Assignment 9
Guideline Solution

From the data set `Assignment 9.dta`:

Requirements:

1. Perform unit root test of series *y* and *x*.

```
. dfuller x, trend lag(1) reg

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                              ---------- Interpolated Dickey-Fuller ---------
                 Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value            Value
------------------------------------------------------------------------------
 Z(t)              0.601           -3.980            -3.420           -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.9970

------------------------------------------------------------------------------
D.x          |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |
         L1. |   .0001061   .0001764     0.60   0.548    -.0002405    .0004526
         LD. |     .46018   .0349881    13.15   0.000     .3914361    .5289239
      _trend |   4.166909    1.14105     3.65   0.000     1.924999    6.408818
       _cons |   2128.626   140.0551    15.20   0.000     1853.449    2403.803
------------------------------------------------------------------------------

. dfuller d.x, trend lag(1) reg

Augmented Dickey-Fuller test for unit root         Number of obs   =       497

                              ---------- Interpolated Dickey-Fuller ---------
                 Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value            Value
------------------------------------------------------------------------------
 Z(t)            -10.657           -3.980            -3.420           -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
D2.x         |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         D.x |
         L1. |  -.4007114   .0376021   -10.66   0.000    -.4745915   -.3268312
         LD. |   -.414172   .0358603   -11.55   0.000    -.4846298   -.3437141
      _trend |   3.508317   .3506567    10.00   0.000     2.819351    4.197283
       _cons |   1596.429   147.0971    10.85   0.000     1307.415    1885.444
------------------------------------------------------------------------------

. dfuller y, trend lag(1) reg

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                              ---------- Interpolated Dickey-Fuller ---------
                 Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value            Value
------------------------------------------------------------------------------
 Z(t)              1.000           -3.980            -3.420           -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 1.0000

------------------------------------------------------------------------------
D.y          |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |   .0001178   .0001179     1.00   0.318    -.0001137    .0003494
         LD. |   .6997015   .0248993    28.10   0.000     .6507799    .7486231
      _trend |   2.897751   1.159296     2.50   0.013      .619992    5.175511
       _cons |   1811.233   147.0426    12.32   0.000     1522.327    2100.139
------------------------------------------------------------------------------
```

```
. dfuller d.y, trend lag(1) reg

Augmented Dickey-Fuller test for unit root         Number of obs   =       497

                                 ---------- Interpolated Dickey-Fuller ---------
                    Test         1% Critical       5% Critical      10% Critical
                Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -10.554            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
D2.y         |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         D.y |
         L1. |  -.2787856   .0264156   -10.55   0.000    -.3306866   -.2268845
         LD. |    -.32127   .0373756    -8.60   0.000    -.3947051   -.2478349
      _trend |   3.631984   .3708318     9.79   0.000     2.903379    4.36059
       _cons |   1678.082   154.4788    10.86   0.000     1374.564    1981.6
------------------------------------------------------------------------------
```

- ***Both x and y are I(1).***

2. Perform cointegration test of series *y* and *x* using set up of (i) linear trend; (ii) restricted trend; (iii) unrestricted constant; (iv) restricted constant; and (v) no trend, with one lag term.

(i) <u>linear trend</u>

```
. vecrank y x, trend(t) lags(1) max
                     Johansen tests for cointegration
Trend: trend                                         Number of obs =     499
Sample:  2 - 500                                           Lags =       1
------------------------------------------------------------------------------
                                                          5%
maximum                                    trace      critical
  rank    parms       LL       eigenvalue  statistic    value
    0       4      -7387.4577            .   790.3357    18.17
    1       7      -6992.3441      0.79477     0.1085*    3.74
    2       8      -6992.2899      0.00022
------------------------------------------------------------------------------
                                                          5%
maximum                                      max      critical
  rank    parms       LL       eigenvalue  statistic    value
    0       4      -7387.4577            .   790.2272    16.87
    1       7      -6992.3441      0.79477     0.1085     3.74
    2       8      -6992.2899      0.00022
------------------------------------------------------------------------------
```

(ii) <u>restricted trend</u>

```
. vecrank y x, trend(rt) lags(1) max
                     Johansen tests for cointegration
Trend: rtrend                                        Number of obs =     499
Sample:  2 - 500                                           Lags =       1
------------------------------------------------------------------------------
                                                          5%
maximum                                    trace      critical
  rank    parms       LL       eigenvalue  statistic    value
    0       2      -8050.4781            .  2116.3764    25.32
    1       6      -7075.2453      0.97993   165.9107    12.25
    2       8      -6992.2899      0.28286
------------------------------------------------------------------------------
                                                          5%
maximum                                      max      critical
  rank    parms       LL       eigenvalue  statistic    value
    0       2      -8050.4781            .  1950.4657    18.96
    1       6      -7075.2453      0.97993   165.9107    12.52
    2       8      -6992.2899      0.28286
------------------------------------------------------------------------------
```

(iii) unrestricted constant

```
. vecrank y x, trend(c) lags(1) max

                    Johansen tests for cointegration
Trend: constant                                         Number of obs =     499
Sample:  2 - 500                                              Lags =       1
------------------------------------------------------------------------------
                                                              5%
maximum                                        trace      critical
  rank    parms       LL       eigenvalue  statistic      value
    0       2      -8050.4781           .   2086.8946     15.41
    1       5      -7083.8611     0.97923    153.6607      3.76
    2       6      -7007.0308     0.26504
------------------------------------------------------------------------------
                                                              5%
maximum                                          max      critical
  rank    parms       LL       eigenvalue  statistic      value
    0       2      -8050.4781           .   1933.2339     14.07
    1       5      -7083.8611     0.97923    153.6607      3.76
    2       6      -7007.0308     0.26504
------------------------------------------------------------------------------
```

(iv) restricted constant

```
. vecrank y x, trend(rc) lags(1) max

                    Johansen tests for cointegration
Trend: rconstant                                        Number of obs =     499
Sample:  2 - 500                                              Lags =       1
------------------------------------------------------------------------------
                                                              5%
maximum                                        trace      critical
  rank    parms       LL       eigenvalue  statistic      value
    0       0      -8811.3759           .   3607.1930     19.96
    1       4      -7092.1887     0.99898    168.8185      9.42
    2       6      -7007.7794     0.28703
------------------------------------------------------------------------------
                                                              5%
maximum                                          max      critical
  rank    parms       LL       eigenvalue  statistic      value
    0       0      -8811.3759           .   3438.3745     15.67
    1       4      -7092.1887     0.99898    168.8185      9.24
    2       6      -7007.7794     0.28703
------------------------------------------------------------------------------
```

(v) no trend

```
. vecrank y x, trend(n) lags(1) max

                    Johansen tests for cointegration
Trend: none                                             Number of obs =     499
Sample:  2 - 500                                              Lags =       1
------------------------------------------------------------------------------
                                                              5%
maximum                                        trace      critical
  rank    parms       LL       eigenvalue  statistic      value
    0       0      -8811.3759           .   3204.1193     12.53
    1       3      -7211.7592     0.99836      4.8860      3.84
    2       4      -7209.3163     0.00974
------------------------------------------------------------------------------
                                                              5%
maximum                                          max      critical
  rank    parms       LL       eigenvalue  statistic      value
    0       0      -8811.3759           .   3199.2333     11.44
    1       3      -7211.7592     0.99836      4.8860      3.84
    2       4      -7209.3163     0.00974
------------------------------------------------------------------------------
```

- ***According to (i), rank = 1. According to (ii), (iii), (iv), and (v), rank = 2. There exists cointegrating equation between x and y.***

3. Perform cointegration test of series *y* and *x* using set up of linear trend with (i) one lag term; (ii) two lag terms; and (iii) three lag terms.

(i) one lag term

```
. vecrank y x, trend(t) lags(1) max
                    Johansen tests for cointegration
Trend: trend                                         Number of obs =     499
Sample:  2 - 500                                           Lags =       1
------------------------------------------------------------------------------
                                                              5%
maximum                                        trace      critical
  rank    parms       LL       eigenvalue  statistic     value
    0       4      -7387.4577           .    790.3357     18.17
    1       7      -6992.3441     0.79477      0.1085*     3.74
    2       8      -6992.2899     0.00022
------------------------------------------------------------------------------
                                                              5%
maximum                                          max      critical
  rank    parms       LL       eigenvalue  statistic     value
    0       4      -7387.4577           .    790.2272     16.87
    1       7      -6992.3441     0.79477      0.1085      3.74
    2       8      -6992.2899     0.00022
------------------------------------------------------------------------------
```

(ii) two lag terms

```
. vecrank y x, trend(t) lags(2) max
                    Johansen tests for cointegration
Trend: trend                                         Number of obs =     498
Sample:  3 - 500                                           Lags =       2
------------------------------------------------------------------------------
                                                              5%
maximum                                        trace      critical
  rank    parms       LL       eigenvalue  statistic     value
    0       8      -6795.5337           .    187.5771     18.17
    1      11      -6703.3826     0.30932      3.2749*     3.74
    2      12      -6701.7451     0.00655
------------------------------------------------------------------------------
                                                              5%
maximum                                          max      critical
  rank    parms       LL       eigenvalue  statistic     value
    0       8      -6795.5337           .    184.3022     16.87
    1      11      -6703.3826     0.30932      3.2749      3.74
    2      12      -6701.7451     0.00655
------------------------------------------------------------------------------
```

(iii) three lag terms

```
. vecrank y x, trend(t) lags(3) max
                    Johansen tests for cointegration
Trend: trend                                         Number of obs =     497
Sample:  4 - 500                                           Lags =       3
------------------------------------------------------------------------------
                                                              5%
maximum                                        trace      critical
  rank    parms       LL       eigenvalue  statistic     value
    0      12       -6756.658           .    140.0434     18.17
    1      15       -6688.113     0.24106      2.9534*     3.74
    2      16      -6686.6363     0.00592
------------------------------------------------------------------------------
                                                              5%
maximum                                          max      critical
  rank    parms       LL       eigenvalue  statistic     value
    0      12       -6756.658           .    137.0900     16.87
    1      15       -6688.113     0.24106      2.9534      3.74
    2      16      -6686.6363     0.00592
------------------------------------------------------------------------------
```

- ***According to (i), (ii) and (iii), rank = 1. There exists cointegrating equation between x and y.***

4. From (2) & (3), determine the most appropriated set up and make the conclusion of the test whether *y* and *x* are cointegrated series? If yes, how many cointegrating equations.

```
. vecrank y x, trend(t) lags(3) max

                    Johansen tests for cointegration
Trend: trend                                         Number of obs =     497
Sample:  4 - 500                                           Lags =       3
------------------------------------------------------------------------------
                                                           5%
maximum                                    trace       critical
  rank    parms       LL       eigenvalue  statistic     value
    0      12      -6756.658           .    140.0434     18.17
    1      15      -6688.113      0.24106      2.9534*     3.74
    2      16     -6686.6363      0.00592
------------------------------------------------------------------------------
                                                           5%
maximum                                      max       critical
  rank    parms       LL       eigenvalue  statistic     value
    0      12      -6756.658           .    137.0900     16.87
    1      15      -6688.113      0.24106      2.9534      3.74
    2      16     -6686.6363      0.00592
------------------------------------------------------------------------------
```

- ***With the highest LL, linear trend with three lag terms is the most appropriated and rank = 1. There is one cointegrating equation between x and y.***

5. Estimated VECM models of *y* and *x* using (i) one lag term; (ii) two lag terms; and (iii) three lag terms. Determine the optimal lags. Specify cointegrating equation and speed of adjustment parameters.

(i) <u>one lag term</u>

```
. vec y x, lag(1)

Vector error-correction model

Sample:  2 - 500                                  Number of obs     =       499
                                                  AIC               =   28.41227
Log likelihood = -7083.861                        HQIC              =   28.42883
Det(Sigma_ml)  =  7.34e+09                        SBIC              =   28.45448

Equation           Parms      RMSE     R-sq      chi2     P>chi2
-------------------------------------------------------------------
D_y                    2     331.479   0.9988   399343.6   0.0000
D_x                    2     430.726   0.9953   105050.2   0.0000
-------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |  -1.303404   .0095397  -136.63   0.000    -1.322102   -1.284707
             |
       _cons |   -107.843   69.40373    -1.55   0.120    -243.8718    28.18583
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |  -.8364345   .0123959   -67.48   0.000    -.8607301    -.812139
             |
       _cons |   168.0502    90.1839     1.86   0.062    -8.706964    344.8074
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                   1   1.71e+10   0.0000
```

```
------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500508   .0000115 -1.3e+05   0.000    -1.500531   -1.500486
       _cons |  -1678.204          .        .       .            .           .
------------------------------------------------------------------------------

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Akaike's information criterion and Bayesian information criterion

------------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+----------------------------------------------------------------
           . |        499         .  -7083.861       4    14175.72   14192.57
------------------------------------------------------------------------------
               Note: N=Obs used in calculating BIC; see [R] BIC note.
```

(ii) <u>two lag terms</u>

```
. vec y x, lag(2)

Vector error-correction model

Sample:  3 - 500                                        Number of obs     =       498
                                                        AIC               =   27.18032
Log likelihood = -6758.899                              HQIC              =   27.21018
Det(Sigma_ml)  =  2.11e+09                              SBIC              =   27.25641

Equation           Parms      RMSE     R-sq      chi2     P>chi2
------------------------------------------------------------------
D_y                    4     213.415   0.9995   964110.3   0.0000
D_x                    4     263.876   0.9982   280728.2   0.0000
------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |   -.427199   .0483639    -8.83   0.000    -.5219905   -.3324075
             |
           y |
         LD. |   .4022605   .0224005    17.96   0.000     .3583563    .4461646
             |
           x |
         LD. |   .4814843   .0598769     8.04   0.000     .3641276    .5988409
             |
       _cons |   171.4546   44.03061     3.89   0.000     85.15624    257.7531
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |   .2903827   .0597995     4.86   0.000     .1731779    .4075875
             |
           y |
         LD. |   .5734396   .0276971    20.70   0.000     .5191544    .6277249
             |
           x |
         LD. |   .3676105   .0740347     4.97   0.000     .2225051    .5127159
             |
       _cons |    252.237   54.44157     4.63   0.000     145.5335    358.9405
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms      chi2     P>chi2
-------------------------------------------
_ce1                   1   8.09e+08   0.0000
-------------------------------------------
```

```
Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500176   .0000527 -2.8e+04   0.000     -1.50028   -1.500073
       _cons |  -577.9622          .        .       .            .           .
------------------------------------------------------------------------------

. estat ic

Akaike's information criterion and Bayesian information criterion

------------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC         BIC
-------------+----------------------------------------------------------------
           . |        498         .  -6758.899       8     13533.8    13567.48
------------------------------------------------------------------------------
               Note: N=Obs used in calculating BIC; see [R] BIC note.
```

(iii) <u>three lag terms</u>

```
. vec y x, lag(3)

Vector error-correction model

Sample:  4 - 500                                  Number of obs     =       497
                                                  AIC               =   27.1245
Log likelihood = -6727.439                        HQIC              =  27.16771
Det(Sigma_ml)  =  1.96e+09                        SBIC              =  27.23459

Equation           Parms      RMSE     R-sq      chi2     P>chi2
------------------------------------------------------------------
D_y                    6     211.673   0.9995   980003.4   0.0000
D_x                    6     253.098   0.9984   305162.2   0.0000
------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |  -.3193909    .058155    -5.49   0.000    -.4333726   -.2054092
             |
           y |
          LD. |   .3553209   .0559202     6.35   0.000     .2457194    .4649225
         L2D. |   .0174858   .0312998     0.56   0.576    -.0438607    .0788323
             |
           x |
          LD. |   .6042668   .0763396     7.92   0.000     .4546439    .7538896
         L2D. |   .0521968   .0634683     0.82   0.411    -.0721988    .1765925
             |
        _cons |   181.5838   45.38519     4.00   0.000     92.63049    270.5372
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |   .4434064   .0695359     6.38   0.000     .3071185    .5796943
             |
           y |
          LD. |   .4713427   .0668638     7.05   0.000     .3402922    .6023932
         L2D. |    .012763   .0374252     0.34   0.733     -.060589    .0861151
             |
           x |
          LD. |   .5220109   .0912792     5.72   0.000     .3431069     .700915
         L2D. |   .0911602   .0758891     1.20   0.230    -.0575797    .2399001
             |
        _cons |    130.797   54.26707     2.41   0.016     24.43548    237.1585
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms      chi2     P>chi2
```

```
------------------------------------------
_ce1                  1   6.64e+08   0.0000
------------------------------------------

Identification:  beta is exactly identified

                Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .       .       .            .           .
           x |  -1.500049   .0000582 -2.6e+04   0.000    -1.500163   -1.499935
       _cons |  -92.13439          .       .       .            .           .
------------------------------------------------------------------------------

. estat ic

Akaike's information criterion and Bayesian information criterion

------------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+----------------------------------------------------------------
           . |        497         .  -6727.439      12    13478.88   13529.38
------------------------------------------------------------------------------
               Note: N=Obs used in calculating BIC; see [R] BIC note.
```

- ***With the lowest BIC, three lag is the most optimal.***
- ***Cointegrating equation*** $y-1.5x-92.13=0$ ***or*** $y=92.13+1.5x$
- ***Speed of adjustment parameter = -0.3194***