Chapter 13 Curve Sketching

In this chapter we will learn how to sketch the curve of relation by considering its properties such as the intercepts, symmetrical, domain, range, and asymptotes.

Guidelines for Curve Sketching

1. **Domain (D) and Range (R)**

   Domain is the set of values of x for which $f(x)$ is defined.

   Range is the set of values of $f(x)$ correspond to the domain.

2. **Intercepts**

   To find y-intercept, we set $x = 0$ and solve for y.

   To find x-intercept, we set $y = 0$ and solve for x.

3. **Symmetry**

   (i) If $f(-x) = f(x)$ for all x in D, then the curve is symmetric about the y-axis (reflectional symmetry).

   (ii) If $f(x) = -f(x)$ for all x in D, then the curve is symmetry about the x- axis (That is, replacing y with $-y$ and get the same function).

   (iii) If $f(-x) = -f(x)$ for all x in D, then f is symmetric about the origin (rotational symmetry).

4. **Asymptotes**

   (i) *Horizontal Asymptotes*.

   If either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$, then the line $y = L$ is a horizontal asymptote of the curve $y = f(x)$.

   (ii) *Vertical Asymptotes*.

   The line $x = a$ is a vertical asymptote if at least one of the following statements is true:

$$\lim_{x\to a^+} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = \infty$$

$$\lim_{x\to a^+} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = -\infty$$

5. **Intervals of Increase or Decrease**

   Compute $f'(x)$ and use the Increasing/Decreasing Test.

   - If $f'(x) > 0$, then f is increasing.
   - If $f'(x) < 0$, then f is decreasing.

   Step 1) Find $f'(x)$

   Step 2) Find critical numbers

   Step 2.1) Find the values of x that make $f'(x)$ does not exist.

   Step 2.2) Set $f'(x) = 0$, then solve for *x*.

   Step 3) Find the sign of $f'(x)$

6. **Local Maximum and Minimum Values**

   - If f' changes from positive to negative at critical number c, then $f(c)$ is a local maximum.
   - If f' changes from negative to positive at critical number c, then $f(c)$ is a local minimum.

7. **Concavity and Points of Inflection**

   Compute $f''(x)$ and use the Concavity Test.

   - If $f''(x) > 0$, then the curve is concave upward.
   - If $f''(x) < 0$, then the curve is concave downward.
   - There exist the inflection points when $f''(x)$ changes concave upward to concave downward and vice versa.

   Step 1) Find $f''(x)$

   Step 2) Find critical numbers

   Step 2.1) Find the values of x that make $f''(x)$ does not exist.

   Step 2.2) Set $f''(x) = 0$, then solve for *x*.

   Step 3) Find the sign of $f''(x)$.

8. **Sketch the Curve** by using the information from step 1 – step 7.

Example 1: Consider the function $f(x) = x^3 + 6x^2 + 9x$. Find the domain, intercepts, symmetry, asymptotes, and sketch the curve of this function.

Example 13.2: Consider the relation $x^2 y - x^2 - y + 4 = 0$. Find the domain, intercepts, symmetry, asymptotes, and sketch the curve of this relation.

Example 13.3: Consider the relation $x^2y+2y=6$. Find the domain, intercepts, symmetry, asymptotes, and sketch the curve of this relation.