Midterm 2010

2. Find all critical points of $f(x,y) = xe^{-x}(y^2 - 4y)$ and use the second derivative test to classify the points whether they are *relative maximum*, *relative minimum*, *saddle point* or *neither*.

Ans: (0,0) and (0,4) are saddle points and (1, 2) is a relative minimum

3. For a two-variable function

$$f(x,y) = x^2 + 2xy^2 + 2y^2$$

Use the extreme value theorem to find the absolute maximum and absolute minimum values of the above function on the closed area bounded by $x \le 2$, $y \le 2$ and $y \ge -x$.

Ans: (2, 2) and (2, -2) are absolute maximum and (-2, 2) is absolute minimum. Other critical points are (0, 0), (-1, 1), (2, 0) and (1/3, -1/3)

Midterm 2009

5. Use the **extreme value theorem** to find the absolute maximum point(s) for

$$h(x,y) = x^2 y\left(x - y - \frac{1}{2}\right)$$

subject to $1 \le x \le 2$, $y \ge \frac{1}{2}$ and $y \le x - 1$.

Ans: f(2, ¾) = 2.25 absolute maximum, other critical points are (3/2, 1/2), (2, 1/2), (2, 1)

More Examples:-

(A) Find all critical points and classify them. The function is $f(x,y) = xy^2 + x^2 y - xy$

Ans: (0, 0), (1, 0), (-1, 0) and (0, 1) are saddle points, (1/√5, 2.5) is relative minimum, and (-1/√5, 2.5) is relative maximum.

(B) Use the **extreme value theorem** to find the absolute maximum and minium point(s) for

$$f(x,y) = x^2 + x^2 y + y^2 - 2y + 2$$

subject to $y \ge x^2$ and $y + 2x \le 3$.

Ans: f(-3, 9) = 155 absolute maximum, f(0, 1) = 1 absolute minimum, other critical points are (2/3, 5/3), (0, 0), (1/2, 1/4), (-1/2, 1/4), and (1, 1)