EE 415 / EE 418 Game theory

Lecture 5 Dynamic Games of Complete information

- Backward induction
- Bank robber or granade game
- Stackelberg model of duopoly
- Threat, commitment and reputation
- Read ch. 2 section 2.1

Let's play: classroom games

- 2. The Ultimatum Game
- Two players
- Player A offers a split of a 100 Baht bill
- If B agrees, then the game is over.
- If B refuses, it is then B's turn to offer a split, but now the bill reduced to 80 Baht
- If A agrees, then the game is over. Both get paid the agreed split.
- If A refuses, the game is over and neither get anything.

Few terminologies

- **Complete information** : games in which the players' payoff functions are common knowledge
- Example of incomplete information is a sealed-bid auction: each bidder knows her own valuation but not other, thus you are uncertain about another player's payoff function
- Another example is Cournot duopoly model in which firms have asymmetric information: firm 1 competes with firm 2 with 2 possible technologies

FEW TERMINOLOGIES

- **Perfect information** : under dynamic game, each move in the game, the player with the move knows the full history of the play of the game till that point.
- Example of dynamic game with perfect and complete information is Chess, the game you played in class (Rubinstein's bargaining model), Stackelberg game. (section 2.1)
- We can have a game of complete but imperfect information: at some point, the player with the move does not know the history of the game. (section 2.2) Example, in some part of the game players play simultaneously, we do not a clear history of the move. If we draw a game tree, some node is not a single node.

Extensive Form of a Game

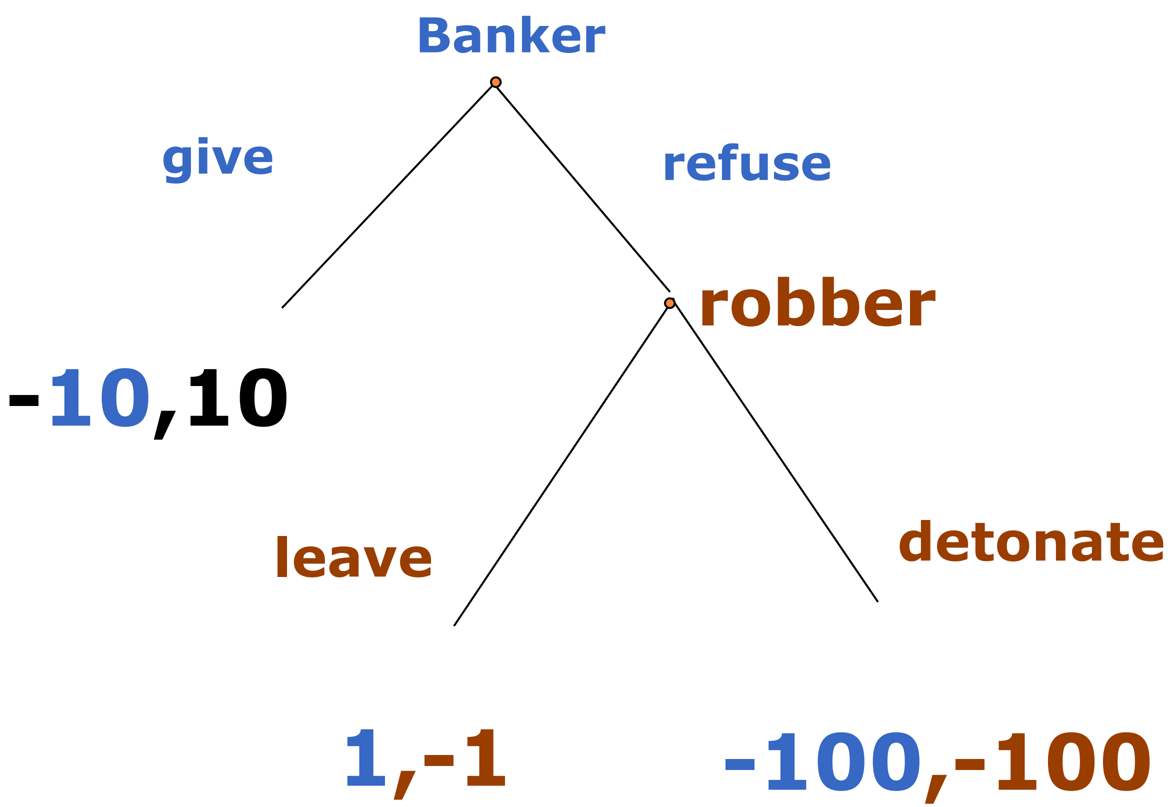
- Extensive Form of a Game (Game tree)
  - Representation of possible moves in a game in the form of a decision tree
  - Initial node
  - Singleton or information set
  - Some rules on drawing a game tree

2.1 Dynamic Games of complete and perfect information

- Players move in turn, responding to each other's actions and reactions
- Player 1 moves first, selecting a_1.
- Player 2 observes a_1, then choose a_2.
- Payoffs are $u_1(a_1,a_2)$ and $u_2(a_1,a_2)$
  - Ex: Bank Robber or grenade game
  - Ex. Stackelberg model
  - Ex. Leontif
  - Ex. Rubinstein's sequential (alternating) bargaining
  - Ex. Entry Deterrence (this topic is optional depending on time availability)

2.1 Dynamic Games of complete and perfect information

- Bank Robber



2.1 Dynamic Games of complete and perfect information

		robber	
		leave	detonate
Banker	give	-10,10	-10, 10
	refuse	1,-1	-100,-100

Sequential Games and empty threat

- There are 2 NE's: (give, detonate) and (refuse, leave)
- Every extensive form game has an corresponding strategic form game.
- Are these two NEs sensible?
- Should a bank robber plays "bomb" if he would have given a chance to play?
- What if we solve this game by looking backward.
- When Robber gets the move at the second stage of the game, he will try to choose a_2 to max $u_2(a_1,a_2)$ given a_1 chosen from the first stage. Robber gets his best reply as $R_2(a_1)$.
- Since the banker can solve the Robber's problem also, knowing that $R_2(a_1)$ will be occurred. Thus, banker will choose a_1 to max $u_1(a_1,a_2)$ given $a_2=R_2(a_1)$

Sequential Games and empty threat

- We call $(a_1^*, R_2(a_1^*))$ the backward induction outcome of this game.

- Key point is that this outcome does not involve noncredible threat.
- Player 1 will not believe the threat by player 2 (in this example, to bomb the bank), because bomb the bank is not the best interest for player 2 when the second stage or the change for him to move arrives.
- In fact, the player 2 does not even have to move in this game, because the backward induction implies that the game ends with the first stage.
- Noting that what makes (give, detonate) as incredible NE because of the logic above, not because the "detonate" is not occurred.

Sequential Games and empty threat

- Backward induction can help eliminating incredible strategy: no intention to do what you say or threat to do.
- So in the grenade game, the sensible NE is just (refuse, leave)
- (give, detonate) is not sensible, and can be eliminated by the backward induction
- NE in each stage of the game (or subgame) can be found by working backward the game tree.
- We do ask for NE in each stage even the game ends at the first stage.
- Later we will develop new refined NE concept known as Subgame perfect NE.

2.1B Sequential Games: Stackelberg

- The Advantage of Moving First
  - The first firm (Leader) can choose a large level of output thereby forcing second firm (follower) to choose a small level.
  - Can show the firms mover advantage by looking at the Stackelberg model and comparing to Cournot model.
  - Setup : firm 1 choose q1; firm2 after seeing q1 choose q2; then the payoffs are given
  - Let work out the generic example

SEQUENTIAL GAMES: STACKELBERG

- Given inverse market demand as P=a-bQ, where $Q = q_1 + q_2$. Assume firm 1 is the leader and moves first by setting q_1, then firm 2, the follower, sees q_1, then choose q_2.
- Assume that Firm 1 knows that firm 2 knows q_1. Firm 2 knows that firm 1knows that firm 2 knows q_1.
- Both firms have an identical marginal cost, c.
- Solve this by working backward

Sequential Games: Stackelberg

q_2^* is the solution from max $\pi^2 = q_2(a - q_1 - q_2 - c)$

$\Rightarrow q_2(q_1) = (a - bq_1 - c)/\ 2b. \quad (*)$

q_1^* is the solution from

$\max \pi^1(q_1,\ q_2(q_1)) = q_1(a - q_1 - q_2(q_1) - c)$

$\text{FromFOC}: q_1^* = (a - c)\ /\ 2b$

and Substitute in $(*)$: $q_2^* = (a - c)\ /\ 4b.$

Total output $Q^* = q_1^* + q_2^* = 3\ (a - c)/4b$

$< Q^*\text{–Cournot} = 2(a - c)/3b.$

Sequential Games: Stackelberg

- Check yourself that the aggregate profits are lower, so the fact that firm 1 the leader makes more money, then firm 2 follower is worse off than in the Cournot game.
- In this game, firm 2 has more information and becomes worse off (contrasting to a single-decision theory)
- Or having firm 1 knows that firm 2 knows q_1 hurts firm 2.
- What if firm 2 announced that he will play tough (producing more output), making both firms worse off if firm 1 is not nice

Threats, Commitments, and Credibility

- In general, there is no rule saying that moving first is always better. In Stackelberg, the leader can set stage for the follower to play, thereby having advantage. We can think of a different game that a second mover has advantage: player 1 Cutting and player 2 allocating pie.
- Strategic Moves or actions that a firm can take to gain advantage in the marketplace
  - Deter entry by potential entrant
  - Induce competitors to reduce output, leave, raise price
  - Implicit agreements that benefit one firm

Threats, Commitments, and Credibility

- Strategic Move
  - Action that gives a player an advantage by constraining his own behavior
  - Firm 1 must constrain his behavior to the extent Firm 2 is convinced that he is committed
- How To Make the First Move
  - Demonstrate Commitment
  - Firm 1 must do more than announce they will produce sweet cereal
    - Invest in expensive advertising campaign
    - Buy large order of sugar and send invoice to firm 2
  - Commitment must be strong enough to induce firm 2 to make the decision firm 1 wants it to make

Threats, Commitments, and Credibility

- Empty Threats
  - If a firm will be worse off if it charges a low price, the threat of a low price is not credible in the eyes of the competitors.
  - When firms know the payoffs of each others actions, firms cannot make threats the other firm knows they will not follow.
- Strategic commitments can be effect but not without risk
  - Rely heavily on accurate knowledge of payoff matrix and industry
  - May have competitors out there that they don't know about and lose sale

Role of Reputation

- Threat might be credible because irrational people don't always make profit maximizing decisions
- A party thought to be crazy can lead to a significant advantage because they can act impossible. For example, notorious kidnapper.
- We will discuss this more later in class of repeated games