



EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

1.1)

Student	Y_i	X_i	$Y_i X_i$	X_i^2	$X_i = X_i - \bar{X}$	$Y_i = Y_i - \bar{Y}$	X_i^2	$X_i Y_i$	$\hat{Y}_i$	$\hat{U}_i$
1	2.8	63	176.4	3969	-14.625	-0.4125	213.89	6.03		-0.4125
2	3.4	72	244.8	5184	-5.625	0.1875	31.64	-1.05		0.1875
3	3.0	78	234	6104	0.375	-0.2125	0.14	-0.08		-0.2125
4	3.5	81	283.5	6561	3.375	0.2875	11.39	0.97		0.2875
5	3.6	87	313.2	7569	9.375	0.3875	87.89	3.63		0.3875
6	3.0	75	225	5625	-2.625	-0.2125	6.89	0.56		-0.2125
7	2.7	75	202.5	5625	-2.625	-0.5125	6.89	2.35		-0.5125
8	3.7	90	333	8100	12.375	0.4875	153.14	6.03		0.4875
Sum	25.7	621	2092.4	48717	0	0	511.87	17.44		0
mean	3.2125	77.625	251.55	6079.69	0	0	63.98	2.18		0

to find $\hat{\beta}_2$ using short-cut = $\frac{\sum X_i Y_i}{\sum X_i^2} = \frac{17.44}{511.87} = \frac{17.44}{511.87} = 0.0341$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$= 3.2125 - (0.0341)(77.625)$$

$$= 3.2125 - 2.647$$

$$= 0.5681$$

$$\therefore \hat{Y}_i = 3.2073 + 0.0000656 X_i \quad \text{can be interpreted that}$$

If exam point increased by 1 point, GPA on average will increase by 0.0341
and when total microeconomics exam point is equal to zero, student's GPA is 0.5681

1.2) • $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 \bar{X}$

$$= 3.2073 + 0.0000656(77.625)$$

$$= 3.2125$$

• $\hat{U}_i = Y_i - \hat{Y}_i$ in the model

to find $\hat{U}_i$, we know that $\hat{Y}_i$ is approximately 3.2125

For example Student 1: $Y_i = 2.8$

$$Y_i - \hat{Y}_i = 2.8 - 3.2125 = -0.4125$$

Student 2: $Y_i = 3.4$

$$Y_i - \hat{Y}_i = 3.4 - 3.2125 = 0.1875$$

• to prove that $\sum_{i=1}^n \hat{U}_i = 0$, we sum $\hat{U}_i$ of each student of total of 8 students such as $[0.4125 + 0.1875 - 0.2125 + 0.2875 + \dots + 0.4875]$ and get -0.003 which approximately equal to 0

1.3)

$$\text{Var}(\hat{\beta}_1) = \frac{\sum_{i=1}^n \hat{u}_i^2}{n-2} = \frac{0.4347253}{8-2} = 0.0725$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum_{i=1}^n x_i^2 \hat{\sigma}^2}{\sum_{i=1}^n x_i^2} = \frac{48213(0.0725)}{8(511.875)} = 0.862$$

$$\text{Var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum_{i=1}^n x_i^2} = \frac{0.0725}{511.875} = 0.00014163$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

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2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y ?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

X_i	Y_i	$Y_i - \bar{Y}$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	$Y_i - \hat{Y}_i$	$(Y_i - \hat{Y}_i)^2$	X_i^2
10	0	-9.1	-10	100	91	-0.1364	0.0186	100
12	2	-7.1	-8	64	56.8	0.0745	0.0055	144
14	5	-4.1	-6	36	24.6	1.2854	1.6523	196
16	6	-3.1	-4	16	12.4	0.4964	0.2464	256
18	7	-2.1	-2	4	4.2	-0.2923	0.0853	324
22	10	0.9	2	4	18	-2.66	7.0756	576
24	10	0.9	4	16	35.4	0.551	0.3036	676
26	15	5.9	6	36	55.2	-0.1381	0.0191	784
28	16	6.9	8	64	70.9	1.9728	3.8919	900
30	20	10.9	10	100				

$$2.1.) \quad \hat{\beta}_2 = \frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2} = \frac{394}{440} = 0.8955$$

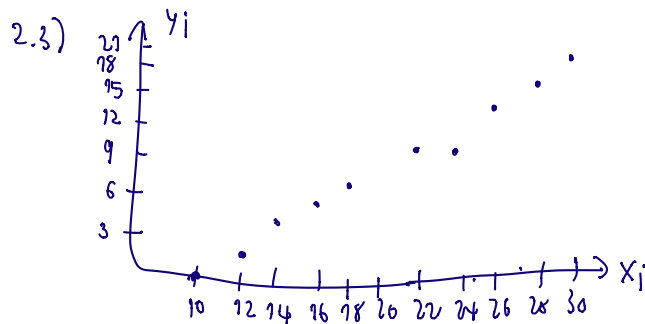
$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8955)(20) = -8.8091$$

$$Y_i = -8.8091 + 0.8955 X_i + \hat{u}_i \rightarrow \text{When } X_i \text{ increase by 1 } Y_i \text{ will increase by } 0.8955 \text{ and } y\text{-intercept is } -8.8091$$

$$2.2) \quad \hat{Y}_i = -8.8091 + 0.8955 X_i$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\sum_{i=1}^{10} \hat{u}_i = -0.1364 + 0.0745 + 1.2854 + \dots + 1.9728 \approx 0$$



The line passes
 $(\bar{x}, \bar{y})$ where
 $\bar{x} = 20, \bar{y} = 9.1$

2.4) $x_i = 16$

$$\begin{aligned} y_i &= -8.8091 + 0.8955 x_i \\ &= -8.8091 + 0.8955(16) \\ &= 5.5189 \end{aligned}$$

2.5) $\text{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} = \frac{1.7679}{440} = 0.004$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum x_i^2}{n \sum x_i^2} \sigma^2 = \frac{4440}{10(440)} (1.7679) = 1.7679$$

$$\text{Var}(\hat{y}_i) = \sigma^2 = 1.7679$$

3. proof: The unbiased estimator of β_1 is $\hat{\beta}_1$ is to find $E(\hat{\beta}_1) = \beta_1$

$$\bar{y} = \hat{\beta}_1 + \hat{\beta}_2 \bar{x}, \quad \hat{\beta}_2 = \sum k_i y_i$$

$$\hat{\beta}_1 = \bar{y} - (\sum k_i y_i) \bar{x}$$

$$\hat{\beta}_1 = \frac{\sum y_i}{n} - \sum \bar{x} k_i y_i$$

$$\hat{\beta}_1 = \sum \left(\frac{1}{n} - \bar{x} k_i \right) y_i$$

$$\hat{\beta}_1 = \sum \left(\frac{1}{n} - \bar{x} k_i \right) y_i$$

$\therefore$ the assumption is that $\hat{\beta}_1$ is linear