Thursday, September 1, 2016 8:12 AM

$$y_{1t} = \boxed{\beta_{10} + \beta_{11}X_{1t} + \beta_{12}X_{2t}} + \varepsilon_{1t}$$

$$\rightarrow y_{2t} = \boxed{\beta_{20} + \beta_{21}X_{3t} + \beta_{22}X_{4t}} + \varepsilon_{2t}$$

$$\begin{bmatrix} \begin{bmatrix} y_{11} \\ y_{12} \\ \vdots \\ y_{1T} \end{bmatrix} \\ \begin{bmatrix} y_{21} \\ y_{22} \\ \vdots \\ y_{2T} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & x_{11} & x_{21} \\ 1 & x_{12} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{1T} & x_{2T} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 & x_{31} & x_{41} \\ 1 & x_{32} & x_{42} \\ \vdots & \vdots & \vdots \\ 1 & x_{3T} & x_{4T} \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} \begin{bmatrix} \beta_{10} \\ \beta_{11} \\ \beta_{12} \end{bmatrix} \\ \begin{bmatrix} \beta_{20} \\ \beta_{21} \\ \beta_{22} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{1T} \end{bmatrix} \\ \begin{bmatrix} \varepsilon_{21} \\ \varepsilon_{22} \\ \vdots \\ \varepsilon_{2T} \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \Rightarrow$$

$$Y = X\beta + \varepsilon$$

$$V = \begin{bmatrix} [V_1] & \\ & [V_2] \end{bmatrix}$$

$$\beta_{\substack{FGLS \\ SUR}} = (X'\hat{V}^{-1}X)^{-1}X'\hat{V}^{-1}Y$$

Unbiased $\rightarrow$ $\boxed{E(\hat{\beta}) = \beta}$ — Finite Sample

Efficient. $\rightarrow$ Min. $Var(\hat{\beta})$ compare to other LUE

Consistent. $\rightarrow$ $\boxed{\text{plim } \hat{\beta} = \beta}$

$n \to \infty$

Asymptotic Property

$$\Downarrow$$

$$\lim_{n \to \infty} \text{Prob}[\hat{\beta} - \beta = \varepsilon] = 0$$

where $\varepsilon \neq 0$, $\varepsilon \to 0$

If there exists relationship between/among error terms across equations, system eq. estimation method (SUR) will lead to a more asymptotically efficient estimators.