

# ENDOGENOUS GROWTH MODELS – ROMER MODEL

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EE 462 Development Macroeconomics

Semester 1/2015

# Topics

- Basic Elements of Romer Model
- Growth in Romer Model
- Special Cases
- Comparative Statics: A Permanent Increase in R&D

# Endogenous Growth Theory

- The *endogenous growth theory* or *new growth theory* focuses on understanding the economic forces underlying technological progress.
- **Technological progress** is driven by profit-maximizing firms or investors' own interests.
- The Romer model endogenizes technological progress by introducing the search for new ideas by researchers interested in profiting from their inventions.
- This model is aimed to explain mainly why and how the advanced countries exhibit sustained growth.
- Technological progress is driven by *research and development (R&D)*.

# Romer Model: Basic Elements (1)

- Romer (1990) and Jones (1995) use a model of firms that invest in R&D.
- The production function is given by:

$$Y = K^\alpha (AL_Y)^{1-\alpha}, \quad (0 < \alpha < 1) \quad \text{-- (1)}$$

where  $A$  = a given level (or stock) of knowledge

$L_Y$  = labor used in the production sector

$L$  = total labor:  $L = L_Y + L_A$  ( $L_A$  = labor used in R&D)

- The production function exhibits CRTS w.r.t.  $K$  and  $L$ , but exhibits IRTS w.r.t. all three inputs ( $K$ ,  $L$ , and  $A$ ).
- Capital accumulations as people forego consumption at rate  $s_K$ .

$$\Delta K = s_K Y - dK \quad \text{-- (2)}$$

## Romer Model: Basic Elements (2)

- Labor (equivalent to population) grows at a constant rate  $n$ :

$$\frac{\Delta L}{L} = n \quad \text{-- (3)}$$

- Romer assumes that the number of new ideas ( $\Delta A$ ) at any given point in time is can be derived from:

$$\Delta A = \bar{\delta} L_A, \quad \text{-- (4)}$$

where  $L_A$  is the number of people attempting to discover new ideas, and  $\bar{\delta}$  is the rate at which new ideas are discovered.

- $\bar{\delta}$  could a function of A:  $\bar{\delta} = \delta A^\phi$  -- (5)
  - $\phi > 0 \rightarrow$  Productivity of research increases with the stock of knowledge
  - $\phi < 0 \rightarrow$  the “fishing out” case
  - $\phi = 0 \rightarrow$  Productivity of research is independent of the stock of knowledge

## Romer Model: Basic Elements (3)

- Also, it is possible that the average productivity of research depends on the number people searching for new ideas.
- The general **production function for ideas** can be written as:

$$\Delta A = \delta A^\phi L_A^\lambda \quad \text{-- (6)}$$

where  $\phi < 1$  and  $\lambda \in (0,1)$ , according to Jones (1995).

- One can think  $\lambda$  as the “originality of idea”, and  $\phi$  as the “knowledge spillover”.
  - $\lambda < 1 \rightarrow$  an externality associated with duplication
  - $\phi > 0 \rightarrow$  positive knowledge spillover in research
- Suppose further that  $L_A/L = s_R \rightarrow L_Y/L = 1 - s_R \quad \text{-- (7)}$

# Growth in Romer Model

- Along the balance growth path, one can show that

$$g_y = g_k = g_A \quad \text{-- (8)}$$

- Question: What determine the growth rate of A (i.e. growth rate of ideas)?

- Define the *growth rate of ideas* as  $g_A = \Delta A/A$ .

- Recall that  $\Delta A = \delta A^\phi L_A^\lambda$ . Hence,

$$\frac{\Delta A}{A} = \delta A^{\phi-1} L_A^\lambda. \quad \text{-- (9)}$$

- Along a balanced growth path,  $\Delta A/A \equiv g_A$  is constant. Taking logs and derivatives of both sides of equation (9) gives:

$$g_A = \frac{\lambda n}{1-\phi}. \quad \text{-- (10)}$$

# Special Cases in Romer Model

- Case 1: Assume  $\lambda = 1$  and  $\phi = 0$ .
  - No duplication problem in research and the productivity of a researcher today is independent of the stock of ideas previously discovered.

$$\rightarrow \Delta A = \delta L_A$$

$$\rightarrow g_A = \frac{\Delta A}{A} = n$$

- Case 2: Assume  $\lambda = 1$  and  $\phi = 1$ .

$$\rightarrow \Delta A = \delta L_A A$$

$$\rightarrow g_A = \frac{\Delta A}{A} = \delta L_A$$

# Comparative Statics: A Permanent Increase in R&D Share

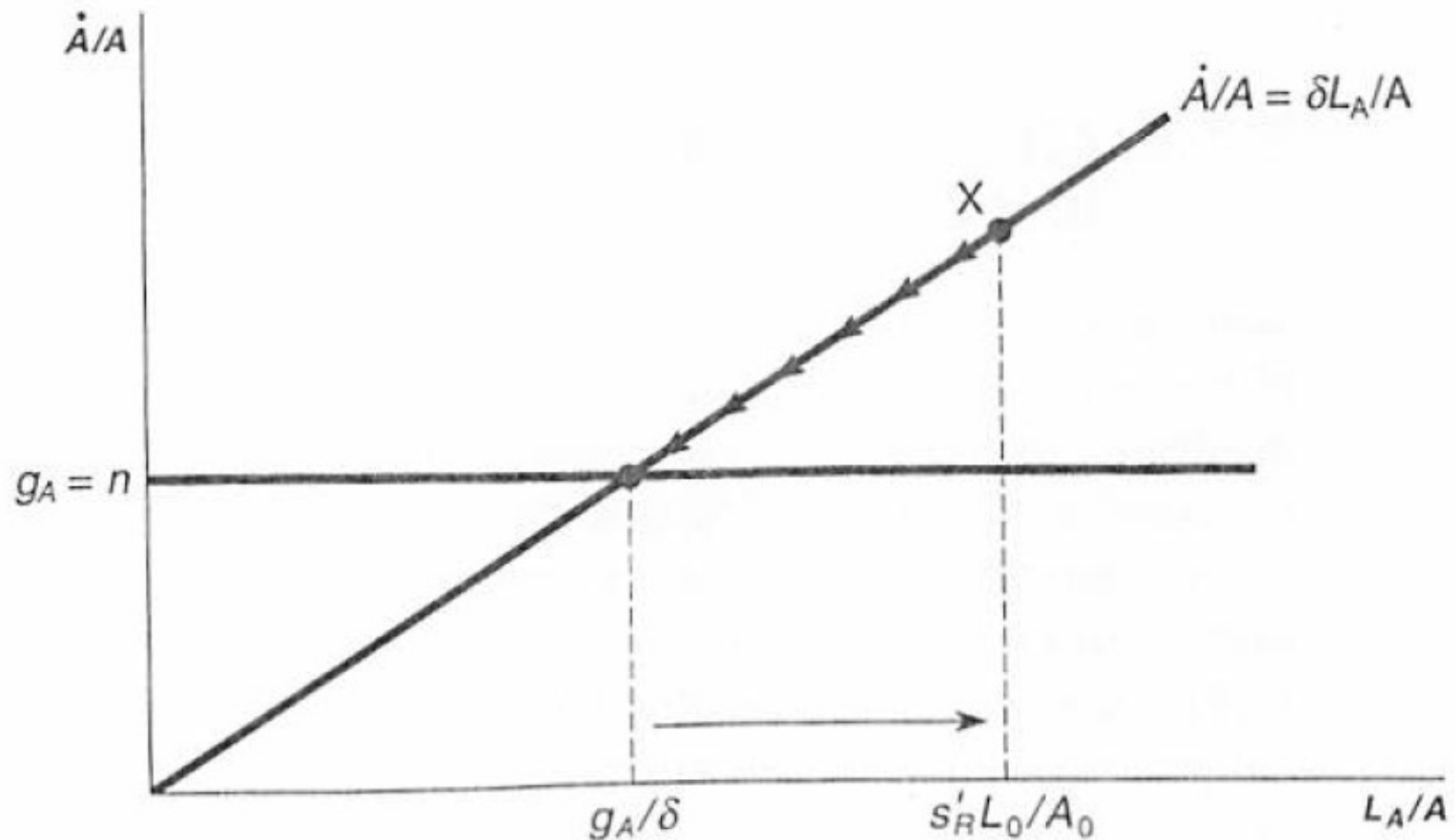
- For simplicity, assume  $\lambda = 1$  and  $\phi = 0$ . Equation (9) can be rewritten as:

$$\frac{\Delta A}{A} = \delta \frac{S_R L}{A}, \quad \text{-- (11)}$$

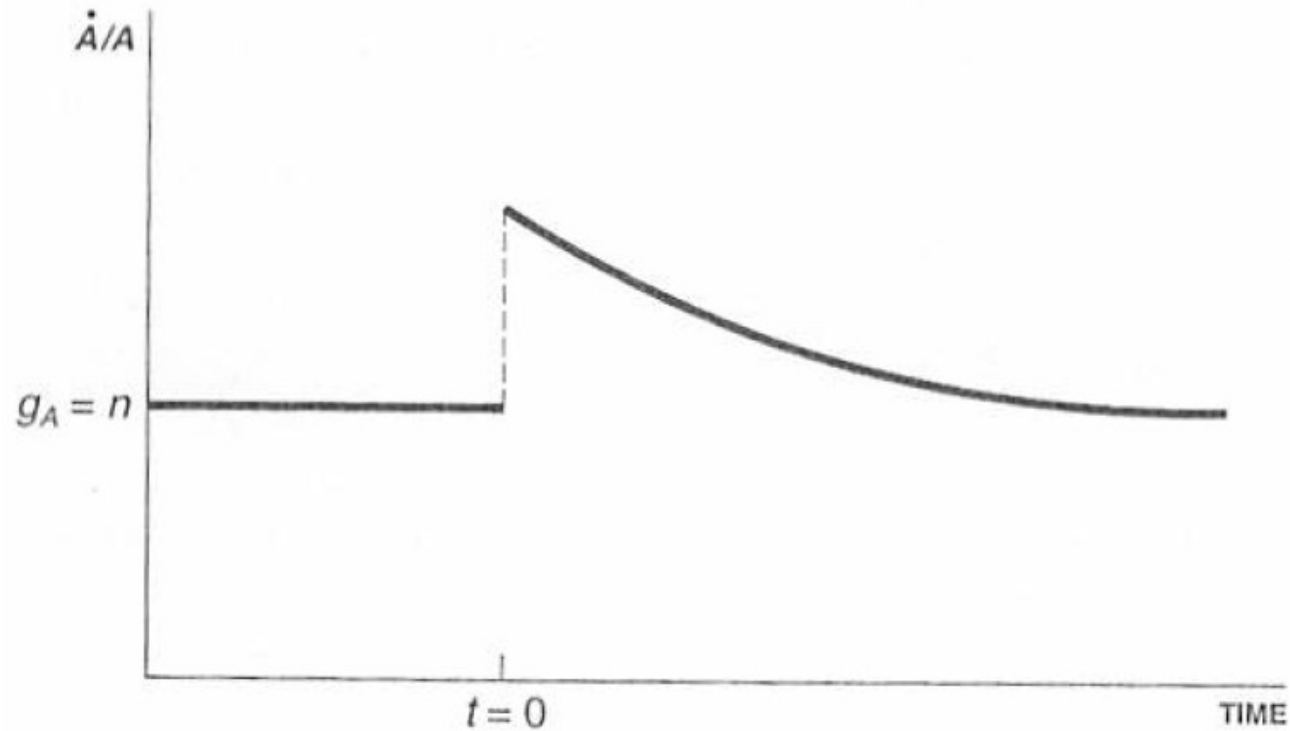
where  $S_R$  is the share of labor engaged in R&D.

- Question: What happens to the per capita output ( $y$ ) if  $S_R$  increases?
  - Need to know the impact of an increase in R&D share on technological progress.
  - Use a Solow framework to derive output per effective labor and output per worker along a balanced growth path.

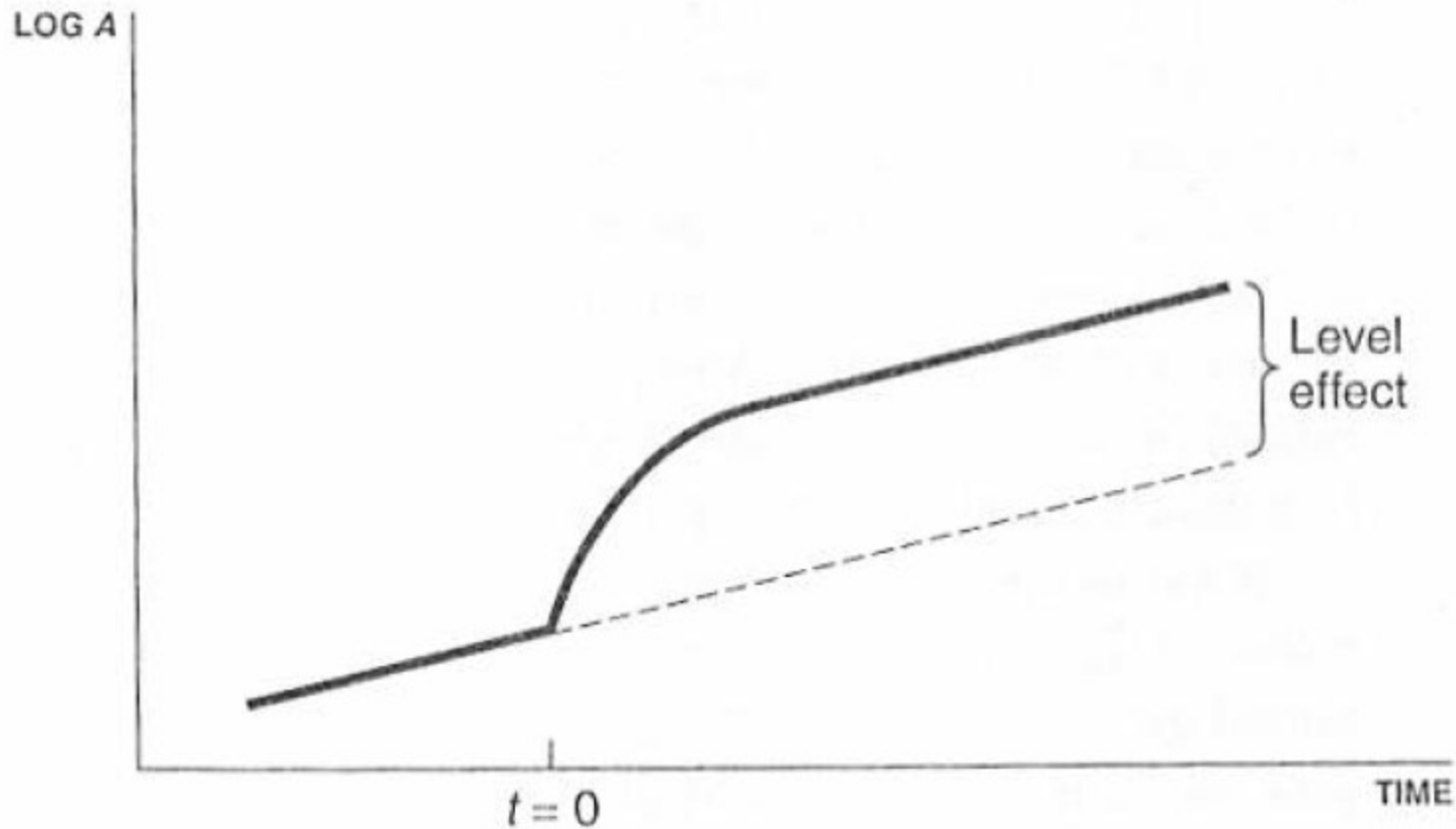
# Technological Progress: An Increase in The R&D Share



# Technological Progress Over Time



# The Level of Technology Over Time



# Steady-State Output Per Capita

- Based on the Solow framework, the ratio  $y/A$  along a balanced growth path is constant and is given by:

$$\left(\frac{y}{A}\right)^* = \left(\frac{s_K}{n+g_A+d}\right)^{\alpha/(1-\alpha)} (1 - s_R) \quad \text{-- (12)}$$

- Also, along a balanced growth path, from equation (11), the level of  $A$  can be solved in terms of the labor force:

$$A = \delta \frac{s_R L}{g_A} \quad \text{-- (13)}$$

- Based on the above information, we can derive:

$$y^* = \left(\frac{s_K}{n+g_A+d}\right)^{\alpha/(1-\alpha)} (1 - s_R) \frac{\delta s_R}{g_A} L \quad \text{-- (14)}$$

- $s_R$  affects  $y^*$  both negatively (more researchers=fewer workers producing output) and positively (more researchers = more ideas).

# Implications

- Population growth is the *only source of* technological progress and any long-run growth in per capita income.
  - A higher population growth in the Romer model means a *higher long-run economic growth rate*.
- Government cannot change the growth rate of per capita income by *any policy* except a policy to increase the fertility rate.
- For instance, a policy of increasing the R&D share of labor  $s_R$  cannot change the long-run growth rate of the economy. Why?
  - They only increase the growth rate during the transition period (but not in the long run).