EE320 Exercise 5

Semester 2, 2014

Chapter 7: Derivatives of More-Than-One Independent Variable Function

1. Determine U_x and U_y from the following functions.

a) $U = xy - \ln xy$

b) $U = 3x^2 + 4y^2 - xy$

c) $U = \frac{24x}{25+3x} + \frac{15y}{10+y}$

d) $U = \frac{x-y}{x+y}$

e) $U = \sqrt{xy}$

2. If $Z = \frac{x^3-y^3}{xy}$, prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

3. If $Z = 2x^2 - 2y^2 - 3x - 4xy^2$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

4. If $f(x, y) = x^3 + 3x^2y + 6xy^2 - y^3$, determine $f_{xx}(2, 3)$, $f_{yy}(2, 3)$ and $f_{xy}(2, 3)$ [30, 6, and 48]

5. Determine dz from the following

a) $z = x^3 + x^2y - y^3$

b) $z = 2x^3 - 4xy^2 + 3y^3$

c) $z = x^3 + y^3 - 3xy$

d) $z = 3x^2 - xy^2$

6. If $U = x + 4x^{\frac{1}{2}}y^{\frac{1}{2}} - 3y, x = t^3$ and $y = \frac{1}{t}$, determine $\frac{dU}{dt}$

7. Determine $\frac{dy}{dx}$ from the following.

a) $x + y^3 = y^5 - x^2 + 2y$

b) $x\sqrt{y} = 2$

c) $x - y + 3xy = 2$

d) $y^6 = x^5$

e) If $x^3 - y^3 - 4xy = -\frac{1}{2}$, find $\frac{dy}{dx}$ when $x = 2, y = -2$.

8. Let$z = 5x^3 - 3x^2y^2 + 7y^5$

a) $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x^2}$

b) $\frac{\partial z}{\partial y}$ and $\frac{\partial^2 z}{\partial y^2}$

c)$\frac{\partial^2 z}{\partial y \partial x}$

d)$\frac{\partial^2 z}{\partial x \partial y}$

9. If $x^3 + y^3 - 3xy = 0$, determine $\frac{dy}{dx}$ $[\frac{dy}{dx} = \frac{y-x^2}{y^2-x}]$

10. If $x^3 - y^3 - 4xy = -\frac{1}{2}$, determine $\frac{dy}{dx}$ when $x = 2, y = -2$ $[\frac{dy}{dx} = 1]$

11. If $z = \frac{x^3-y^3}{xy}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

12. If $f(x, y) = 2x^2 - 3xy + 4y^2$, determine $f_x(1, -1)$ and$f_x(1, -1)$. [7 and -11]

13. If $U = x + 4x^{0.5}y^{0.5} - 3y, x = t^3$ and $y = \frac{1}{t}$, determine $\frac{dU}{dt}[3t^2 + 4 + \frac{3}{t^2}]$

14. Let $Q = K^{\frac{1}{3}}L^{\frac{2}{3}}$ and $K = 2t, L = 4t$, determine growth rate of Q overtime.

15. Let $Q_x^d = 400 - 40P_x + 50P_y + 0.0051I$

when Q_x^d = Quantity demanded for goods X

P_x, P_y = Price of goods X and Y respectively and I = Income

a) If $P_x = 10, P_y = 6$ and $I = 20,000$, Q_x^d =?

b) From a., determine own price, cross price, income elasticity and state economic meanings

16. Let $C = a + bY_d = 20 + 0.8Y_d$

$I = I_0 + iY = 6 + 0.2Y$

$G = G_0 = 4$

$X = X_0 + xY = 2 + 0.1Y$

$T = T_0 + tY = 1 + 0.2Y$

(Unit: Million Baht)

a) Determine DAE equation and equilibrium national income(Y_E).

[DAE $= 31.2 + 0.94Y$ and $Y_E = 520$]

b) Determine investment multiplier and import multiplier.

[16.67, −13.33]

c) At equilibrium national income,

(1)Which kind of balance of trade does occur in this economy?(surplus/deficit)and how much? [surplus by 54 Million Baht]

(2) How much is government revenue? How much is government spending? How does government budget balance?(surplus/deficit) and how much? [105 Million Baht, 4 Million Baht and Surplus by 101 Million Baht]

17. Let the production function be $Q = 4K^{\frac{1}{5}}L^{\frac{3}{5}}$

a) Show that production function is decreasing returns to scale.

b) Determine MP_L, MP_K together with describe and explain
how the line $MP_L$and MP_K look like.

c) Determine and explain the effect of increase in L to MP_K.

d) Determine Marginal Rate of Technical Substitution of capital for labour ($MRTS_{KL}$)

$[MRTS_{KL} = \frac{1L}{3K}]$

e) Determine Elasticity of Production with respect to capital and labour.

$[\varepsilon_K = \frac{1}{5}, \varepsilon_L = \frac{3}{5}]$