Topic 5

Consumption under Uncertainty

Tools for Describing Risky Outcomes

Definition: A **lottery** is any event with an uncertain outcome.

Examples: Investment, Roulette, Football Game.

Definition: A **probability** of an outcome *(of a lottery)* is the likelihood that this outcome occurs.

Example: The probability often is estimated by the historical frequency of the outcome.

Tools for Describing Risky Outcomes

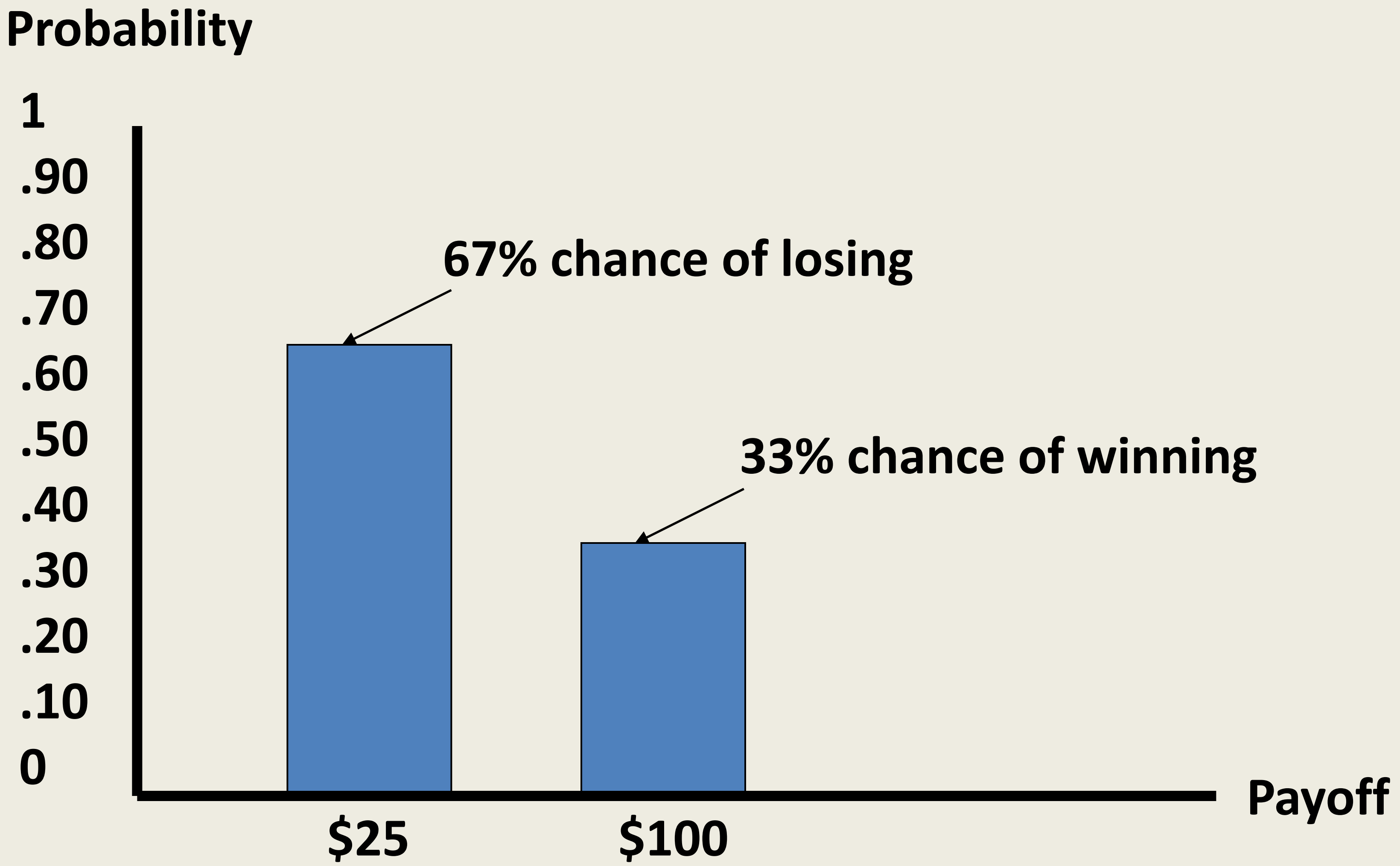
Definition: The **probability distribution** of the lottery depicts all possible payoffs in the lottery and their associated probabilities.

Property:

- The probability of any particular outcome is between 0 and 1.
- The sum of the probabilities of all possible outcomes equals 1.

Definition: Probabilities that reflect subjective beliefs about risky events are called **subjective probabilities**.

Probability Distribution



Expected Value and Expected Utility

Definition: The **expected value** of a lottery is a measure of the average payoff that the lottery will generate.

Let X be a lottery with three possible outcomes A, B, and C.

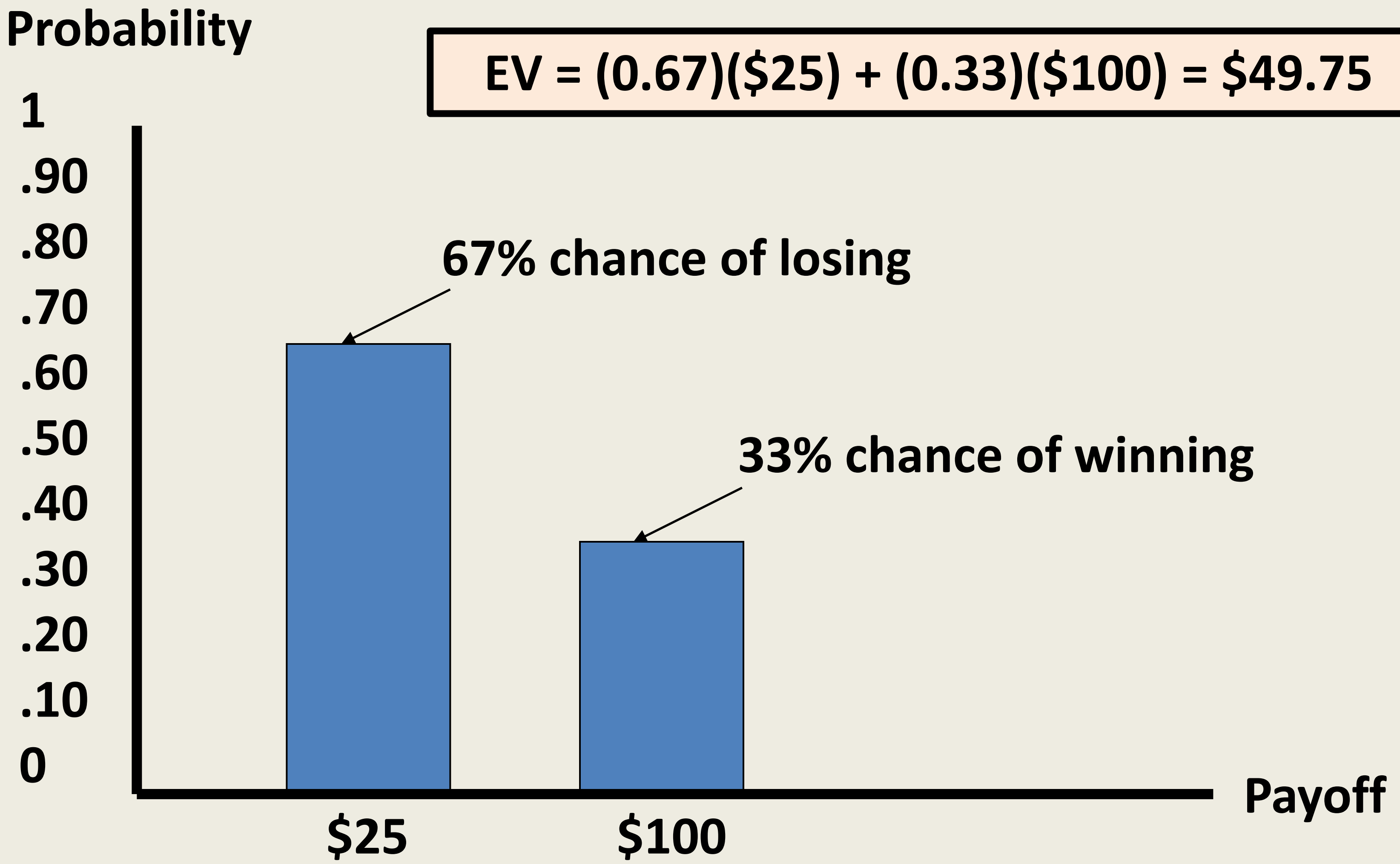
$$E[X] = Pr(A)\cdot A + Pr(B)\cdot B + Pr(C)\cdot C$$

Definition: The **expected utility** of a lottery is the average value of the utility levels that the lottery will generate.

$$E[U(X)] = Pr(A)\cdot U(A) + Pr(B)\cdot U(B) + Pr(C)\cdot U(C)$$

Expected Value

EV = (0.67)($25) + (0.33)($100) = $49.75



Variance & Standard Deviation

Definition: The **variance** of a lottery is a measure of the lottery's riskiness.

$\mathrm{Var}[X] = E[X^2] - (E[X])^2$

$\mathrm{Var}[X] = (x - E[X])^2 \cdot \Pr(X = x)$

Definition: The **standard deviation** of a lottery is the square root of the variance. It is an alternative measure of risk.

Example

X = x	X = 0	X = 1	X = 2	X = 3
Pr(X = x)	0.1	0.2	0.4	0.3

E[X] = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.3) = 1.9

Var[X] $= (x - E[X])^2 \cdot Pr(X = x)$

$= (0.1)(0-1.9)^2 + (0.2)(1-1.9)^2 + (0.4)(2-1.9)^2 + (0.3)(3-1.9)$

$= 0.89$

Var[X] $= E[X^2] - (E[X])^2$

$E[X^2] = (0^2)(0.1) + (1^2)(0.2) + (2^2)(0.4) + (3^2)(0.3) = 4.5$

$Var[X] = 4.5 - 1.9^2 = 0.89$

Risk Preferences

Notes:

- Utility as a function of income only

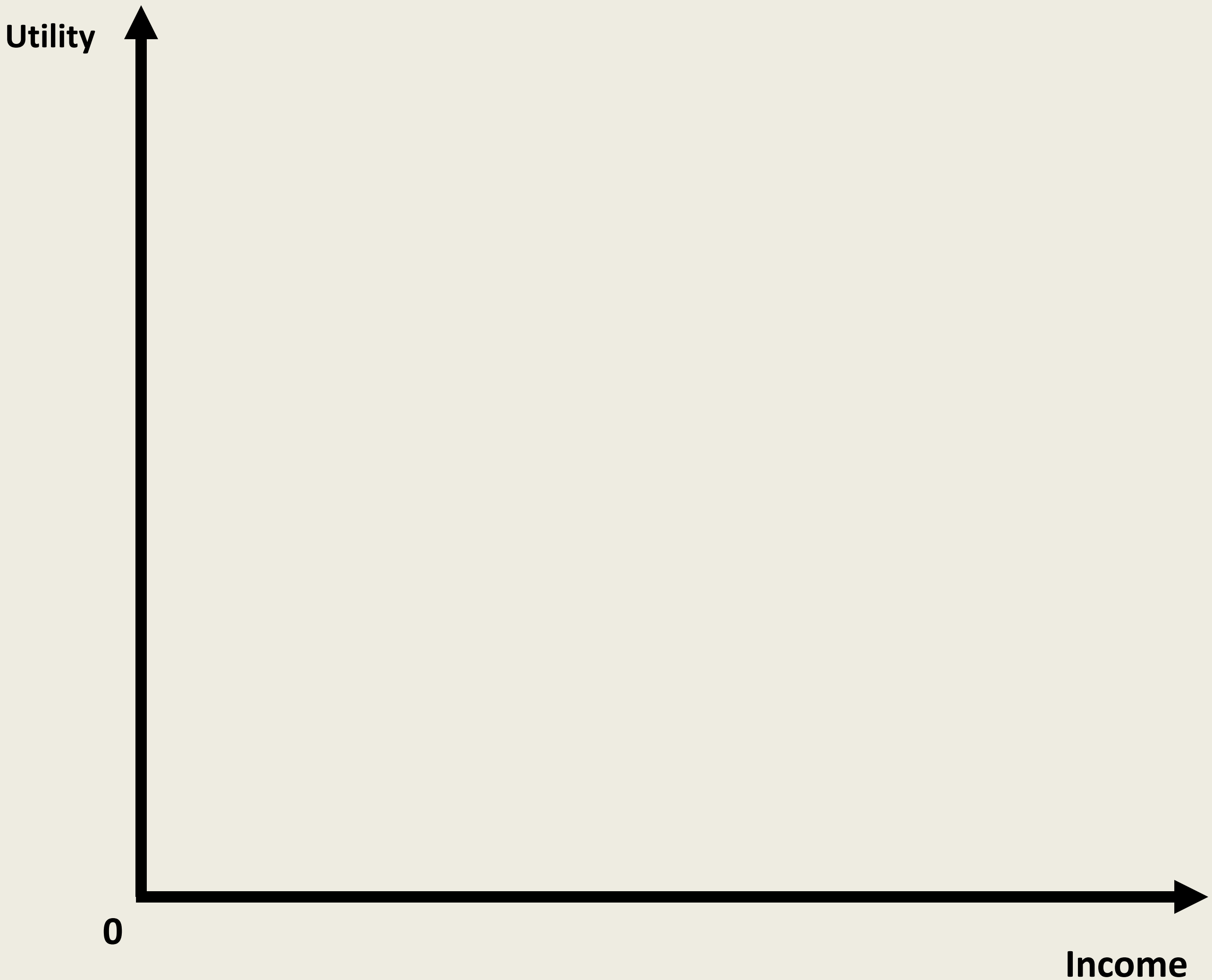
Definition: The risk preferences can be classified as follows:

An individual who prefers a sure thing to a lottery with the same expected value is **risk-averse.**

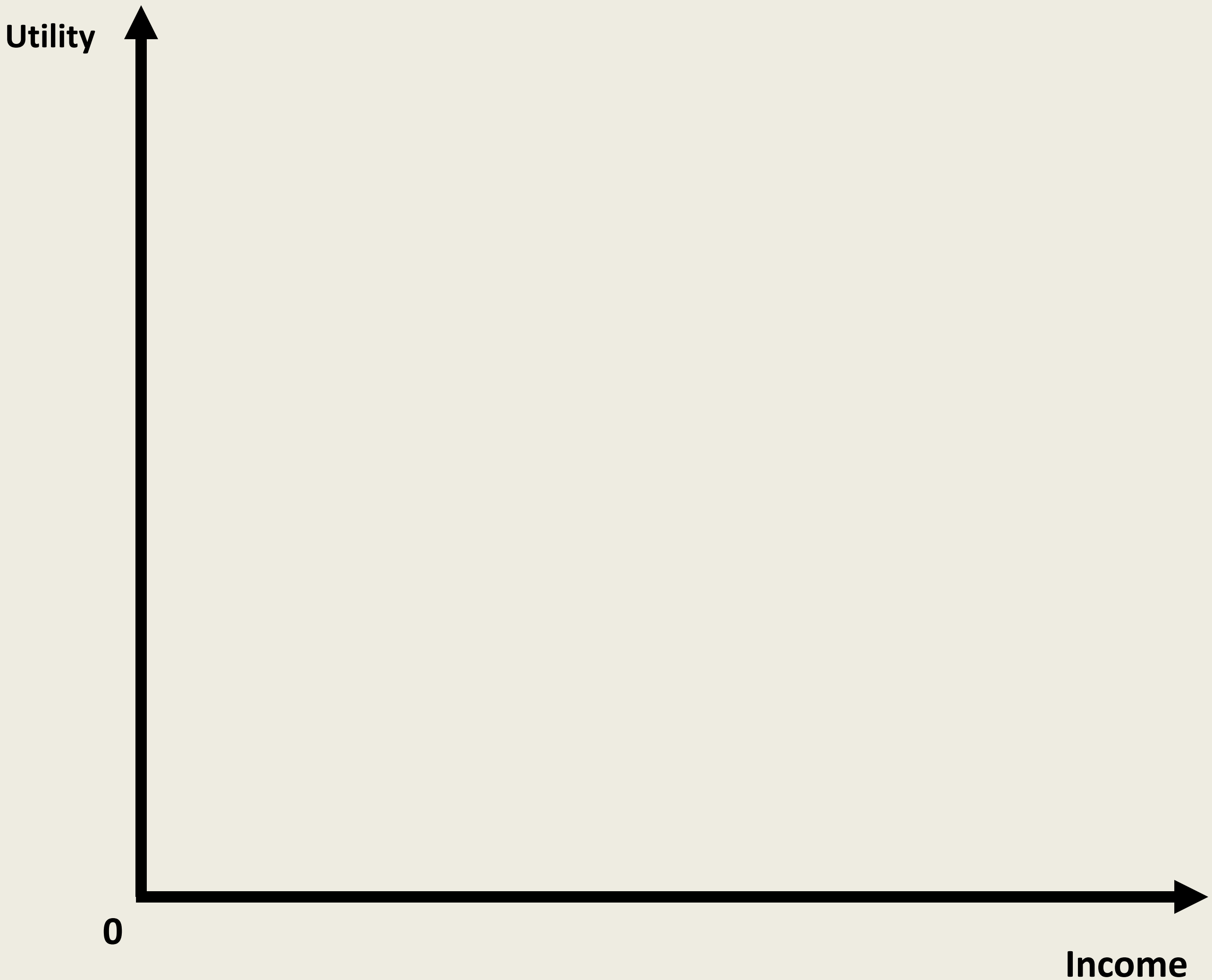
An individual who is indifferent about a sure thing or a lottery with the same expected value is **risk-neutral.**

An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is **risk-loving.**

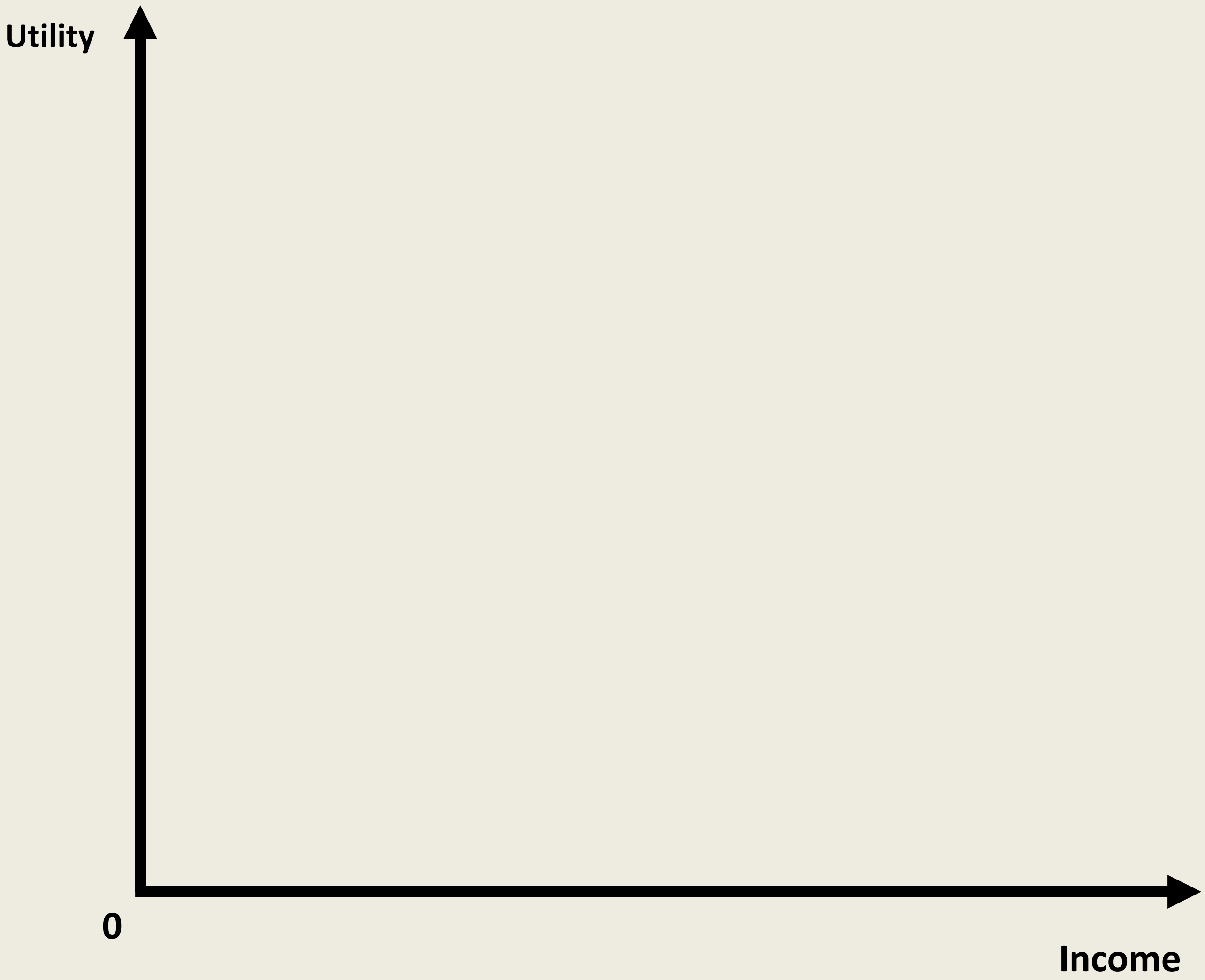
Risk-Averse Individuals



Risk-Neutral Individuals



Risk-Loving Individuals



Risk Preferences

Let X be a lottery.

U(E[X]) denotes **Utility of the EV (sure thing) of the lottery X.**

E[U(X)] denotes **Expected Utility of the lottery X.**

risk-averse: U(E[X]) > E[U(X)]

happiness from sure thing > expected happiness from lottery

risk-neutral: U(E[X]) = E[U(X)]

happiness from sure thing = expected happiness from lottery

risk-loving: U(E[X]) < E[U(X)]

happiness from sure thing < expected happiness from lottery

LEARNING-BY-DOING EXERCISE 15.1

Computing the Expected Utility for Two Lotteries

Consider the two lotteries depicted in Figure 15.3. They have the same expected value, but the first (investing in the Internet company's stock) has a larger variance than the second (investing in the public utility company's stock). This tells us that the first lottery is riskier than the second lottery. Suppose that a risk-averse decision maker has the utility function $U(I) = \sqrt{100I}$, where I denotes the payoff of the lottery.

Problem Which lottery does the decision maker prefer—that is, which one has the bigger expected utility?

One decision maker is risk neutral, with the utility function $U(I) = 100I$, while the other is risk loving, with the utility function $U(I) = 100I^2$, where I denotes the payoff of the lottery.

Problem

(a) Which lottery does the risk-neutral decision maker prefer?

(b) Which lottery does the risk-loving decision maker prefer?

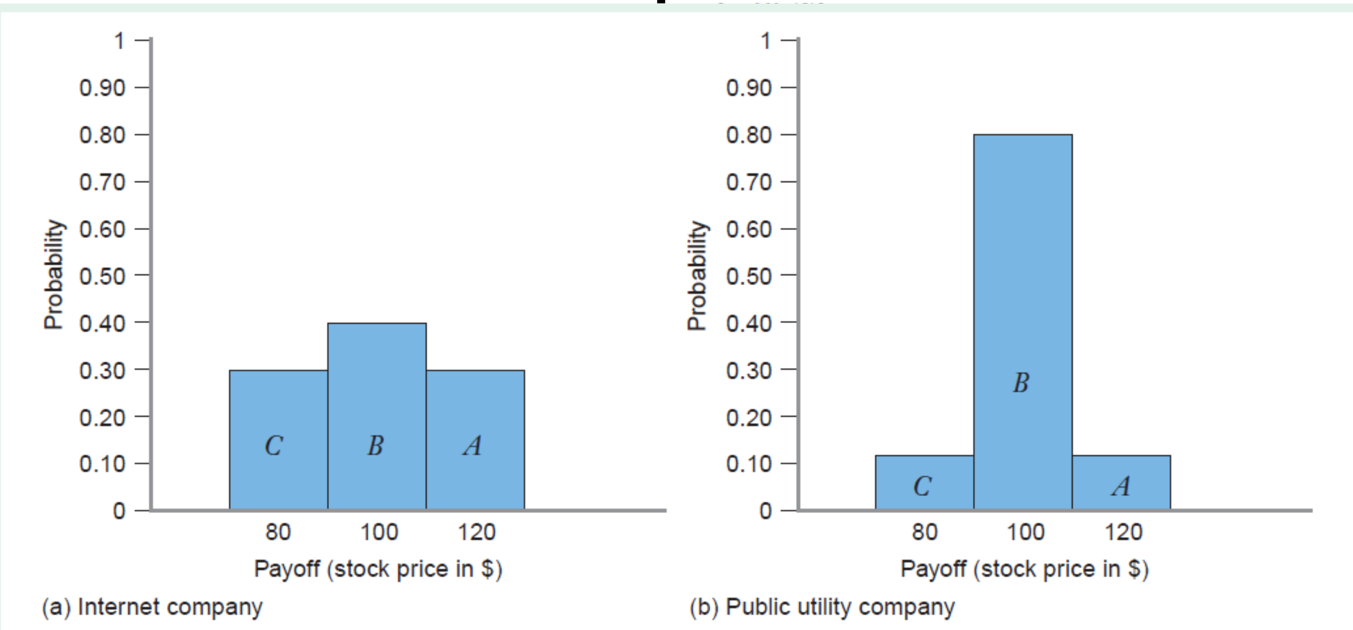


FIGURE 15.3 Probability Distributions, Riskiness, and Variances

Risk Premium

Most people are risk-averse, but some are willing to take risks if they are fairly compensated. The money needed as the compensation is called the **risk premium**.

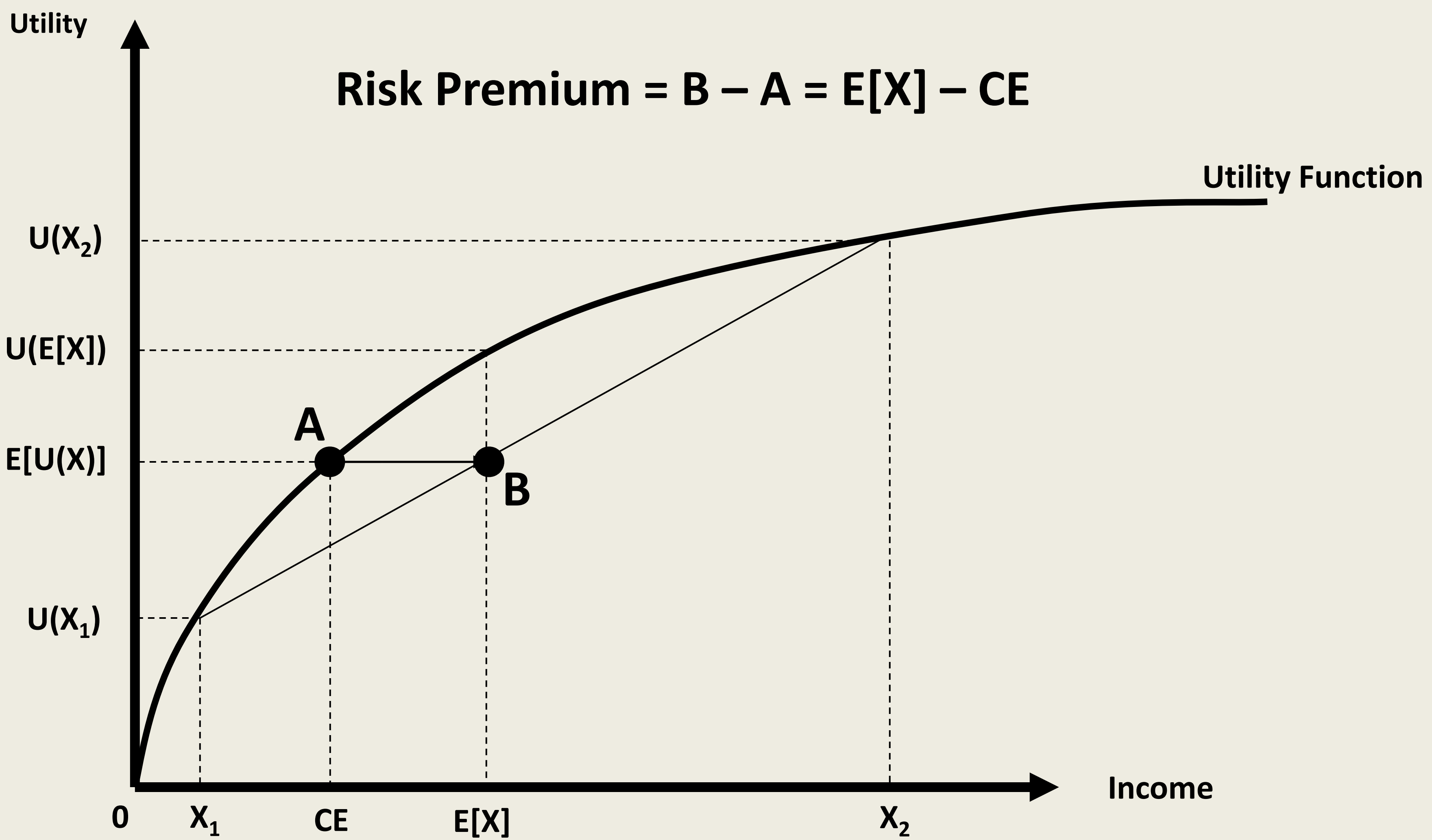
Definition: The **certainty equivalent (CE) i**s the guaranteed amount of money (sure thing) such that an individual is indifferent between the lottery and the said money.

$$U(CE) = E[U(X)]$$

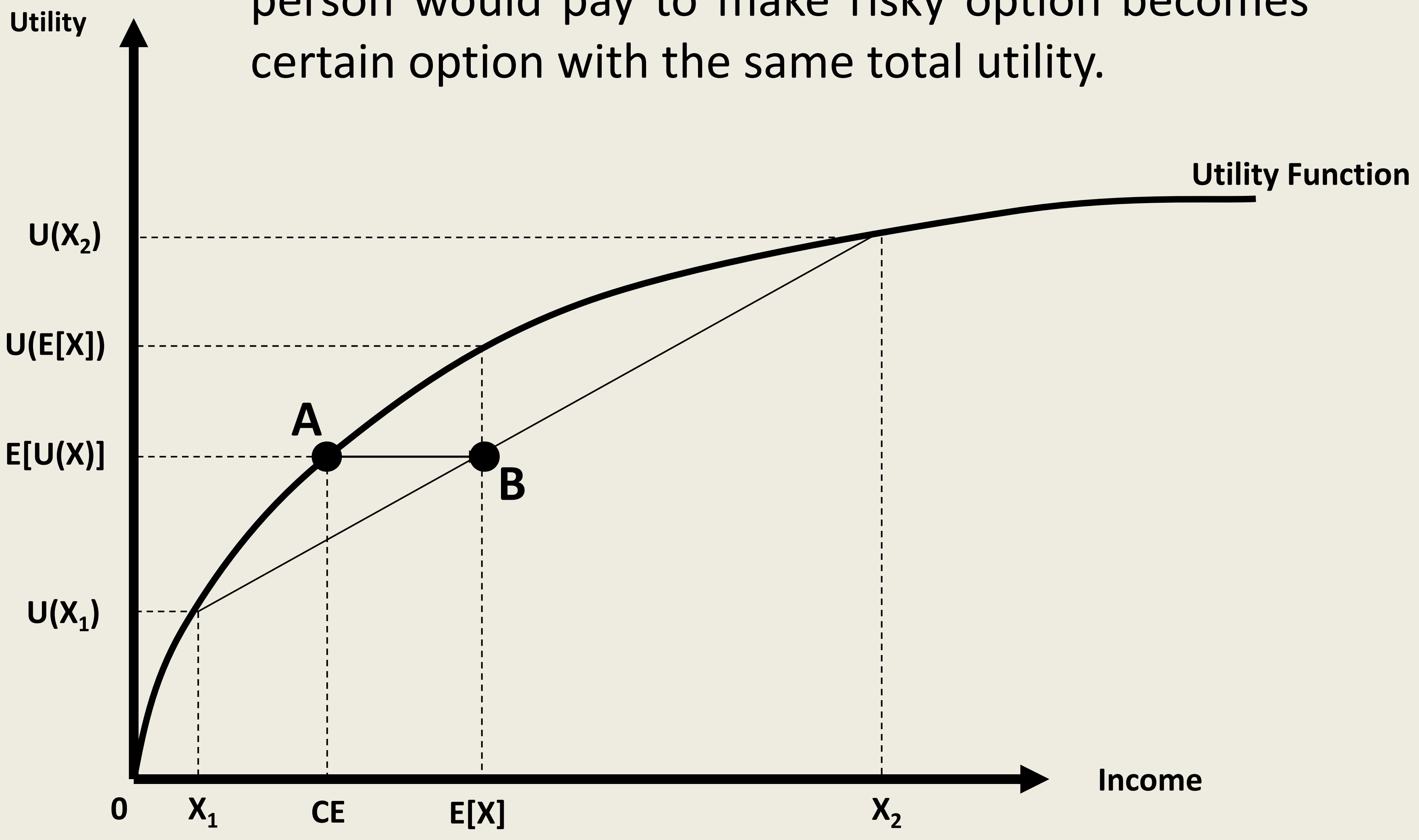
Definition: The **risk premium (RP)** of a lottery is the difference between the EV of the lottery and its CE.

$$RP = E[X] - CE$$

Risk Premium



Risk Premium (B-A)
The maximum amount of money that a risk-averse person would pay to make risky option becomes certain option with the same total utility.



Risk Premium

Risk-Averse: **CE < E[X]** **Risk Premium is positive**

They are willing to accept a smaller sure thing rather than the lottery.

Risk-Loving: **CE > E[X]** **Risk Premium is negative**

They require a larger sure thing because they love the lottery.

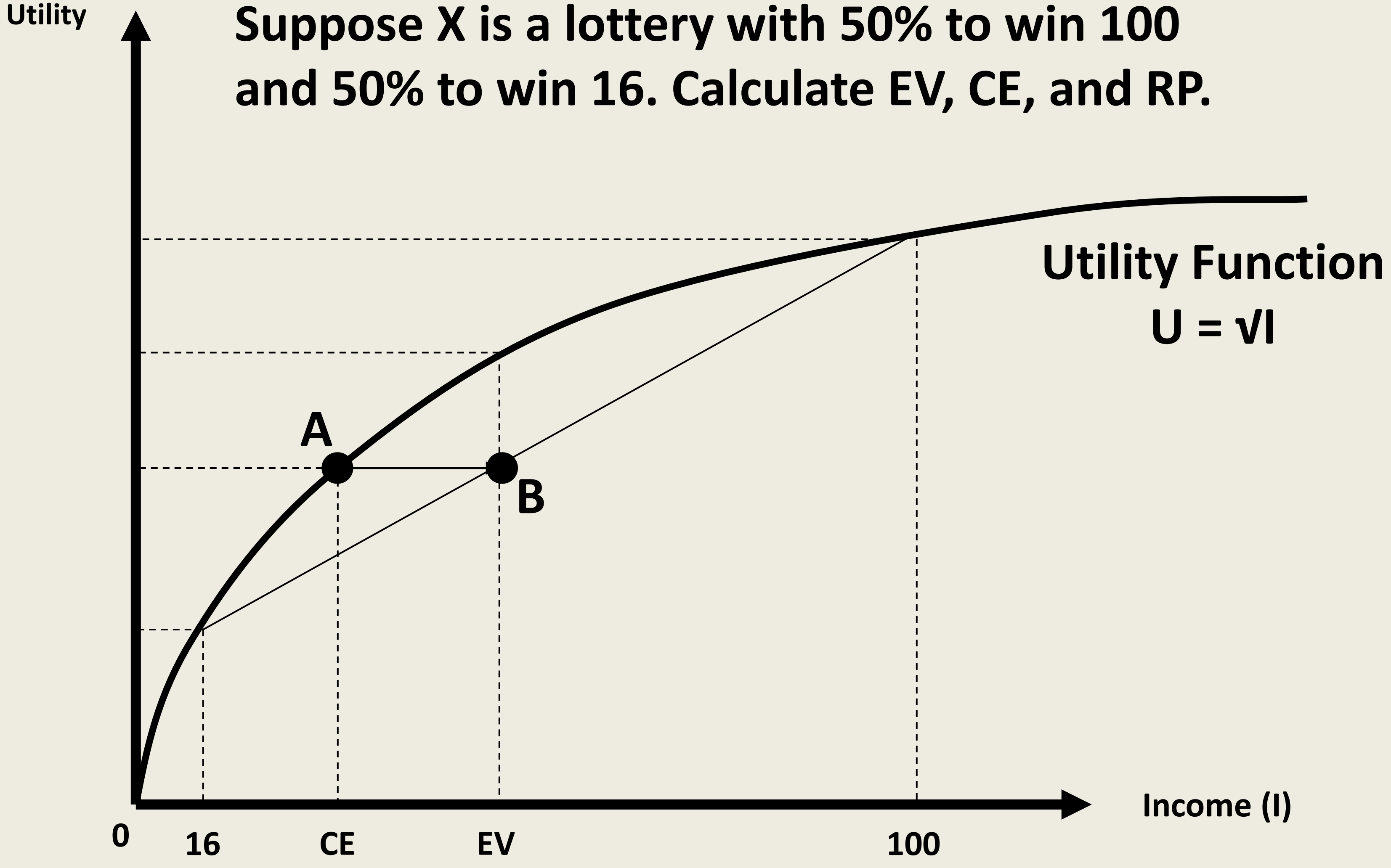
Risk-Neutral: **CE = E[X]** **Risk Premium is zero**

They are indifferent between the sure thing and the lottery.

The larger the variance of the lottery, the riskier the lottery, and the larger the risk premium

Risk Premium – Example

Suppose X is a lottery with 50% to win 100 and 50% to win 16. Calculate EV, CE, and RP.



Insurance

Risk Premium is the money that risk-averse people accept to take risks.

Insurance Premium is the "price" of the insurance that risk-averse people buy in order to eliminate the risks.

Fairly-Priced Insurance Policy is a policy where the insurance premium is equal to the expected value of the insurance payment.

Fairly-Priced Insurance – Example

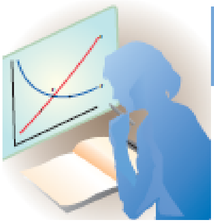
Example

Insurance Premium (price) = $500

It promises to pay $10,000 if Bad Thing happens.
It promises to pay $0 if Nothing happens.

Pr(BadThings) = 5% Pr(Nothing) = 95%

500 = .05(10,000) + .95(0)
That is, insurance premium = expected payment.
Thus, <u>this insurance is fairly priced</u>.



LEARNING-BY-DOING EXERCISE 15.4

The Willingness to Pay for Insurance

Your current disposable income is $90,000. Suppose that there is a 1 percent chance that your house may burn down, and if it does, the cost of repairing it will be $80,000, reducing your disposable income to $10,000. Suppose, too, that your utility function is $U = \sqrt{I}$.

Problem

(a) Would you be willing to spend $500 to purchase an insurance policy that fully insures you against your loss?

(b) What is the highest price that you would be willing to pay for an insurance policy that fully insures you in the event that your house burns down?

Adverse Selection & Moral Hazard

Definition: **Asymmetric Information** is a situation in which one party knows more about its own actions or characteristics than another party.

Definition: **Adverse Selection** is *opportunism* characterized by an informed person's taking advantage of a less informed person who does not know about an *unobserved characteristic* of the informed person.

Definition: **Moral Hazard** is *opportunism* characterized by an informed person's taking advantage of a less informed person through an *unobserved action.*

Adverse Selection & Moral Hazard

Adverse Selection / Hidden Information:

- Good and Bad cars in second-hand garages.
- Healthy and Unhealthy people in health insurance markets.

Moral Hazard / Hidden Action:

- People drive carelessly after buying car insurances.
- Employers fails to observe lazing employees.
- Doctors over-prescribe treatments for patients with health insurances.