B.E. International Program
Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 1/2019

Assignment# 4

Question 1

Consider the function f defined for all (x,y) such that

$$f\ (x, y; a) = \frac{1}{2}x^2\ - x + ay(x-1)\ - \frac{1}{3}y^3 + a^2y^2\ ,$$

where a is a constant.

a. Prove that $(x^*, y^*) = (1 - a^3, a^2)$ is a stationary point of $f\ (x, y; a)$.

b. Given that $G(a) = f\ (x^*, y^*; a)$, show the derivative of G with respect to a.

c. Calculate $\frac{\partial f\ (x,y;a)}{\partial a}$ and evaluate its value where $(x^*, y^*) = (1 - a^3, a^2)$. Compare your answer with the answer obtained in b. Are they the same?

d. Where in the xy-plan is f convex.

Question 2: Price discrimination (Moderate)

Suppose that there are three groups of people who take Sky train to commute in Bangkok. The first group is students (s), the second group is senior citizens (old), and the third group is working-aged people (w). The demand for each group is given by the following equations:

Demand of students: $p_s = 8 - Q_s/2$
Demand of senior citizens: $p_{old} = 16 - 2Q_{old}$
Demand of working-aged people: $p_w = 20 - Q_w$

The Sky train operator has a constant marginal cost at MC = \$4, and total cost at TC = 4Q + 10. Consider the following problems.
a) Determine the profit-maximizing level of output/price under the third-degree price discrimination. Calculate the level of maximized profit.
b) Confirm your result in (a) using the Hessian-matrix method.

Question 3 (Moderate)

Consider a simple two-market model where demand and supply for each market is given below. (Notationally, let's name the two markets as A and B, respectively.)

Market A:
Demand: $p_A = 10 - 2Q_A$
Supply: $p_A = 1 + Q_A$

Market B:
Demand: $p_B = 20 - Q_B$
Supply: $p_B = 2 + 2Q_B$

a. Derive the market equilibrium
b. Suppose the government imposes unit tax on consumers in both markets at the rate of t_A and t_B. Solve for the after-tax equilibrium as the function of t_A and t_B.
c. How much revenue can the government collect from the taxation?
d. Determine the level of t_A and t_B that maximizes government's revenue.

Question 4 (Monopolist and the Optimal advertising)

Consider a linear demand equation faced by a monopolist.

$$Q = 2000 + 4\sqrt{A} - 20P,$$

where A is the dollar amount of the expenditure on advertisement, P is the unit price, and Q is the amount of quantity demanded by consumers. Both θ and β are strictly positive.

The demand equation is an extended version of the traditional one. The proposed function captures an idea that advertising can boost the total demand in the market as potential customers get more information about the product.

Suppose that the monopolist cost function is given by $C(Q, A) = c(Q) + A$ where $c(Q) = 2Q + 1000$

Consider the following problem.

a) Construct the profit function.
b) Determine the optimal pricing (P*) and optimal advertisement (A*) that maximize the profit of the monopolist.
c) Confirm the result with the second-order differential test, i.e. hessian-matrix method.

Question 5 (Optimal input decision)

Suppose that the output Q of a firm depends on two inputs of the quantities K and L. The output level is determined by the production function

$$Q = 36K + 16L - 3K^2 - 2KL - L^2$$

a. Is the firm's production function strictly concave? Explain.

b. Determine the optimal input (K*, L*) that maximizes the output level.
c. Write down the firm's profit function when the price of Q is P and the per-unit factor prices of K and L are r and w, respectively, where both r and w are positive numbers. Find the levels of K^* and L^* that maximize the firm's profits.
d. Verify that the second-order sufficient conditions for maximum profits are satisfied.
e. Determine the effect of an increase in r on the firm's use of each input. (i.e. determine $\frac{\partial K^*}{\partial r}$ and $\frac{\partial L^*}{\partial r}$).

Question 6

Each of two skin clinics, W and N, provides its own brand of a skin treatment in the amounts Q_w and Q_n at the prices P_w and P_n, respectively. Each clinic independently chooses the quantity of treatment that maximizes its own profit, where the demand functions for the two brands are given by:

$$P_w = 60 - 6Q_w + 2Q_n$$

$$P_n = 30 + 2Q_w - 4Q_n$$

Suppose that the total cost functions for clinics W and N are:

$$C_w = 18 + 8Q_w$$

$$C_n = 24 + 8Q_n$$

a. Initially each clinic maximizes its own profit, taking another clinic's optimal quantity as given. Determine the amounts ${Q^*}_w$ and ${Q^*}_n$ that maximize each clinic's profit, and calculate the corresponding profit level of each clinic.

b. Suppose now that the two clinics collude and act as a monopolist whose total revenue is the combined revenues and total cost is the combined costs. Determine the new amounts ${Q^*}_w$ and ${Q^*}_n$ that maximize the combined profit, and calculate the new maximum profit. Discuss the change in the overall profits, if any, after the two clinics collude.