

EE325 Section 1 HW 2 Due Thursday February 20<sup>th</sup> (23:00 hr.), 2020

Use 4 decimal places for numerical answers

1. In Table 1.a.  $X_i$  is total microeconomics exam point (total points are 100) and  $Y_i$  is GPA of each student.

Table 1.a

Student	$Y_i$	$X_i$
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\bar{x} = \left[ \frac{63+72+78+9+87+75+75+90}{8} \right]$$

$$\bar{x} = 77.625$$

$$\bar{y} = \frac{2.8+3.4+3+3.5+3.6+3+2.7+3.7}{8}$$

$$\bar{y} = 3.2125$$

1.1 Now consider the two-variable  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Use OLS to find the estimator of  $\beta_0$  and  $\beta_1$ . (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$$

$$\hat{\beta}_1 = \frac{2012.4 - 8(77.625)(3.2125)}{48717 - 8(77.625)^2} = \frac{2012.4 - 1994.9625}{48717 - 48205.125} = \frac{31}{910}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_0 = 3.2125 - 0.034066(77.625) = 0.5681$$

1.2 For each observation  $i$ , find  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$$\hat{u}_i = y_i - \hat{y}_i$$

$$\begin{aligned} i=1, \hat{u}_1 &= 2.8 - 2.714 = 0.086 \\ i=2, \hat{u}_2 &= 3.4 - 3.021 = 0.379 \\ i=3, \hat{u}_3 &= 3 - 3.225 = -0.225 \\ i=4, \hat{u}_4 &= 3.5 - 3.327 = 0.173 \\ i=5, \hat{u}_5 &= 3.6 - 3.532 = 0.068 \\ i=6, \hat{u}_6 &= 3 - 3.123 = -0.123 \\ i=7, \hat{u}_7 &= 2.7 - 3.123 = -0.423 \\ i=8, \hat{u}_8 &= 3.7 - 3.634 = 0.066 \end{aligned}$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\begin{aligned} i=1, \hat{y}_1 &= 0.5681 + 0.034(63) = 2.714 \\ i=2, \hat{y}_2 &= \text{-----} (72) = 3.021 \\ i=3, \hat{y}_3 &= \text{-----} (78) = 3.225 \\ i=4, \hat{y}_4 &= \text{-----} (81) = 3.327 \\ i=5, \hat{y}_5 &= \text{-----} (87) = 3.532 \\ i=6, \hat{y}_6 &= \text{-----} (75) = 3.123 \\ i=7, \hat{y}_7 &= \text{-----} (75) = 3.123 \\ i=8, \hat{y}_8 &= \text{-----} (90) = 3.634 \end{aligned}$$

$$\sum_{i=0}^N \hat{u}_i = 0.086 + 0.379 + (-0.225) + 0.173 + 0.068 + (-0.123) + (-0.423) + 0.066 = 0$$

1.3 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

$$\sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.435}{8-2} = 0.0725$$

$$var(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum (X_i - \bar{X})^2} (\sigma^2) = \frac{48717 (0.07245)}{8 (511.875)} = 0.863$$

$$var(\hat{\beta}_0) = \frac{\sigma^2}{\sum (X_i - \bar{X})^2} = \frac{0.0725}{511.875} = 0.000141$$

2. Data is listed in the table

$X_i$	$Y_i$
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Find estimators of  $\beta_0$  and  $\beta_1$  from the OLS method and interpret the meaning.

$$\begin{aligned} \hat{\beta}_1 &= \frac{\sum X_i Y_i - n \bar{X} \bar{Y}}{\sum X_i^2 - n \bar{X}^2} \\ &= \frac{2214 - 10(20)(9.1)}{4440 - 10(20)^2} \\ &= \frac{2214 - 1820}{4440 - 4000} = 0.895 \end{aligned}$$

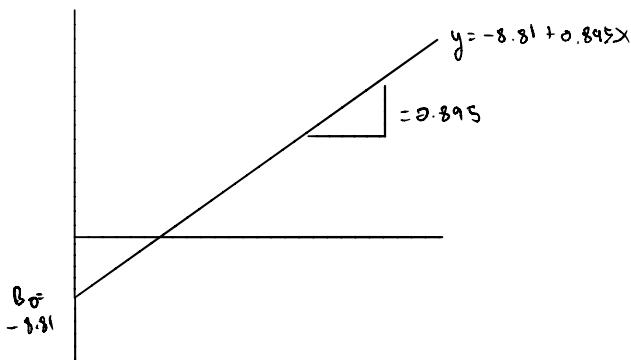
$$\begin{aligned} \hat{\beta}_0 &= \bar{Y} - \hat{\beta}_1 \bar{X} \\ &= 9.1 - 0.895(20) \\ &= -0.81 \end{aligned}$$

2.2 Find the value of  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$$\begin{aligned} i=1, \hat{y}_1 &= 0.1453, \hat{u}_1 = -0.145 \\ i=2, \hat{y}_2 &= 1.9364, \hat{u}_2 = 0.064 \\ i=3, \hat{y}_3 &= 3.727, \hat{u}_3 = 1.273 \\ i=4, \hat{y}_4 &= 5.518, \hat{u}_4 = 0.482 \\ i=5, \hat{y}_5 &= 7.309, \hat{u}_5 = -0.309 \\ i=6, \hat{y}_6 &= 9.1, \hat{u}_6 = -0.891 \\ i=7, \hat{y}_7 &= 10.892, \hat{u}_7 = -2.682 \\ i=8, \hat{y}_8 &= 12.682, \hat{u}_8 = 0.527 \\ i=9, \hat{y}_9 &= 14.4727, \hat{u}_9 = -0.264 \\ i=10, \hat{y}_{10} &= 16.2636, \hat{u}_{10} = 1.945 \end{aligned}$$

$$\begin{aligned} \sum_{i=0}^N \hat{u}_i &= (-0.145) + 0.064 + 1.273 + 0.482 + (-0.309) \\ &+ (-0.891) + (-2.682) + 0.527 + (-0.264) \\ &+ 1.945 = 0 \end{aligned}$$

2.3 Plot graph and draw regression line. Does the line pass  $(\bar{X}, \bar{Y})$ ?



sub  $\bar{x}$  to regression fn.

$$\begin{aligned} y &= -8.81 + 0.895(20) \\ &= 9.1 = \bar{y} \end{aligned}$$

$\therefore$  line passes  $(\bar{x}, \bar{y})$

2.4 If  $X_i = 16$ , what is the predicted Y?

$$\begin{aligned} \hat{y}_i &= 0_0 + \beta_1 x_i = -8.81 + 0.895(16) \\ &= 5.51 \end{aligned}$$

2.5 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

$$\sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.091}{10-2} = 1.7614$$

$$var(\hat{\beta}_0) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = \frac{1.7614}{440} = 0.004$$

$$var(\hat{\beta}_1) = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2} (\sigma^2) = \frac{4440(1.7614)}{10(440)} = 1.777$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where  $u_i \sim \text{NIID}(0, \sigma^2)$ . Find an OLS estimator of  $\beta_1$ . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

SLR 1-4,  $\hat{\beta}_1 = \text{unbiased}$

$$\hat{\beta}_1 = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

$$\begin{aligned} \hat{\beta}_1 &= \sum_{i=1}^n (y_i - \bar{y}) k_i \\ &= \sum_{i=1}^n (\beta_0 + \beta_1 x_i + u_i - \beta_0 - \beta_1 \bar{x}) k_i \\ &= \sum_{i=1}^n (\beta_1 (x_i - \bar{x}) + u_i) k_i \\ &= \beta_1 \sum_{i=1}^n (x_i - \bar{x}) k_i + \sum_{i=1}^n u_i k_i \\ &= \beta_1 \sum_{i=1}^n k_i \left( \frac{\sum_{i=1}^n x_i}{\sum_{i=1}^n k_i} - \bar{x} \right) + \sum_{i=1}^n u_i k_i \\ &= \beta_1 \frac{\sum_{i=1}^n x_i k_i}{\sum_{i=1}^n k_i} + \sum_{i=1}^n u_i k_i \end{aligned}$$

$$E(\hat{\beta}_1) = E\left[\beta_1 + \sum_{i=1}^n u_i k_i\right] = \beta_1 + E\left[\sum_{i=1}^n u_i k_i\right]$$

$$E(\hat{\beta}_1) = \beta_1 + \sum_{i=1}^n k_i E(u_i)$$

$$E(\hat{\beta}_1) = \beta_1$$

$\therefore$  unbiased.