Lecture 7
Bond Prices and Yields

Pym Manopimoke, PhD, Bank of Thailand

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Bond Characteristics

- Bonds are a vehicle by which borrowing and lending is facilitated
- Bonds are debt securities = claims on a specified periodic stream of income (or a stream of income that is determined according to a specified formula or contract)
- *Issuers* (i.e. sellers) are borrowers
  They are required to return the principal amount borrowed (face value) at maturity of the contract
  Underwriters administer the public issuance in exchange for fees
- *Holders* are creditors (i.e. lenders)
  They become the recipient of the payments
- These securities are relatively easy to understand because the payment formulas are specified in advance

Bond Characteristics

- Maturity
- Face value or par
- Coupon rate and frequency of coupon payments
- Issuer
- Tax treatment
- Bond indenture

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Bond Indenture

Valuing Bonds

- Cash flow is contractually specified
  - → A *'zero-coupon bond'* is issued that makes no coupon payment but sold at price below par
  - → A typical *'coupon bond'* obligates the issuer to make semiannual payments of interest to the bondholder for the life of the bond
- Determine cash flow from contract terms

cash flow of a 2-year zero coupon bond

				1000	
issue	0.5	1.0	1.5	2.0	years

2. Coupon bonds (par $1000): semi-annual coupon payments

cash flow of a 6% coupon 2-year bond

	30	30	30	1000+30	
issue	0.5	1.0	1.5	2.0	years

- Compute present value of cash flow

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Zero Coupon Bonds

- Issues no interest/ coupon payments
- Referred to as pure discount bonds: they pay a predetermined face value amount at a specified date in the future (e.g. $1,000 or $10,000)
- Purchased at a price today that is below the face value
- The increase in value between purchase and redemption represents the interest earned

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Types of Zero Coupon Bonds

- US Treasury Bills
  - 1 month, 3 month, 6 month and 1 year maturities
- US Treasury Separate Trading of Registered Interest and Principal Securities (STRIPS)
  - 6 months to 10 year maturities

Pricing Zero Coupon Bonds

- The price is the present value of the future face value

Suppose the government promises to pay you $1,000 in exactly 5 years and assume the relevant discount rate is 5%. What is the value or price of this commitment the government is making to you?

Interest Rate Sensitivity (ZCB)

- 1 year zero coupon bond: $P = FV(1+r)^{-1}$

$$\begin{aligned}\frac{dP}{dr} &= \frac{d}{dr} FV(1+r)^{-1} \\ &= -FV(1+r)^{-2} = -FV(1+r)^{-1}(1+r)^{-1} \\ &= -P(1+r)^{-1} \\ \Rightarrow \frac{dP}{P} &= -(1+r)^{-1} dr \approx \%\Delta P\end{aligned}$$

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What about for a T year ZCB?

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Example: Interest Rate Sensitivity

Bond Maturity	Yield	Price	dP/P	dP/P = -T/(1+r) × dr
1	0.01	$ 990.10		
	0.02	$ 980.39	-0.98%	-0.98%
	0.03	$ 970.87	-0.97%	-0.97%
	0.04	$ 961.54	-0.96%	-0.96%
	0.05	$ 952.38	-0.95%	-0.95%
	0.06	$ 943.40	-0.94%	-0.94%
	0.07	$ 934.58	-0.93%	-0.93%
	0.08	$ 925.93	-0.93%	-0.93%
	0.09	$ 917.43	-0.92%	-0.92%

Bond Maturity	Yield	Price	dP/P	dP/P = -T/(1+r) × dr
10	0.01	$ 905.29		
	0.02	$ 820.35	-9.38%	-9.80%
	0.03	$ 744.09	-9.30%	-9.71%
	0.04	$ 675.56	-9.21%	-9.62%
	0.05	$ 613.91	-9.13%	-9.52%
	0.06	$ 558.39	-9.04%	-9.43%
	0.07	$ 508.35	-8.96%	-9.35%
	0.08	$ 463.19	-8.88%	-9.26%
	0.09	$ 422.41	-8.80%	-9.17%
	0.1	$ 385.54	-8.73%	-9.09%

Coupon Bonds

- Coupon bonds makes periodic coupon interest payments based on the coupon rate, face value and payment frequency
- Coupon bonds have a face value (e.g. $1,000 or $10,000) and is paid at maturity

Pricing Coupon Bonds

- Consider the following contractual terms of a bond
  - Face value = $1000
  - 3 years to maturity
  - Coupon rate = 7%
  - Annual coupon payment = Coupon rate * Face Value = $70
  - Relevant discount rate is 5%

  Cash flow from coupon bond:
  Year 1 7% * $1000 = $70
  Year 2 $70
  Year 3 $70 +$1000

Considering previous coupon bond, suppose the coupon frequency is semi-annual or twice per year:

- Face Value = $1,000
- 3 years to maturity
- Annual coupon rate = 7%
- Semi-Annual coupon payment = (Annual coupon rate/2)*Face Value
- Relevant discount rate is 5%

Cash flow:

Date 0.5	(7% * $1,000)/2 = $35
Date 1	$35 (Year 1 cash flows = $35 + $35 = $70)
Date 1.5	$35
Date 2	$35 (Year 2 cash flows = $35 + $35 = $70)
Date 2.5	$35
Date 3	$35 + $1,000 (Year 3 cash flows = $35 + $35 +$1,000 = $1,070)

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Short Cut Formula

Coupon bond price = Finite annuity plus discounted face value

$$= \left[\frac{C}{\frac{r}{n}}\right] \times \left[1 - \left(\frac{1}{1+\frac{r}{n}}\right)^{nT}\right] + \frac{FV}{\left(1+\frac{r}{n}\right)^{nT}}$$

$$= C \times PVA\left(\frac{r}{n}, nT\right) + \frac{FV}{\left(1+\frac{r}{n}\right)^{nT}}$$

where C = periodic coupon payment, n is the number of compounding periods pear year and T is the maturity date in years.

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Interest Rate Sensitivity (Coupon Bonds)

$$P = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^T} + \frac{FV}{(1+r)^T}$$

= portfolio of zero coupon bonds

$$\frac{d}{dr}P = \frac{d}{dr}\frac{C}{1+r} + \frac{d}{dr}\frac{C}{(1+r)^2} + \cdots + \frac{d}{dr}\frac{C}{(1+r)^T} + \frac{d}{dr}\frac{FV}{(1+r)^T}$$

For zero coupon bond with maturity M: $\frac{d}{dr}P = -M \times P \times \frac{1}{1+r}$

$$\frac{d}{dr}P = \frac{-1}{1+r}\left[\frac{1\times C}{1+r} + \frac{2\times C}{(1+r)^2} + \cdots + \frac{T\times C}{(1+r)^T} + \frac{T\times FV}{(1+r)^T}\right]$$

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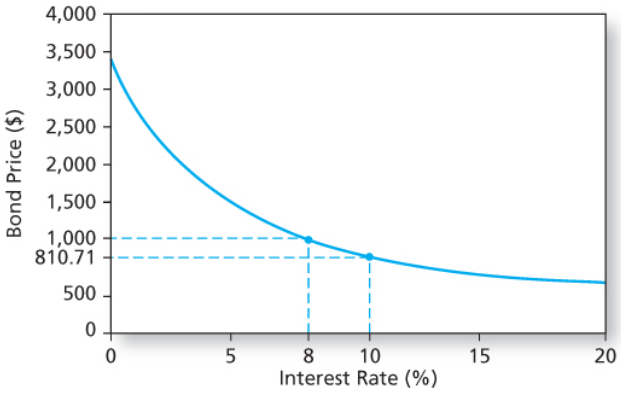
Prices and Interest Rates

- Bond prices fall as market interest rates rise
- Consider the price of a 30 year, 8% coupon bond for a range of interest rates



➢ The curve is convex – an increase in an interest rate results in a smaller price change than a decrease of equal magnitude

- The longer the maturity, the more sensitive the bond's price to changes interest rates

Time to maturity	2%	4%	6%	8%	10%
1 year	1059.11	1038.83	1019.13	1000	981.41
10 years	1541.37	1327.03	1148.77	1000	875.35
20 years	1985.04	1547.11	1231.15	1000	828.41
30 years	2348.65	1695.22	1276.76	1000	810.71

Bond Pricing Between Coupon Dates

- Buyer pays and seller receives accrued interest
- Accrued interest assumes that interest is earned continually (although paid only every six months)
- Invoice Price = Flat Price + Accrued Interest

Yield to Maturity (YTM)

- The interest rate that makes the present value of the bond's payments equal to its price
- Observe the (market) price, and solve (numerically) the formula for *r*

- YTM is the bond's internal rate of return
- YTM differs from current yield of a bond
- Example: An 8% coupon, 30-year bond is selling for $1,276.76

Calculating YTM

- For T=1 or T=2 you can solve using simple algebra
- For T=2 you need to use the formula for the solution to a quadratic equation
- For T>2 utilize numerical methods: plug & chug! (spreadsheets are a great tool)

Bond Price Conventions

- Price is often quoted as a percentage of "par value" = "face value"
- If coupon rate = yield, PV = Face value and bond sells at 100% of par
- If coupon rate > yield, PV > Face value and bond sells at more than 100% of par
- If coupon rate < yield, PV < Face value and bond sells at less than 100% of par

Price Convergence

- Bond price converges to par (but it is not necessarily a smooth path)

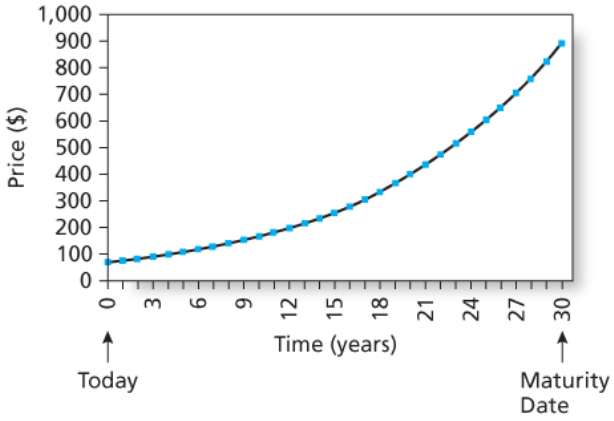
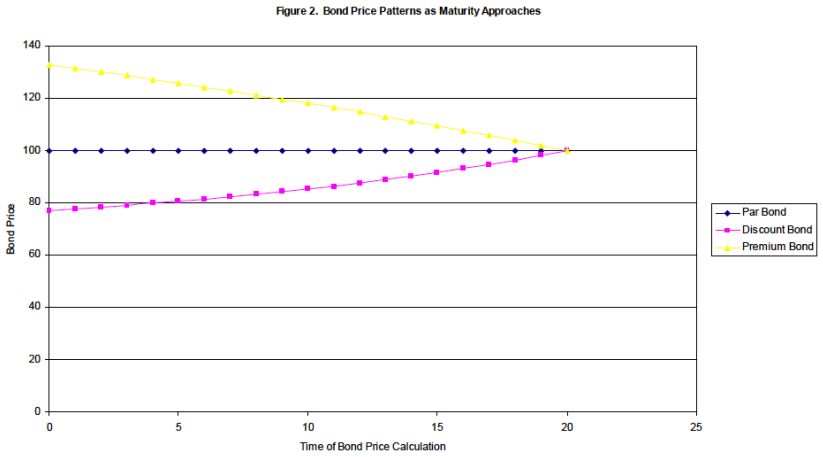


Figure 14.7 The price of a 30-year zero-coupon bond over time at a yield to maturity of 10%. Price equals $1,000/(1.10)^T$, where T is time until maturity.

YTM



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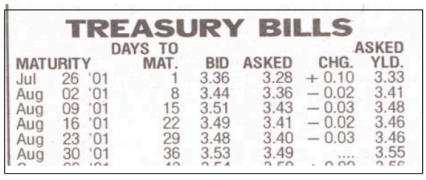
Bond Prices Over Time

- The price of a premium bond declines to par at maturity. An investor of this bond is compensated for the capital loss by a higher coupon rate than the required rate of return.
- The price of a discount bond increases to par at maturity. This capital gain is compensation to an investor due to the lower coupon rate than the required rate of return.

Bonds by Issuer: Treasury Bills

- Issued by the US government and has no credit risk
- Pays par value at maturity and pays no coupon
- Matures in less than one year



TREASURY BILLS

MATURITY	DAYS TO MAT.	BID	ASKED	CHG.	ASKED YLD.
Jul 26 '01	1	3.36	3.28	+ 0.10	3.33
Aug 02 '01	8	3.44	3.36	– 0.02	3.41
Aug 09 '01	15	3.51	3.43	– 0.03	3.48
Aug 16 '01	22	3.49	3.41	– 0.02	3.46
Aug 23 '01	29	3.48	3.40	– 0.03	3.46
Aug 30 '01	36	3.53	3.49	...	3.55

Bank discount yield convention for T-Bills

$$r_{BD} = \frac{10{,}000 - P}{10{,}000} * \frac{360}{n}$$

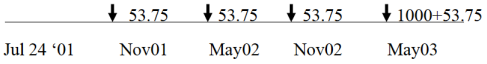
$$P = 10{,}000 * [1 - r_{BD} * (n/360)]$$

Bonds by Issuer: Treasury Notes and Bonds

- Both pay semi annual coupons
- Notes are issued with maturities of 1-10 years
- Bonds are issued with maturities longer than 10 years
- Price quotes are in "$:32nds"

Rate	Maturity	Bid	Ask	Ask yld.
10 ¾	May 03	111:21	111:25	3.93

- Cash flow $1000 face value



- Ask price = (111+25/32)% *1000 = $1117.8125

Bonds by Issuer: Municipal bonds

- Issued by state and local government
- Interest is tax-free, capital gains are not
- Yields typically lower than Treasury issues

Bonds by Issuer: Corporate Bonds

- Debt issued by corporations to finance operations
- Most are semi-annual coupon bonds
- Give investors a premium in terms of higher yield to compensate for the default risk (credit risk)
- Credit risk is rated by Moody's investor service, Standard and Poor's Corporation, Duff and Phelp, and Fitch Investor Service
- Bonds are categorized based on their credit ratings as
  - Investment grade bonds (BBB (or BBa) or above)
  - Speculative-grade bonds (Junk) (below BBB)
- Highest rating is AAA or Aaa
- Some entities such as insurance can only invest in investment grade bonds.
- Credit spread = Corporate bond yield – US Treasury bond yield

Types of Corporate Bonds

- Straight bonds: pay the promised cash flow until maturity
- Callable bonds: the issuer has the right to buy the bond back
- Puttable bonds: the investor (buyer of the bond) has the right to sell the bond back
- Convertible bonds: give the bondholders an option to exchange bond for stock
- Example: A 6% coupon 5-year convertible bond issued by GE is convertible at a conversion ratio of 25:1 (i.e. one bond for 25 GE shares)

Determinants of Bond Safety

- Coverage ratios: Ratios of company earnings to fixed cost
- Leverage ratio: Debt-to-equity ratio
- Liquidity ratios: Current ratio and quick ratio
- Profitability ratios: Measures of rates of returns on assets or equity
- Cash flow-to-debt-to-equity ratio: Total cash flow to debt

Homework

- BKM Chapter 14, Qu 4-7; 11, 15-17