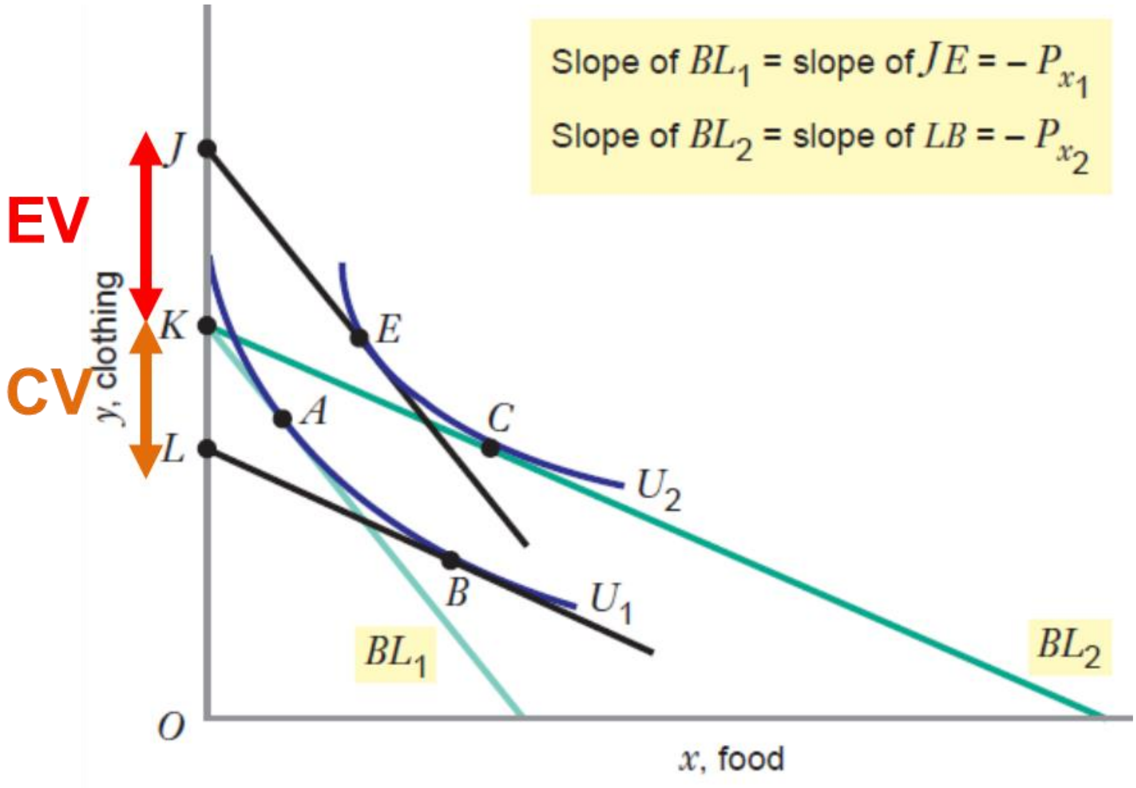
Extra Note

Compensating Variation and Equivalent Variation

- When the price of a good changes, your utility (welfare) changes.
- You can think of CV and EV as a "monetary" measure of the welfare change due to the price change.



CV: <u>At your ORIGINAL utility (IC_1)</u>, CV is the difference between **"the income required to buy the optimal bundle BEFORE the price change, Bundle A"** and **"the income required to buy the optimal bundle AFTER the price change, Bundle B".**

CV คือ ส่วนต่างของ เงินที่ใช้ซื้อ Bundle เก่า และเงินที่ใช้ซื้อ Bundle ใหม่ โดยที่ทั้งสอง Bundles ให้ Utility ในระดับก่อนราคาเปลี่ยน กล่าวคือให้ U_1

EV: <u>At your NEW utility (IC_2)</u>, EV is the difference between **"the income required to buy the optimal bundle BEFORE the price change, Bundle E"** and **"the income required to buy the optimal bundle AFTER the price change, Bundle C"**

EV คือ ส่วนต่างของ เงินที่ใช้ซื้อ Bundle เก่า และเงินที่ใช้ซื้อ Bundle ใหม่ โดยที่ทั้งสอง Bundles ให้ Utility ในระดับหลังราคาเปลี่ยน กล่าวคือให้ U_2

In the case of Price FALL, CV and EV are "positive". This is because **AFTER the price falls, you need LESS money** to buy the optimal bundle AFTER the price change, (Bundle B for CV and Bundle C for EV).

Positive values of CV and EV imply that we "GAIN" after the price falls. That is, our welfare, measured in monetary terms, rises.

In the case of Price RISE, CV and EV are "negative". This is because **AFTER the price rises, you need MORE money** to buy the optimal bundle AFTER the price change, (Bundle B for CV and Bundle C for EV).

Negative values of CV and EV imply that we "LOSE" after the price rises. That is, our welfare, measured in monetary terms, falls.

Example

LEARNING-BY-DOING EXERCISE 5.9

Compensating and Equivalent Variations with an Income Effect

As in Learning-By-Doing Exercise 5.4, a consumer purchases two goods, food x and clothing y. He has the utility function $U(x, y) = xy$. He has an income of \$72 per week, and the price of clothing is \$1 per unit. His marginal utilities are $MU_x = y$ and $MU_y = x$. Suppose the price of food falls from \$9 to \$4 per unit.

Problem

(a) What is the compensating variation of the reduction in the price of food?

(b) What is the equivalent variation of the reduction in the price of food?

STEP 1: Find Bundles A (initial bundle), C (final bundle), and B (substitution bundle).

We can derive the demand function, using the optimality condition and the BL.

$MU_X/MU_Y = P_X/P_Y$

$Y/X = P_X/P_Y$ (We don't substitute P_X because we are deriving the demand.)

$P_YY = P_XX$ Then we substitute it into the BL: $P_XX + P_YY = I$.

$2P_XX = I$

$X = I/(2P_X)$ This is our demand for X, as a function of I and P_X.

At Point A (optimal bundle before the price fall), I = 72 and $P_X = 9$.

$X_A = 72/(2x9) = 4$ $Y_A = 36$ (from $Y = I - P_XX = 72 - 9(4)$)

At Point C (optimal bundle after the price fall), I = 72 and $P_X = 4$.

$X_C = 72/(2x4) = 9$ $Y_C = 36$ (from $Y = I - P_XX = 72 - 4(9)$)

At Point B (substitution bundle), we know that $U_A = U_B$.

Recall that U = XY.

$U_A(X_A, Y_A) = 4 \text{ x } 36 = 144$

$U_B = 144 = X_BY_B$ (We will use this afterwards.)

Point B is where the optimality condition holds at the NEW price ($P_X = 4$).

$MU_X/MU_Y = P_X'/P_Y$

$Y/X = 4/1$

$Y = 4X$ Solve this with $U_B = 144 = X_BY_B$.

$144 = 4X_B^2$

$X_B = 6$ $Y_B = 144/6 = 24$

STEP 2: Find the income required to buy Bundle B and hence CV.

Income required to buy Point B is $P_X'X_B + P_YY_B = 4(6) + 24 = 48$.

CV = Income to buy A – Income to buy B = 72 – 48 = 24.

STEP 3: Find Bundle E.

Bundle E is the optimal bundle at the OLD price ($P_X = 9$), but gives the utility equal to Bundle C (final bundle where $P_X = 4$). That is, $U_C = U_E$.

$U_C(X_C, Y_C) = U_C(9, 36) = 9 \text{ x } 36 = 324$

$U_E = 324 = X_EY_E$ (We will use this afterwards.)

Point E is where the optimality condition holds at the OLD price ($P_X = 9$).

$MU_X/MU_Y = P_X/P_Y$

$Y/X = 9/1$

$Y = 9X$ Solve this with $U_E = 324 = X_EY_E$.

$324 = 9X_E^2$

$X_E = 6$ $\qquad\qquad$ $Y_E = 324/6 = 54$

STEP 4: Find the income required to buy Bundle E and hence EV.

Bundle E is the optimal bundle at the OLD price ($P_X = 9$).

Income required to buy Point E is $\quad P_X X_E + P_Y Y_E = 9(6) + 54 = 108$.

EV = Income to buy E – Income to buy C = 108 – 72 = 36.

FIGURE 5.19 Compensating and Equivalent Variation with an Income Effect The consumer's income is \$72, and the price of the clothing *y* is \$1 per unit. When the price of food is \$9 per unit, the consumer's budget line is BL_1, and he buys basket *A*, with utility U_1. After the price of food falls to \$4 per unit, his budget line is BL_2, and he buys basket *C*, with utility U_2. To reach utility U_1 *after* the price decrease, he would need an income of \$48 to buy basket *B*, so his compensating variation is \$72 − \$48 = \$24. To reach utility U_2 *before* the price decrease, he would need an income of \$108 to buy basket *E*, so his equivalent variation is \$108 − \$72 = \$36. When there is an income effect (basket *E* is not directly above basket *A*, and basket *C* is not directly above basket *B*), the compensating variation and the equivalent variation are generally not equal.