2-{sigma2}) (((dr-{alpha})^2-
{sigma2}*r) winitial(identity)
```

```
warning: 1 missing value returned for equation 1 at initial values
...
warning: 1 missing value returned for equation 4 at initial values
```

```

Step 1
Iteration 0: GMM criterion Q(b) = .00005673
Iteration 1: GMM criterion Q(b) = 6.386e-07
Iteration 2: GMM criterion Q(b) = 6.382e-07

```

```

Step 2
Iteration 0: GMM criterion Q(b) = .00760718
Iteration 1: GMM criterion Q(b) = .00652536
Iteration 2: GMM criterion Q(b) = .00652532

```

GMM estimation

```

Number of parameters = 2
Number of moments = 4
Initial weight matrix: Identity
GMM weight matrix: Robust
Number of obs = 1429

```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
/alpha	.0021651	.0007043	3.07	0.002	.0007847 .0035456
/sigma2	.0008218	.0000853	9.63	0.000	.0006546 .000989

```

Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

```

```
. estat overid
```

Test of overidentifying restriction:

Hansen's J chi2(2) = 9.32468 (p = 0.0094)

```
. est store Merton
```

Vasicek model:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}) (((dr-{alpha}-{beta}*r)^2-{sigma2})*r) winitial(identity)
```

warning: 1 missing value returned for equation 1 at initial values

warning: 1 missing value returned for equation 4 at initial values

```

Step 1
Iteration 0: GMM criterion Q(b) = .00005673
Iteration 1: GMM criterion Q(b) = 3.000e-09
Iteration 2: GMM criterion Q(b) = 2.107e-09

```

```

Step 2
Iteration 0: GMM criterion Q(b) = .00263124
Iteration 1: GMM criterion Q(b) = .00226079
Iteration 2: GMM criterion Q(b) = .00226074

```

GMM estimation

```

Number of parameters = 3
Number of moments = 4
Initial weight matrix: Identity
GMM weight matrix: Robust
Number of obs = 1429

```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
/alpha	-.0003515	.0012376	-0.28	0.776	-.002777 .0020741
/beta	.0012479	.0005106	2.44	0.015	.0002471 .0022487
/sigma2	.0007951	.0000862	9.23	0.000	.0006262 .0009641

```

Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons

```

```
. estat overid
Test of overidentifying restriction:
Hansen's J chi2(1) = 3.23059 (p = 0.0723)
. est store Vasicek
```

CIR SR model:

```
. gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
{sigma2}*r) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r)*r) winitial(identity) nolog
```

warning: 1 missing value returned for equation 1 at initial values

...
warning: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) = .0117512

GMM estimation

```
Number of parameters = 3
Number of moments = 4
Initial weight matrix: Identity
GMM weight matrix: Robust
Number of obs = 1429
```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/alpha	-.0031028	.0011644	-2.66	0.008	-.005385	-.0008207
/beta	.0018442	.0005066	3.64	0.000	.0008514	.0028371
/sigma2	.0002351	.0000278	8.45	0.000	.0001805	.0002896

```
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons
```

```
. estat overid
Test of overidentifying restriction:
Hansen's J chi2(1) = 16.7924 (p = 0.0000)
. est store CIR_SR
```

```
. est table Unrestricted Merton Vasicek CIR_SR, star(0.1 0.05 0.01)
```

Variable	Unrestricted	Merton	Vasicek	CIR_SR
alpha				
_cons	-.00096793	.00216513***	-.00035145	-.00310284***
beta				
_cons	.00140354***		.00124786**	.00184424***
sigma2				
_cons	.00064191***	.00082182***	.00079513***	.00023507***
gamma				
_cons	.12739949*			

Legend: * p<.1; ** p<.05; *** p<.01

```
. *Test Unrestricted vs Merton
. test (_b[/beta]=0) (_b[/gamma]=0)
( 1) [beta]_cons = 0
( 2) [gamma]_cons = 0
      chi2( 2) = 8.96
      Prob > chi2 = 0.0113
```

GMM Examples Simulated Data

Example 1

```
set obs 500
capture drop y x z u
mat mCorr = (1, .7, .5 \ .7, 1, 0 \ .5, 0, 1)
corr2data x z u, mean(10 10 0) sd(3 4 20) corr(mCorr) n(500) clear
corr
g y=1+2*x+u
reg y x
ivregress gmm y (x=z)
```

```
. set obs 500
number of observations (_N) was 500, now 500

. capture drop y x z u

. mat mCorr = (1, .7, .5 \ .7, 1, 0 \ .5, 0, 1)

. corr2data x z u, mean(10 10 0) sd(3 4 20) corr(mCorr) n(500) clear
(obs 500)

. corr
(obs=500)
```

	x	z	u
x	1.0000		
z	0.7000	1.0000	
u	0.5000	0.0000	1.0000

```
. g y=1+2*x+u
. reg y x
```

Source	SS	df	MS	Number of obs	=	500
Model	127744.002	1	127744.002	F(1, 498)	=	424.96
Residual	149700.001	498	300.602411	Prob > F	=	0.0000
Total	277444.003	499	556.000006	R-squared	=	0.4604
				Adj R-squared	=	0.4593
				Root MSE	=	17.338

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x	5.333333	.2587168	20.61	0.000	4.825022 5.841644
_cons	-32.33333	2.70086	-11.97	0.000	-37.63982 -27.02685

```
. ivregress gmm y (x=z)
```

```
Instrumental variables (GMM) regression
Number of obs = 500
Wald chi2(1) = 19.64
Prob > chi2 = 0.0000
R-squared = 0.2806
Root MSE = 19.98

GMM weight matrix: Robust
```

y	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
x	2	.4513481	4.43	0.000	1.115374 2.884626
_cons	.9999991	4.62114	0.22	0.829	-8.057269 10.05727

```
Instrumented: x
Instruments: z
```

Example 2

```

set obs 500
capture drop y x z* u
mat mCorr = (1, .7, .7, .5 \ .7, 1, .4, 0 \ .7, .4, 1, 0 \ .5, 0, 0, 1)
corr2data x z1 z2 u, mean(10 20 15 0) sd(3 4 5 20) corr(mCorr) n(500) clear
corr
g y=1+2*x+u
reg y x
ivregress gmm y (x=z1)
ivregress gmm y (x=z1 z2)
ivregress 2sls y (x=z1 z2)

```

```

. set obs 500
number of observations (_N) was 0, now 500

. capture drop y x z* u

. mat mCorr = (1, .7, .7, .5 \ .7, 1, .4, 0 \ .7, .4, 1, 0 \ .5, 0, 0, 1)

. corr2data x z1 z2 u, mean(10 20 15 0) sd(3 4 5 20) corr(mCorr) n(500) clear
(obs 500)

. corr
(obs=500)

```

	x	z1	z2	u
x	1.0000			
z1	0.7000	1.0000		
z2	0.7000	0.4000	1.0000	
u	0.5000	-0.0000	-0.0000	1.0000

```

. g y=1+2*x+u
. reg y x

```

Source	SS	df	MS	Number of obs	=	500
Model	127743.999	1	127743.999	F(1, 498)	=	424.96
Residual	149700	498	300.602411	Prob > F	=	0.0000
				R-squared	=	0.4604
				Adj R-squared	=	0.4593
Total	277443.999	499	555.999998	Root MSE	=	17.338

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
x	5.333333	.2587168	20.61	0.000	4.825022 5.841644
_cons	-32.33333	2.70086	-11.97	0.000	-37.63982 -27.02685

```

. ivregress gmm y (x=z1)

```

```

Instrumental variables (GMM) regression
Number of obs = 500
Wald chi2(1) = 24.63
Prob > chi2 = 0.0000
R-squared = 0.2806
Root MSE = 19.98

GMM weight matrix: Robust

```

y	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
x	2	.4030215	4.96	0.000	1.210092 2.789908
_cons	.9999999	4.004532	0.25	0.803	-6.848738 8.848738

```

Instrumented: x
Instruments: z1

```

```
. ivregress gmm y (x=z1 z2)
Instrumental variables (GMM) regression      Number of obs   =      500
                                             wald chi2(1)    =      35.10
                                             Prob > chi2     =      0.0000
                                             R-squared       =      0.2806
                                             Root MSE       =      19.98

GMM weight matrix: Robust
```

```
-----+-----
          y |          Coef.   Robust
            |          Std. Err.   z   P>|z|   [95% Conf. Interval]
-----+-----
          x |             2     .3375669   5.92  0.000   1.338381   2.661619
        _cons |             1     3.373732   0.30  0.767  -5.612393   7.612393
-----+-----
```

```
Instrumented:  x
Instruments:  z1 z2
```

```
. estat overid
Test of overidentifying restriction:
Hansen's J chi2(1) = 1.1e-14 (p = 1.0000)
```

```
. ivregress 2sls y (x=z1 z2)
Instrumental variables (2SLS) regression      Number of obs   =      500
                                             wald chi2(1)    =      31.50
                                             Prob > chi2     =      0.0000
                                             R-squared       =      0.2806
                                             Root MSE       =      19.98
```

```
-----+-----
          y |          Coef.   Std. Err.   z   P>|z|   [95% Conf. Interval]
-----+-----
          x |             2     .3563483   5.61  0.000   1.30157   2.69843
        _cons |             1     3.673801   0.27  0.785  -6.200517   8.200518
-----+-----
```

```
Instrumented:  x
Instruments:  z1 z2
```

Example 3

```
set obs 500
capture drop y x z* u
mat mCorr = (1, .7, .3, .5 \ .7, 1, .4, .2 \ .3, .4, 1, .3 \ .5, .2, .3, 1)
corr2data x z1 z2 u, mean(10 20 15 0) sd(3 4 5 20) corr(mCorr) n(5000) clear
corr
g y=1+2*x+u
reg y x
ivregress gmm y (x=z1)
ivregress gmm y (x=z1 z2)
estat overid
ivregress 2sls y (x=z1 z2)
```

```
. set obs 500
number of observations (_N) was 0, now 500

. capture drop y x z* u

. mat mCorr = (1, .7, .3, .5 \ .7, 1, .4, .2 \ .3, .4, 1, .3 \ .5, .2, .3, 1)

. corr2data x z1 z2 u, mean(10 20 15 0) sd(3 4 5 20) corr(mCorr) n(5000) clear
(obs 5,000)

. corr
(obs=5,000)
```

```
-----+-----
          |          x          z1          z2          u
-----+-----
          x |      1.0000
          z1 |      0.7000      1.0000
          z2 |      0.3000      0.4000      1.0000
          u |      0.5000      0.2000      0.3000      1.0000
-----+-----
```

```
. g y=1+2*x+u
```

```
. reg y x
```

Source	SS	df	MS	Number of obs	=	5,000
Model	1279743.99	1	1279743.99	F(1, 4998)	=	4264.96
Residual	1499700	4,998	300.060025	Prob > F	=	0.0000
				R-squared	=	0.4604
				Adj R-squared	=	0.4603
Total	2779444	4,999	556	Root MSE	=	17.322

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
y					
x	5.333333	.081666	65.31	0.000	5.173232 5.493435
_cons	-32.33333	.852611	-37.92	0.000	-34.00483 -30.66184

```
. ivregress gmm y (x=z1)
```

Instrumental variables (GMM) regression	Number of obs	=	5,000
	wald chi2(1)	=	1044.34
	Prob > chi2	=	0.0000
	R-squared	=	0.4274
	Root MSE	=	17.841

GMM weight matrix: Robust

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
y					
x	3.904762	.1208297	32.32	0.000	3.66794 4.141584
_cons	-18.04762	1.229494	-14.68	0.000	-20.45738 -15.63785

```
Instrumented: x
Instruments: z1
```

```
. ivregress gmm y (x=z1 z2)
```

Instrumental variables (GMM) regression	Number of obs	=	5,000
	wald chi2(1)	=	1103.18
	Prob > chi2	=	0.0000
	R-squared	=	0.4315
	Root MSE	=	17.776

GMM weight matrix: Robust

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
y					
x	3.997248	.1203476	33.21	0.000	3.761371 4.233125
_cons	-19.00297	1.224456	-15.52	0.000	-21.40286 -16.60308

```
Instrumented: x
Instruments: z1 z2
```

```
. estat overid
```

Test of overidentifying restriction:

Hansen's J chi2(1) = 293.132 (p = 0.0000)

```
. ivregress 2sls y (x=z1 z2)
```

Instrumental variables (2SLS) regression	Number of obs	=	5,000
	wald chi2(1)	=	1100.94
	Prob > chi2	=	0.0000
	R-squared	=	0.4305
	Root MSE	=	17.792

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
y					
x	3.97411	.119773	33.18	0.000	3.739359 4.208861
_cons	-18.7411	1.223875	-15.31	0.000	-21.13985 -16.34235

```
Instrumented: x
Instruments: z1 z2
```

Example

Griliches (1976)'s wage model can be stated as:

$$LW = \alpha + \beta S + \gamma IQ + \delta' h + \varepsilon \tag{2}$$

and $h \equiv (EXPR, TENURE, RNS, SMSA, YEAR DUMMIES)$

- where: *LW* = Log wages
S = Completed years of schooling
IQ = IQ Score
EXPR = Experience in years
TENURE = Tenure in years
RNS = Dummy for residency in the southern states
SMSA = Dummy for residency in the metropolitan areas
YEAR DUMMIES = Dummy for years

IQ score and years of completed schooling might be correlated with error term, thus, instrumental variables can be employed. Instrumental variables to be used include:

- MED* = Mother's education in years
KWW = Score on the "Knowledge of the World of Work" test
MRT = Dummy for marital status (1 if married)
AGE = Age of the individual

Create Dummy variable

```
. xi i.year
i.year      _Iyear_66-73      (naturally coded; _Iyear_66 omitted)
```

OLS

```
. reg lw s iq expr tenure rns smsa _I*
```

Source	SS	df	MS	Number of obs = 758		
Model	59.9127611	12	4.99273009	F(12, 745)	=	46.86
Residual	79.3733888	745	.106541461	Prob > F	=	0.0000
Total	139.28615	757	.183997556	R-squared	=	0.4301
				Adj R-squared	=	0.4210
				Root MSE	=	.32641

	lw	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
s		.0619548	.0072786	8.51	0.000	.0476658 .0762438
iq		.0027121	.0010314	2.63	0.009	.0006873 .0047369
expr		.0308395	.0065101	4.74	0.000	.0180592 .0436198
tenure		.0421631	.0074812	5.64	0.000	.0274763 .0568498
rns		-.0962935	.0275467	-3.50	0.001	-.1503719 -.0422151
smsa		.1328993	.0265758	5.00	0.000	.0807268 .1850717
_Iyear_67		-.0542095	.0478522	-1.13	0.258	-.1481506 .0397317
_Iyear_68		.0805808	.0448951	1.79	0.073	-.0075551 .1687168
_Iyear_69		.2075915	.0438605	4.73	0.000	.1214867 .2936963
_Iyear_70		.2282237	.0487994	4.68	0.000	.132423 .3240245
_Iyear_71		.2226915	.0430952	5.17	0.000	.1380889 .307294
_Iyear_73		.3228747	.0406574	7.94	0.000	.2430579 .4026915
_cons		4.235357	.1133489	37.37	0.000	4.012836 4.457878

2SLS with S IQ as endogenous & MED, KWW, MRT, AGE as IV

```
. ivregress 2sls lw expr tenure rns smsa _I* (s iq = med kww mrt age)
```

```
Instrumental variables (2SLS) regression      Number of obs =    758
                                             wald chi2(12) =  459.55
                                             Prob > chi2    =  0.0000
                                             R-squared     =  0.2280
                                             Root MSE     =  .37665
```

lw	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
s	.1724253	.0207381	8.31	0.000	.1317794 .2130712
iq	-.0090988	.0047044	-1.93	0.053	-.0183193 .0001216
expr	.0492895	.0081546	6.04	0.000	.0333068 .0652722
tenure	.0422171	.0088429	4.77	0.000	.0248854 .0595488
rns	-.1017935	.0341765	-2.98	0.003	-.1687781 -.0348088
smsa	.1261109	.0309275	4.08	0.000	.0654942 .1867277
_Iyear_67	-.0596171	.0552955	-1.08	0.281	-.1679942 .04876
_Iyear_68	.0486796	.0520161	0.94	0.349	-.0532701 .1506292
_Iyear_69	.1528176	.051563	2.96	0.003	.051756 .2538792
_Iyear_70	.1744361	.0597576	2.92	0.004	.0573133 .2915588
_Iyear_71	.091666	.054144	1.69	0.090	-.0144543 .1977863
_Iyear_73	.0932398	.0571819	1.63	0.103	-.0188347 .2053142
_cons	4.03351	.3154215	12.79	0.000	3.415295 4.651725

```
Instrumented: s iq
Instruments: expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
              _Iyear_71 _Iyear_73 med kww mrt age
```

```
. estat overid
```

Tests of overidentifying restrictions:

```
Sargan (score) chi2(2) = 13.2683 (p = 0.0013)
Basmann chi2(2)      = 13.2375 (p = 0.0013)
```

GMM with S IQ as endogenous & MED, KWW, MRT, AGE as IV

```
. ivregress gmm lw expr tenure rns smsa _I* (s iq = med kww mrt age)
```

```
Instrumental variables (GMM) regression      Number of obs =    758
                                             wald chi2(12) =  506.92
                                             Prob > chi2    =  0.0000
                                             R-squared     =  0.2168
                                             Root MSE     =  .37936

GMM weight matrix: Robust
```

lw	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
s	.1757958	.0208514	8.43	0.000	.1349279 .2166637
iq	-.0092862	.0049182	-1.89	0.059	-.0189256 .0003533
expr	.0502828	.008104	6.20	0.000	.0343992 .0661664
tenure	.0425214	.0095601	4.45	0.000	.0237838 .0612589
rns	-.1040931	.03373	-3.09	0.002	-.1702026 -.0379835
smsa	.1247512	.0309873	4.03	0.000	.0640172 .1854853
_Iyear_67	-.0530432	.0517876	-1.02	0.306	-.1545449 .0484586
_Iyear_68	.0459546	.0500069	0.92	0.358	-.0520572 .1439664
_Iyear_69	.1554801	.0480126	3.24	0.001	.0613772 .249583
_Iyear_70	.1669875	.0613341	2.72	0.006	.0467748 .2872001
_Iyear_71	.0846485	.055817	1.52	0.129	-.0247508 .1940477
_Iyear_73	.0996068	.0612394	1.63	0.104	-.0204201 .2196338
_cons	4.003924	.3364754	11.90	0.000	3.344445 4.663404

```
Instrumented: s iq
Instruments: expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
              _Iyear_71 _Iyear_73 med kww mrt age
```

```
. estat overid
```

Test of overidentifying restriction:

```
Hansen's J chi2(2) = 11.6015 (p = 0.0030)
```

Alternative Command `ivreg2` and `ivgmm0`

```
. ivreg2 lw expr tenure rns smsa _I* (s iq=med kww age mrt)
```

IV (2SLS) estimation

```
-----
Total (centered) SS      = 139.2861498
Total (uncentered) SS  = 24652.24662
Residual SS            = 107.5313411

Number of obs =      758
F( 12, 745) =    37.64
Prob > F      =    0.0000
Centered R2   =    0.2280
Uncentered R2 =    0.9956
Root MSE     =    .3766
```

lw	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
s	.1724253	.0207381	8.31	0.000	.1317794	.2130712
iq	-.0090988	.0047044	-1.93	0.053	-.0183193	.0001216
expr	.0492895	.0081546	6.04	0.000	.0333068	.0652722
tenure	.0422171	.0088429	4.77	0.000	.0248854	.0595488
rns	-.1017935	.0341765	-2.98	0.003	-.1687781	-.0348088
smsa	.1261109	.0309275	4.08	0.000	.0654942	.1867277
_Iyear_67	-.0596171	.0552955	-1.08	0.281	-.1679942	.04876
_Iyear_68	.0486796	.0520161	0.94	0.349	-.0532701	.1506292
_Iyear_69	.1528176	.051563	2.96	0.003	.051756	.2538792
_Iyear_70	.1744361	.0597576	2.92	0.004	.0573133	.2915588
_Iyear_71	.091666	.054144	1.69	0.090	-.0144543	.1977863
_Iyear_73	.0932398	.0571819	1.63	0.103	-.0188347	.2053142
_cons	4.03351	.3154215	12.79	0.000	3.415295	4.651725

```
Anderson canon. corr. LR statistic (underidentification test):      49.564
Chi-sq(3) P-val =      0.0000
```

```
-----
Cragg-Donald F statistic (weak identification test):      12.552
Stock-Yogo weak ID test critical values:
5% maximal IV relative bias      11.04
10% maximal IV relative bias     7.56
20% maximal IV relative bias     5.57
30% maximal IV relative bias     4.73
10% maximal IV size              16.87
15% maximal IV size              9.93
20% maximal IV size              7.54
25% maximal IV size              6.28
```

Source: Stock-Yogo (2005). Reproduced by permission.

```
-----
Sargan statistic (overidentification test of all instruments):      13.268
Chi-sq(2) P-val =      0.0013
```

```
-----
Instrumented:      s iq
Included instruments: expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69
                  _Iyear_70 _Iyear_71 _Iyear_73
Excluded instruments: med kww age mrt
-----
```

```
. ivreg2 lw expr tenure rns smsa _I* (s iq=med kww age mrt), gmm
```

2-Step GMM estimation

Statistics robust to heteroskedasticity

```
-----
Total (centered) SS      = 139.2861498
Total (uncentered) SS  = 24652.24662
Residual SS            = 109.0846511

Number of obs =      758
F( 12, 745) =    41.98
Prob > F      =    0.0000
Centered R2   =    0.2168
Uncentered R2 =    0.9956
Root MSE     =    .3794
```

lw	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
s	.1757958	.0206766	8.50	0.000	.1352703	.2163212
iq	-.0092862	.0048824	-1.90	0.057	-.0188555	.0002832
expr	.0502828	.0080438	6.25	0.000	.0345171	.0660484
tenure	.0425214	.0094549	4.50	0.000	.0239901	.0610526
rns	-.1040931	.0335239	-3.11	0.002	-.1697986	-.0383875

smsa	.1247512	.0307747	4.05	0.000	.0644338	.1850686
_Iyear_67	-.0530432	.0514609	-1.03	0.303	-.1539047	.0478184
_Iyear_68	.0459546	.0495735	0.93	0.354	-.0512077	.1431169
_Iyear_69	.1554801	.0476311	3.26	0.001	.0621249	.2488352
_Iyear_70	.1669875	.0610006	2.74	0.006	.0474285	.2865464
_Iyear_71	.0846485	.0554035	1.53	0.127	-.0239404	.1932373
_Iyear_73	.0996068	.0607034	1.64	0.101	-.0193696	.2185833
_cons	4.003924	.3348423	11.96	0.000	3.347645	4.660203

Anderson canon. corr. LR statistic (underidentification test): 49.564
 Chi-sq(3) P-val = 0.0000

Test statistic(s) not robust

Cragg-Donald F statistic (weak identification test): 12.552

Stock-Yogo weak ID test critical values:	5% maximal	IV relative bias	11.04
	10% maximal	IV relative bias	7.56
	20% maximal	IV relative bias	5.57
	30% maximal	IV relative bias	4.73
	10% maximal	IV size	16.87
	15% maximal	IV size	9.93
	20% maximal	IV size	7.54
	25% maximal	IV size	6.28

Test statistic(s) not robust

Source: Stock-Yogo (2005). Reproduced by permission.

Hansen J statistic (overidentification test of all instruments): 11.601
 Chi-sq(2) P-val = 0.0030

Instrumented: s iq
 Included instruments: expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69
 _Iyear_70 _Iyear_71 _Iyear_73
 Excluded instruments: med kww age mrt

. ivgmm0 lw expr tenure rns smsa _I* (s iq = med kww mrt age)

Instrumental Variables Estimation via GMM
 Number of obs = 758
 Root MSE = 0.3794
 Hansen J = 11.6015
 Chi-sq(2) P-val = 0.00303

lw	Coef.	GMM Std. Err.	z	P> z	[95% Conf. Interval]	
s	.1757958	.0206766	8.50	0.000	.1352703	.2163212
iq	-.0092862	.0048824	-1.90	0.057	-.0188555	.0002832
expr	.0502828	.0080438	6.25	0.000	.0345171	.0660484
tenure	.0425214	.0094549	4.50	0.000	.0239901	.0610526
rns	-.1040931	.0335239	-3.11	0.002	-.1697986	-.0383875
smsa	.1247512	.0307747	4.05	0.000	.0644338	.1850686
_Iyear_67	-.0530432	.0514609	-1.03	0.303	-.1539047	.0478184
_Iyear_68	.0459546	.0495735	0.93	0.354	-.0512077	.1431169
_Iyear_69	.1554801	.0476311	3.26	0.001	.0621249	.2488352
_Iyear_70	.1669875	.0610006	2.74	0.006	.0474285	.2865464
_Iyear_71	.0846485	.0554035	1.53	0.127	-.0239404	.1932373
_Iyear_73	.0996068	.0607034	1.64	0.101	-.0193696	.2185833
_cons	4.003924	.3348423	11.96	0.000	3.347645	4.660203

Instrumented: s iq
 Instruments: expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
 _Iyear_71 _Iyear_73 med kww mrt age

. ivgmm0 lw expr tenure rns smsa _I* (s iq = med kww mrt age), gres

Instrumental Variables Estimation via GMM
 Number of obs = 758
 Root MSE = 0.3794
 Hansen J = 11.6015
 Chi-sq(2) P-val = 0.00303

lw	Coef.	GMM Std. Err.	z	P> z	[95% Conf. Interval]	
s	.1757958	.0208513	8.43	0.000	.1349279	.2166637
iq	-.0092862	.0049182	-1.89	0.059	-.0189256	.0003533
expr	.0502828	.008104	6.20	0.000	.0343992	.0661663
tenure	.0425214	.0095601	4.45	0.000	.0237839	.0612589

rns		-.1040931	.0337299	-3.09	0.002	-.1702025	-.0379836
smsa		.1247512	.0309873	4.03	0.000	.0640172	.1854852
_Iyear_67		-.0530432	.0517876	-1.02	0.306	-.1545449	.0484586
_Iyear_68		.0459546	.0500068	0.92	0.358	-.0520569	.1439661
_Iyear_69		.1554801	.0480124	3.24	0.001	.0613775	.2495827
_Iyear_70		.1669875	.0613341	2.72	0.006	.0467749	.2872
_Iyear_71		.0846485	.0558168	1.52	0.129	-.0247505	.1940474
_Iyear_73		.0996068	.0612394	1.63	0.104	-.0204201	.2196338
_cons		4.003924	.3364752	11.90	0.000	3.344445	4.663404

Instrumented: s iq
Instruments: expr tenure rns smsa _Iyear_67 _Iyear_68 _Iyear_69 _Iyear_70
_Iyear_71 _Iyear_73 med kww mrt age
