

(a) (5 Marks) Use the above information to compute OLS estimates of the intercept coefficient β_1 and that of the slope coefficient β_2 .

From the above information:

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{20.58}{211.00} = 0.0975$$

On the average, when the SAT score increases (decreases) 1 score, the grade point average will go up (goes down) by 0.0975.

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = \frac{50.90}{16} - 0.0975 \left(\frac{388}{16} \right) = 0.8169$$

when SAT score is zero, the grade point average is 0.8169 on average.

(b) (5 Marks) Interpret the slope coefficient estimate you calculated in part(a)—i.e., explain in words what the numeric value you calculated for $\hat{\beta}_2$ means.

(see 4.a).

(c) (10 Marks) Calculate an estimate of σ^2 .

We know that

$$TSS = ESS + RSS$$

$$\sum y_i^2 = \sum \hat{y}_i^2 + \sum \hat{u}_i^2$$

$$\therefore \sum \hat{u}_i^2 = 2.5844 - 2.0063 = 0.5781$$

$\therefore$ the estimated σ^2 is $\hat{\sigma}^2$

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.5781}{16-2} = 0.0413$$

(d) (5 Marks) Compute the value of r^2 . Briefly explain what the calculated value of r^2 means.

By the definition

$$r^2 = \frac{\sum \hat{y}_i^2}{\sum y_i^2} = \frac{2.0063}{2.5844} = 0.7763$$

The variation of the grade point average (y_i) can be explained by the variation of the SAT scores (x_i) by 77.63%.

(e) (5 Marks) Compute the estimated variance of $\hat{\beta}_1$ and the estimated variance of $\hat{\beta}_2$.

We know that

$$\text{Var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{0.0413}{211.00} = 0.000195$$

$$\text{Var}(\hat{\beta}_1) = \frac{\sum x_i^2 \hat{\sigma}^2}{n \sum x_i^2} = \frac{0.0413 \times (9620)}{16 \times (211.00)} = 0.1177$$

(f) (5 Marks) Compute the two-sided 90% confidence interval for the intercept coefficient β_1 . Briefly explain what the two-sided 90% confidence interval means.

A 90% confidence interval for β_1 is

$$\hat{\beta}_1 \pm t_{\frac{0.1}{2}, 16-2} \cdot \text{se}(\hat{\beta}_1)$$

$$0.8169 \pm t_{0.05, 14} \sqrt{0.1177}$$

$$0.8169 \pm 1.761 \times \sqrt{0.1177}$$

$$0.8169 \pm 0.6042 \quad - \textcircled{1}$$

In the long-run, in 90 out of 100 cases intervals like $\textcircled{1}$

will contain the true β_1 .

(g)(5 Marks) Perform a test of the null hypothesis $H_0 : \beta_1 = 0$ against the alternative hypothesis $H_1 : \beta_1 \neq 0$ at the 1 % significance level (i.e., for significance level $\alpha = 0.01$). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.

Step ①

$$H_0: \beta_1 = 0$$

$$H_1: \beta_1 \neq 0$$

Step ②

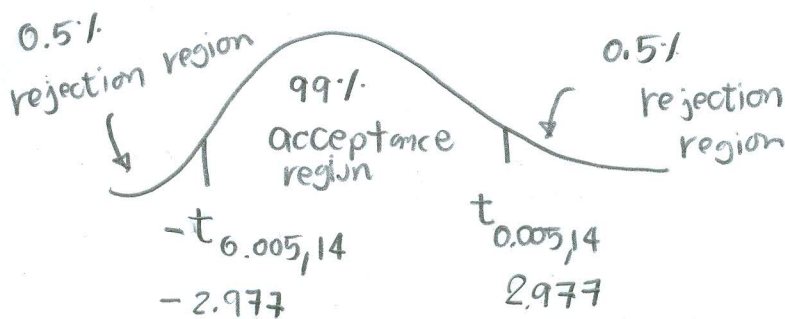
$$\alpha = 0.01$$

Step ③

$$t_{cal} = \frac{0.8169}{0.3431} = 2.3809$$

Step ④

$$t_{critical} = t_{\frac{0.01}{2}, 14} = 2.977$$



Step ⑤ compare b/w t_{cal} and $t_{critical}$ value

$$|t_{cal}| < t_{0.005, 14}$$

We cannot reject the null hypothesis that $H_0: \beta_1 = 0$ with 99% confidence interval.

(h) (5 Marks) Perform a test of the null hypothesis $H_0 : \sigma^2 \geq 0.025$ against the alternative hypothesis $H_1 : \sigma^2 < 0.025$ at the 5 % significance level (i.e., for significance level $\alpha = 0.05$). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.

Step 1:

$$H_0: \sigma^2 \geq 0.025$$

$$H_1: \sigma^2 < 0.025$$

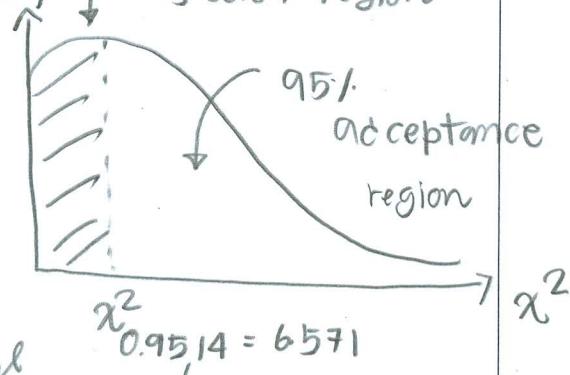
Step 2: $\alpha = 0.05$.

Step 3:

$$\chi^2_{cal} = (n-2) \cdot \frac{\hat{\sigma}^2}{\sigma^2}$$

$$= (16-2) \cdot \frac{0.0413}{0.025} = 93.072$$

Step 4: $\chi^2_{critical} = \chi^2_{1-0.05, 14}$



Step 5: Compare χ^2_{cal} and $\chi^2_{critical}$

$$\text{Since } \chi^2_{cal} > \chi^2_{critical}$$

$\therefore$ With $\alpha = 0.05$ we cannot reject the null hypothesis

that $\sigma^2 \geq 0.025$ #.