Selected Questions: EE 325

Q 3.9

(a) $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 X_i$ and $\hat{\alpha}_1 = \bar{Y} - \hat{\beta}_2 \bar{x}$ [Note: $x_i = (X_i - \bar{X})$]

$$= \bar{Y}, \text{ since } \sum x_i = 0$$

$$\text{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n\sum x_i^2}\sigma^2 \text{ and var}(\hat{\alpha}_1) = \frac{\sum x_i^2}{n\sum x_i^2}\sigma^2 = \frac{\sigma^2}{n}$$

Therefore, neither the estimates nor the variances of the two estimators are the same.

(b) $\hat{\beta}_2 = \dfrac{\sum x_i y_i}{\sum x_i^2}$ and $\hat{\alpha}_1 = \dfrac{\sum x_i y_i}{\sum x_i^2}$, since $x_i = (X_i - \bar{X})$

It is easy to verify that $\text{var}(\hat{\beta}_2) = \text{var}(\hat{\alpha}_2) = \dfrac{\sigma^2}{\sum x_i^2}$

That is, the estimates and variances of the two slope estimators are the same.

(c) Model II may be easier to use with large X numbers, although

with high speed computers this is no longer a problem.

Q3.10

Since $\sum x_i = \sum y_i = 0$, that is, the sum of the deviations from mean value is always zero, $\bar{x} = \bar{y} = 0$ are also zero. Therefore, $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 0$. The point here is that if both Y and X are expressed as deviations from their mean values, the regression line will pass through the origin.

$$\hat{\beta}_2 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{\sum x_i y_i}{\sum x_i^2}$$, since means of the two variables are zero. This is equation (3.1.6).

Q5.5

(*a*) Use the *t* test to test the hypothesis that the true slope coefficient is one. That is obtain: $t = \frac{\hat{\beta}_2 - 1}{se(\hat{\beta}_2)} = \frac{1.0598 - 1}{0.0728} = 0.821$

For 238 df this *t* value is not significant even at $\alpha = 10\%$. The conclusion is that over the sample period, IBM was not a volatile security.

(*b*) Since $t = \frac{0.7264}{0.3001} = 2.4205$, which is significant at the two percent level of significance. But it has little economic meaning. Literally interpreted, the intercept value of about 0.73 means that even if the market portfolio has zero return, the security's return is 0.73 percent.

Q5.5

Under the normality assumption, $\hat{\beta}_2$ is normally distributed. But since a normally distributed variable is continuous, we know from probability theory that the probability that a continuous random variable takes on a specific value is zero. Therefore, it makes no difference if the equality is strong or weak.

Q5.6 (Only a and b)

(*a*) There is positive association in the LFPR in 1972 and 1968, which is not surprising in view of the fact since WW II there has been a steady increase in the LFPR of women.

(*b*) Use the one-tail *t test.*

$$t = \frac{0.6560 - 1}{0.1961} = -1.7542$$. For 17 df, the one-tailed *t* value at α=5% is 1.740. Since the estimated t value is significant, at this level of significance, we can reject the hypothesis that the true slope coefficient is 1 or greater.

(c) The mean LFPR is : 0.2033 + 0.6560 (0.58) $\approx$ 0.5838. To establish a 95% confidence interval for this forecast value, use the formula: 0.5838 $\pm$ 2.11(se of the mean forecast value), where 2.11 is the 5% critical *t* value for 17 df. To get the standard error of the forecast value, use Eq. (5.10.2). But note that since the authors do not give the mean value of the LFPR of women in 1968, we cannot compute this standard error.

(*d*) Without the actual data, we will not be able to answer this question because we need the values of the residuals to plot them and obtain the Normal Probability Plot or to compute the value of the Jarque-Bera test.