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Assignment 7

Q1) a) $Y = zF(K,N) = zK^{\alpha}N^{1-\alpha} = zK^{\alpha}N \cdot N^{-\alpha}$

$\frac{Y}{N} = zK^{\alpha}N^{-\alpha} = z\left(\frac{K}{N}\right)^{\alpha}$

$\therefore\ y = zk^{\alpha}$; $y = \frac{Y}{N}$, $k = \frac{K}{N}$

b) $K' = (1-d)K + I$

$= (1-d)K + sY$

$\frac{K'}{N} = (1-d)\frac{K}{N} + s\frac{Y}{N}$

$\frac{N'}{N'} \cdot \frac{K'}{N} = (1-d)k + sy$

$\frac{N'}{N} \cdot \frac{K'}{N'} = (1-d)k + s(zk^{\alpha})$

$(1+n)k' = (1-d)k + szk^{\alpha}$; $1+n = \frac{N'}{N}$, $k' = \frac{K'}{N'}$

$\therefore\ k' = \frac{szk^{\alpha} + (1-d)k}{(1+n)}$

c) At steady-state, $k' = k = k^*$

$k^* = \frac{szk^{*\alpha} + (1-d)k^*}{(1+n)}$

$(1+n)k^* = szk^{*\alpha} + (1-d)k^*$

$szk^{*\alpha} + (1-d-1-n)k^* = 0$

$szk^{*\alpha} - (n+d)k^* = 0$

$k^*(szk^{*\alpha-1} - (n+d)) = 0$

$k^* = 0$ or $szk^{*\alpha-1} = n+d$

$k^* = \sqrt[\alpha-1]{\frac{n+d}{sz}}$; $k > 0$

$\therefore$ If $k^* > 0$, $k^* = \sqrt[\alpha-1]{\frac{n+d}{sz}}$

d) $C = (1-s)Y$

$\frac{C}{N} = (1-s)\frac{Y}{N}$

$c = (1-s)y = (1-s)zk^{\alpha}$

$c^* = (1-s)zk^{*\alpha} = (1-s)(z)\left(\frac{n+d}{sz}\right)^{\frac{\alpha}{\alpha-1}}$

e) $y^* = zk^{*\alpha} = \frac{Y}{N} = \frac{Y'}{N'}$

$\frac{Y'}{Y} = \frac{N'}{N} = 1+n$

$C = (1-s)Y$

$C' = (1-s)Y'$

$\frac{C'}{C} = \frac{Y'}{Y} = 1+n$

$\therefore$ Y, C grow at rate of n at steady state.

f) $$k^* = \left(\frac{n+d}{sz}\right)^{\frac{1}{\alpha-1}}$$

$$y^* = z k^{*\alpha} = z\left(\frac{n+d}{sz}\right)^{\frac{\alpha}{\alpha-1}}$$

$$c^* = (1-s)(z)\left(\frac{n+d}{sz}\right)^{\frac{\alpha}{\alpha-1}} \quad ; \ 0 < \alpha < 1$$

$$\frac{\partial k^*}{\partial z} = \left(\frac{1}{\alpha-1}\right)\left(\frac{n+d}{sz}\right)^{\frac{-\alpha+2}{\alpha-1}}\left(\frac{n+d}{s}\right)\left(-\frac{1}{z^2}\right) > 0$$

$$\frac{\partial y^*}{\partial z} = z\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{n+d}{sz}\right)^{\frac{1}{\alpha-1}}\left(\frac{n+d}{s}\right)\left(-\frac{1}{z^2}\right) + \left(\frac{n+d}{sz}\right)^{\frac{\alpha}{\alpha-1}} > 0$$

$$\frac{\partial c^*}{\partial z} = (1-s)\left[z\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{n+d}{sz}\right)^{\frac{1}{\alpha-1}}\left(\frac{n+d}{s}\right)\left(-\frac{1}{z^2}\right) + \left(\frac{n+d}{sz}\right)^{\frac{\alpha}{\alpha-1}}\right] > 0$$

$$\frac{\partial k^*}{\partial n} = \left(\frac{1}{\alpha-1}\right)\left(\frac{n+d}{sz}\right)^{\frac{-\alpha+2}{\alpha-1}}\left(\frac{1}{sz}\right) < 0$$

$$\frac{\partial y^*}{\partial n} = z\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{n+d}{sz}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{sz}\right) < 0$$

$$\frac{\partial c^*}{\partial n} = (1-s)(z)\left(\frac{\alpha}{\alpha-1}\right)\left(\frac{n+d}{sz}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{sz}\right) < 0$$

✓ okay

When z decreases, MP_k will decrease, so the firm need to cut k^* to raise MP_k to keep up with n and d. As k^* decreases, y^* will drop and thus c^* drops given s.

When n rises, the firm also need to reduce k^* to increase MP_k. Therefore, y^* and c^* drop as well. To explain more, k^* drops so that investment in each year can expand y at the same rate of n and d lowing y and k'. In each year, k, y, c remain the same.

g) $$c^* = (1-s) z k^{*\alpha}$$
$$= z k^{*\alpha} - s z k^{*\alpha} \qquad \because \ s z f(k^*) = (n+d) k^*$$
$$= z k^{*\alpha} - (n+d) k^*$$

$$\frac{dc^*}{dk^*}(k^{**}) = z\alpha k^{**\alpha-1} - (n+d) = 0$$

$$\therefore \ k^{**} = \sqrt[\alpha-1]{\frac{n+d}{z\alpha}} \quad ; \ k^{**} > 0$$

In fact I only ask about steady state Consumption/N Not the golden rule one.

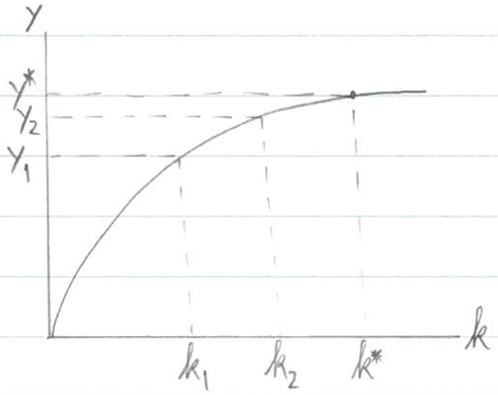
$$s_g z k^{**\alpha} = (n+d) k^{**}$$

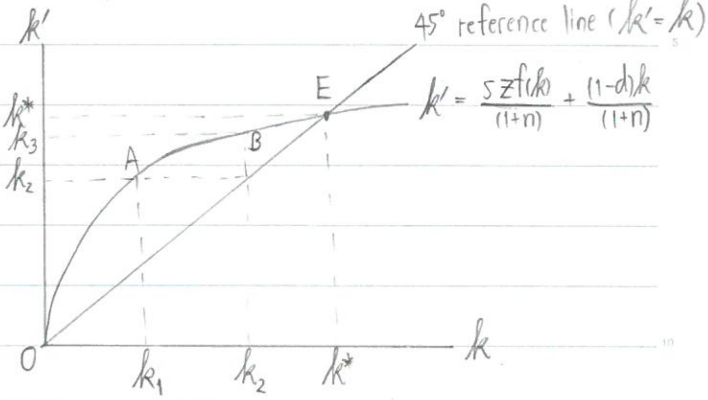
$$\therefore \ s_g = \frac{(n+d)}{z} k^{**1-\alpha} = \frac{(n+d)}{z}\left(\frac{n+d}{z\alpha}\right)^{\frac{1-\alpha}{\alpha-1}} = \frac{n+d}{z}\left(\frac{z\alpha}{n+d}\right) = \alpha$$

$$\therefore \ c_g = (1-\alpha) z \sqrt[\alpha-1]{\frac{n+d}{z\alpha}}$$

golden Rule Consumption!

Q2) In the initial situation, the economy is in a steady state. When a disaster or a war happen, current capital stock will decrease. However, this situation will not affect s, n, d, and z. In the long run, the capital stock per worker at the steady state will not change. Then output per worker and consumption per worker remain unchanged.





In the short run, the capital per worker will drop from k^* to k_1 causing y to drop. Since MP_k at k_1 is very high, the worker can invest a lot to increase y. k_1 will rise to k_2 causing y to increase. This will stimulate higher investment and higher growth of capital stock. From the graph, the growth factor of aggregate capital stock can be represented by slope of the straight line from origin to point A. While slope of reference line has growth factor of 1+n. Slope of $\overline{OA}$ is higher than that of reference line meaning that K grows at the higher rate than labor grows. The worker will continue high investment until they reach k^* due to law of diminishing marginal productivity of k. Therefore, in long term, k will converge to k^* and y will rise to y^*.

After World War II, growth in GDP in Germany and Japan can be very high since the worker add capital in a large amount resulting in high growth in y. However, these countries are subject to steady-state k. Growth of GDP will be lower and stay constant at some point as long as there is no effect on s, n, d, and z.

Good

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