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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90
	25.7	621

$\bar{y} = 3.2125$

$\bar{x} = 77.625$

$var(x) : (x_i - \bar{x})^2$

	$X_i - \bar{X}$	$(x_i - x)^2$
1	-14.625	213.89
2	-5.625	31.64
3	0.375	0.140625
4	3.375	11.3906
5	9.375	87.89
6	-2.625	6.89
7	-2.625	6.89
8	12.375	153.14

511.87

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\beta_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} = \left[\frac{(-0.4125)(-14.625) + (0.1875)(-5.625) + (-0.2125)(0.375) + (0.2875)(3.375) + (0.3875)(9.375) + (-0.2125)(-2.625) + (-0.5125)(-2.625) + (0.4875)(12.375)}{511.87}\right] = 0.034$$

$$\beta_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 3.2125 - 0.034(77.625) = 0.57325$$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = 0.5733 + 0.034 X_i$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

	$\hat{Y}_i$	$\hat{u}_i$
1	2.7153	0.0847
2	3.0213	0.3787
3	3.2253	-0.2253
4	3.3273	0.1725
5	3.5313	0.0687
6	3.1233	-0.1233
7	3.1233	-0.4233
8	3.6333	0.0667

$\sum_{i=0}^{N} \hat{u}_i = -0.0006 \approx 0$

-0.0693

1.3 Find $var(\hat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4346}{6} = 0.0724$$

$$var(\hat{\beta}_1) = \frac{var(\hat{u}_i)}{\sum_i (x_i - \bar{x})^2} = \frac{0.0724}{511.87} = 0.0014$$

$$var(\hat{\beta}_0) = \frac{\hat{\sigma}^2}{\sum X_i^2} = \frac{0.0724}{48717} = 0.00000148613$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$\bar{x} = \frac{200}{10} = 20$

$\bar{y} = \frac{91}{10} = 9.1$

	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$y_i - \bar{y}$
1	-10	100	-9.1
2	-8	64	-7.1
3	-6	36	-4.1
4	-4	16	-3.1
5	-2	4	-2.1
6	2	4	0.9
7	4	16	0.9
8	6	36	5.9
9	8	64	6.9
10	10	100	10.9

440

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

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$$\beta_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{(-10)(-9.1) + (-8)(-7.1) + (-6)(-4.1) + (-4)(-3.1) + (-2)(-2.1) + (2)(0.9) + (4)(0.9) + (6)(5.9) + (8)(6.9) + (10)(10.9)}{440} = 0.8955$$

$$\beta_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 9.1 - (0.8955)(20) = -8.81$$

2.2 Find the value of $\widehat{Y_i}$ and $\widehat{u_i}$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i \quad = -8.81 + 0.8955 X_i$

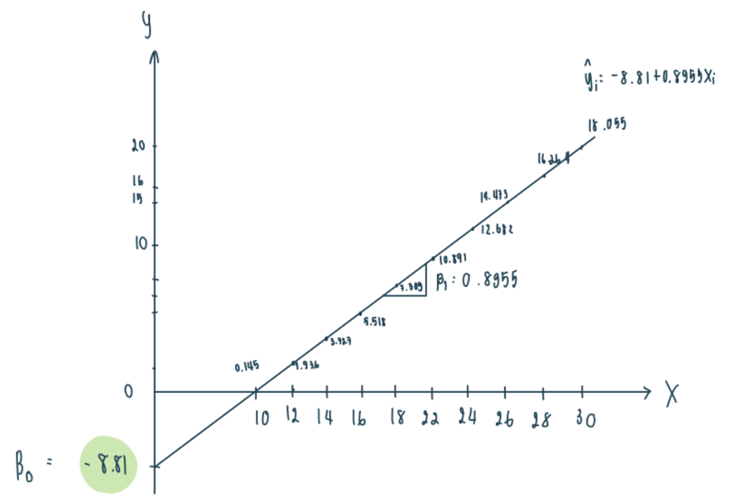
$\hat{u}_i = Y_i - \hat{Y}_i$

	$\hat{Y}_i$	Y_i	$\hat{u}_i$
1	0.145	0	-0.145
2	1.936	2	0.064
3	3.727	5	1.273
4	5.518	6	0.482
5	7.309	7	-0.309
6	10.891	10	-0.891
7	12.682	10	-2.682
8	14.473	15	0.527
9	16.264	16	-0.264
10	18.055	20	1.945

$\sum \hat{u}_i = 0$

14.09091

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.4 If X_i =16, what is the predicted Y?

$\hat{Y}_4 = 5.518$

2.5 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0909}{8} = 1.76136$

$var(\hat{\beta}_1) = \frac{var(\hat{u}_i)}{\sum_i (X_i - \bar{X})^2} = \frac{1.76136}{440} = 0.004$

$var(\hat{\beta}_0) = \frac{\hat{\sigma}^2}{\sum X_i^2} = \frac{1.76136}{4440} = 0.000396^3 \approx 0.0004$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \, ,$$

Where $u_i \sim NIID(0, \sigma^2)$**.** Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

$$u_i = x_i - \bar{x} \qquad k = \frac{u_i}{\sum u_i^2}$$

$$\hat{\beta}_1 = \sum (y_i - \bar{y}) k$$

$$= \sum (\beta_0 + \beta_1 X_i + u_i - \beta_0 - \beta_1 \bar{x}) k$$

$$= \beta_1 \sum (x_i - \bar{x}) k + u_i k$$

$$= \beta \sum u_i \frac{u_i}{\sum u_i^2} + u_i k$$

$$= \beta_1 + u_i k$$

$$E(\hat{\beta}_1) = E(\beta_1 + \sum u_i k)$$

$$= \beta_1 + E(\sum u_i k) \rightarrow SLR\,4: \; E(u_i | x_i) = 0$$

$$= \beta_1 + \sum k \; E(u_i)$$

$$E(\hat{\beta}_1) = \beta_1$$