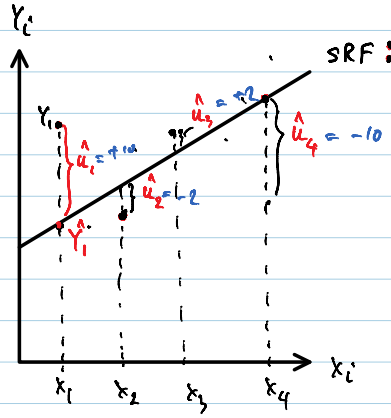
TASK: WE AIM AT ESTIMATING β_1 AND β_2 FROM SRF.



SRF: $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$Y_i = \hat{Y}_i + \hat{u}_i$

OBJECTIVE: WE WANT THE SRF LINE TO BE AS CLOSE TO Y_i AS POSSIBLE.

OBJECTIVE: $\sum \hat{u}_i = \hat{u}_1 + \hat{u}_2 + \hat{u}_3 + \hat{u}_4 + \dots = 0$ X

OBJECTIVE: $\sum_{i=1}^{n} \hat{u}_i^2$ MUST BE MINIMIZED.

$$\text{MIN} \sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

Y_i: ACTUAL Y_i; $\hat{Y}_i$: ESTIMATED Y_i OR PREDICTED Y_i OR FITTED Y_i

$$= \sum_{i=1}^{n} (Y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i))^2$$

$$\text{MIN} \left(\sum_{i=1}^{n} \hat{u}_i^2 \right) = \sum_{i=1}^{n} (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2$$

$\sum_{i=1}^{n} \hat{u}_i^2$: SUM OF RESIDUALS SQUARED

OR $\text{MIN} \sum_{i=1}^{n} \hat{u}_i^2 = f(\hat{\beta}_1, \hat{\beta}_2)$ $\Rightarrow$ OBJECTIVE FUNCTION

$\hat{\beta}_1, \hat{\beta}_2$

WE USE A METHOD CALLED "LAGRANGIAN METHOD"

$$\frac{\partial \sum_{i=1}^{n} \hat{u}_i^2}{\partial \hat{\beta}_1} = 0.$$

$$\frac{\partial \sum_{i=1}^{n} \hat{u}_i^2}{\partial \hat{\beta}_2} = 0.$$

OBJECTIVE FUNCTION: $\sum_{i=1}^{n} \hat{u}_i^2 = \sum (Y_i - \underbrace{\hat{\beta}_1 - \hat{\beta}_2 X_i}_{= \hat{Y}_i})^2$

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_1} = 2\left(\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)\right)(-1) = 0$$

$$\equiv$$

$$\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) = 0 \qquad \text{— (1)}$$

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_2} = 2\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)(-X_i) = 0 \qquad \text{— (2)}$$

w.r.t

FROM (1) :

$$\sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} \hat{\beta}_1 - \sum_{i=1}^{n} \hat{\beta}_2 X_i = 0 \qquad \text{— (3)}$$

$$\sum_{i=1}^{n} Y_i - n \cdot \hat{\beta}_1 - \hat{\beta}_2 \sum_{i=1}^{n} X_i = 0$$

$$\sum_{i=1}^{n} Y_i = n \cdot \hat{\beta}_1 + \hat{\beta}_2 \sum_{i=1}^{n} X_i$$

OR

$$n\hat{\beta}_1 = \sum_{i=1}^{n} Y_i - \hat{\beta}_2 \sum_{i=1}^{n} X_i$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} Y_i - \hat{\beta}_2 \sum_{i=1}^{n} X_i}{n}$$

$$= \frac{\sum_{i=1}^{n} Y_i}{n} - \hat{\beta}_2 \frac{\sum_{i=1}^{n} X_i}{n} = \bar{X}$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

FROM (2)

$$2\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)(-X_i) = 0$$

$$-2\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)(X_i) = 0$$

THIS REDUCES TO

$$\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) X_i = 0$$

$$\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} \hat{\beta}_1 X_i - \sum_{i=1}^{n} \hat{\beta}_2 X_i^2 = 0$$

$$\sum_{i=1}^{n} \hat{\beta}_2 X_i^2 = \sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} \hat{\beta}_1 X_i$$

$$\sum \hat{\beta}_2 X_i^2 = \sum X_i Y_i - \sum (\bar{Y} - \hat{\beta}_2 \bar{X}) X_i$$

$$\sum_{i=1}^{n} \hat{\beta}_2 x_i^2 = \sum_{i=1}^{n} x_i Y_i - \sum_{i=1}^{n} (\bar{Y} - \hat{\beta}_2 \bar{X}) x_i$$

$$\sum_{i=1}^{n} \hat{\beta}_2 x_i^2 = \sum_{i=1}^{n} x_i Y_i - \sum_{i=1}^{n} X_i \left(\frac{\sum_{i=1}^{n} Y_i}{n} - \hat{\beta}_2 \frac{\sum_{i=1}^{n} X_i}{n} \right)$$

$$\sum_{i=1}^{n} \hat{\beta}_2 x_i^2 = \sum_{i=1}^{n} x_i Y_i - \frac{\sum_{i=1}^{n} X_i Y_i}{n} + \hat{\beta}_2 \frac{\left(\sum_{i=1}^{n} X_i \right)^2}{n}$$