Example 3.J: Excess burden *formula under linear model & Tax-Revenue-maxizing tax rate*

Demand: $p^d = a - bQ^d \quad ; \; a \geq 0, \quad b \leq 0.$

Supply : $p^s = c + dQ^s \quad ; \quad d \geq 0.$

- Solve for quantity and prices equilibrium when the unit tax is imposed. Analyze the result

Equilibrium befor tax:

$$p_d = p_s$$
$$a - bQ = c + dQ$$
$$dQ + bQ = a - c$$
$$Q^* = \frac{a-c}{d+b}$$

$$p^* = c + d\left(\frac{a-c}{d+b}\right)$$

Equilibrium after tax:

$$p_s + t = c + dQ$$
$$p_s = c + dQ - t$$

$$a - bQ = c + dQ - t$$
$$dQ + bQ = a - c - t$$
$$Q_t = \frac{a-c-t}{d+b}$$

$$p_t = c + d\left(\frac{a-c-t}{d+b}\right)$$

Unit tax makes equilibrium quantity decreases by $\frac{t}{d+b}$ units but rises equilibrium price by $\frac{dt}{d+b}$.

- Derive the excess burden formula for buyers and sellers

$$\text{Consumer burden} = (P_t - P^*)Q_t$$

$$= \frac{dt}{d+b} \times \frac{a-c-t}{d+b}$$

$$= \frac{dt(a-c-t)}{(d+b)^2}$$

$$\text{Producer burden} = (P^* - P_s)Q_t$$

$$P_s = c + dQ_t$$

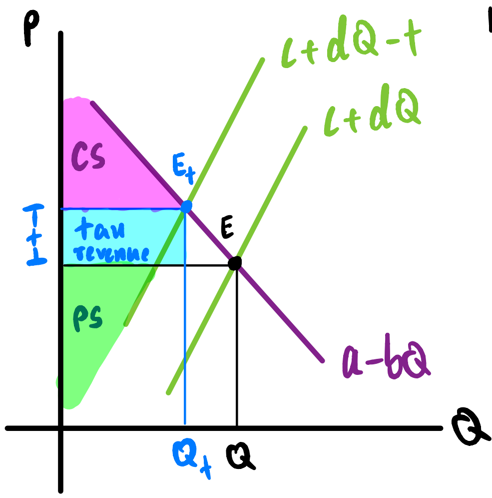
$$= c + d\left(\frac{a-c-t}{d+b}\right)$$

$$(P^* - P_s)Q_t = \left(c + d\left(\frac{a-c}{d+b}\right) - c + d\left(\frac{a-c-t}{d+b}\right)\right)Q_t$$

$$= \left(d\left(-\frac{t}{d+b}\right)\right)Q_t$$

$$= -\frac{dtQ_t}{d+b}$$

- Calculate the tax rate that maximizes the tax revenue of government.



maximize: $R = tQ_t$

$$= t\left(\frac{a-c-t}{d+b}\right)$$

$$= \frac{at-ct-t^2}{d+b}$$

$$\frac{dR}{dt} = \frac{a-c-2t}{d+b} = 0$$

$$a-c-2t = 0$$

$$t = \frac{a-c}{2} \#$$