EE431 Economics of Financial Markets and Institutions, 2/2016

Problem Sets 9 : Bank Run, Systemic Risk and Deposit Insurance

Solution

1. The Diamond-Dybvig model of bank runs

   Consider an economy where

   - "Primary investment" costs 1 in date 0, and yields 1 if cashed in date 1, 1.44 if cashed at date 2.
   - At date 0, an investor does not know when he/she needs to consume, but each investors has a probability of 0.2 of being type 1 and 0.8 of being type 2.
   - (The other settings are the same as The Diamond-Dybvig (2007))

   (a) Caluclate the optimal deposit contract. What are d_1 and d_2?
   ANSWER.

   - $c_1 = d_1$ and $c_2 = d_2$

$$\underset{d_1, d_2}{\text{Max}} \quad \pi U(d_1) + (1 - \pi) U(d_2)$$

   subject to $d_2 \leq \dfrac{(1 - \pi d_1) R}{1 - \pi}$

   - $\pi = 0.2$, $R = 1.44$
   - The optimal values satisfy

$$\begin{aligned} 0.2U'(d_1) + (0.8)U'(d_2)\frac{(-0.2)1.44}{0.8} &= 0 \\ \frac{U'(d_1)}{U'(d_2)} &= 1.44 \end{aligned}$$

   - $U(c) = 1 - \dfrac{1}{C}$. $U'(C) = \dfrac{1}{C^2}$. $R = 1.44$.
   - Then $\left(\dfrac{d_2}{d_1}\right)^2 = 1.44$, $\dfrac{d_2}{d_1} = \sqrt{1.44}$.
   - $\dfrac{d_2}{d_1} = 1.2$. (1)
   - $d_2 = \dfrac{(1 - 0.2d_1)1.44}{0.8}$.
   - $(08 \times 1.2)d_1 = (1 - 0.2d_1)1.44$. (2)
   - Substitute (1) into (2) and get

$$\begin{aligned} \frac{0.96}{1.44} d_1 &= 1 - 0.2d_1 \\ d_1 &= \frac{1}{0.8\dot{6}} \approx 1.1538 \end{aligned}$$

   - Substitute $d_1 = 1.1538$ into (1). $d_2 = 1.2 \times 1.1538 \approx 1.3846$.
   - **ANSWER**. $d_1 = 1.1538$ and $d_2 = 1.3846$.

(b) What is expected payoff on the primary investment? What is the payoff on the optimal deposit contracts in question (a)? Which one is more liquid, the primary investment of the opitmal deposit contract? Compare liquidity risk ad return of the two assets.

ANSWER.

- To compare Expected payoff
  - Expected payoff = (probability of liquidate early × Payoff received if liquidated early) + (probability of liquidate late × Payoff received if liquidated late)
  - Expected payoff of the primary investment = (0.2× 1) + (0.8× 1.44) = 1.352.
  - Expected payoff of the deposits = (0.2× 1.1538) + (0.8× 1.3846) = 1.3384.
  - Thus, expected payoff of the primary investment is higher than expected payoff of the optimal deposit contract.
- To compare liquidity
  - Liquidity ratio = $\frac{r_2}{r_1} = \frac{\text{return on date 2}}{\text{return on date 1}}$. The lower the liquidity ratio, the higher liquidty it is.
  - Liquidity ratio of the primary investment = $\frac{1.44}{1} = 1.44$.
  - Liquidity ratio of the deposit contract = $\frac{1.3846}{1.1538} \approx 1.2$.
  - Thus, the deposit contract is more liquid than the primary investment.

(c) Why do investors would prefer the deposit contract to the primary investment? Explain economic reasons. Show numerically and graphically .

Answer. To know which one investor would prefer, we will compare expected utility gained from each asset **numerically**.

$$\begin{aligned} EU = & \text{[probability of being type 1}\times\text{utility gained from being type 1]} \\ & + \text{[probability of being type 2}\times\text{utility gained from being type 2]} \end{aligned}$$

If an investor decide to invest in **the primary investment**, then the investor will have 1 to consume at date 1 if he/she is type 1 (with probability 0.2) and the investor will have 1.44 to consume at date 2 if he/she is type 2 at date 2. Therefore, the investor's expected utility is as follows.

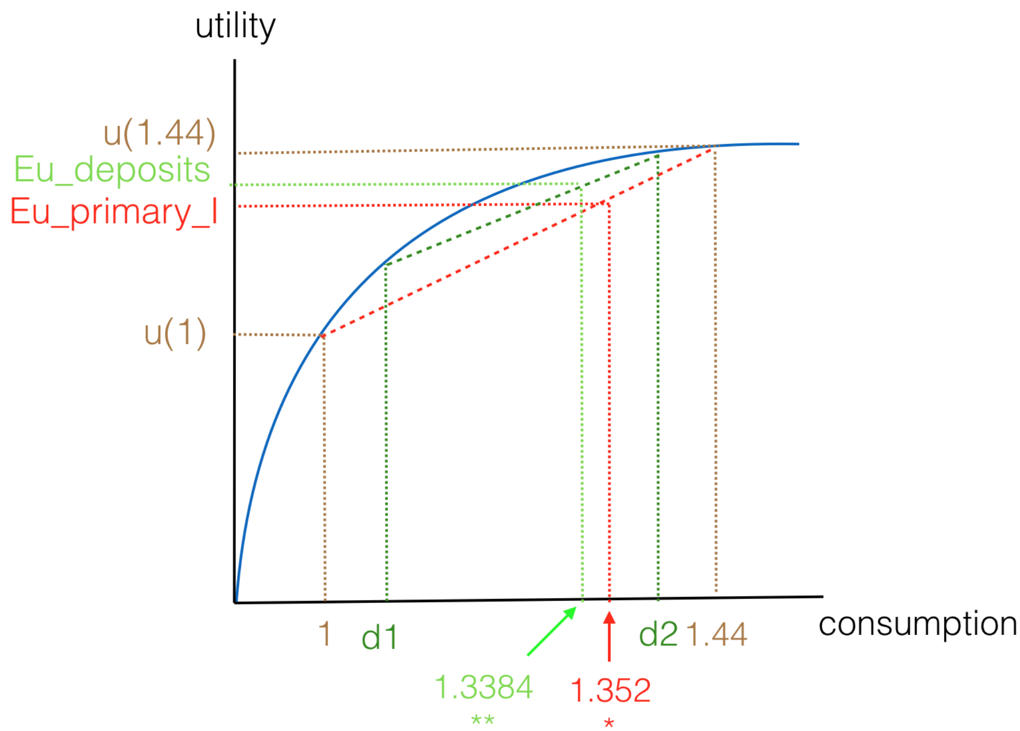
$$\begin{aligned} EU_{\text{primary investment}} &= \text{prob.being type } 1 \times U(C_1) \\ &\quad + \text{prob.being type } 2 \times U(C_2) \\ &= 0.2 \times \left[1 - \frac{1}{1}\right] + 0.8 \times \left[1 - \frac{1}{1.44}\right] \\ &= 0.2444 \end{aligned}$$

If an investor decide to deposit into the bank or invest in **the deposit contract**, then the investor will have 1.1538 to consume at date 1 if he/she is type 1 (with probability 0.2) and the investor will have 1.384632 to consume at date 2 if he/she is type 2 at date 2. Therefore, the investor's expected utility is as follows.

$$
\begin{aligned}
EU_{\text{deposit contract}} &= \text{prob.being type } 1 \times U(C_1) \\
&\quad + \text{prob.being type } 2 \times U(C_2) \\
&= 0.2 \times \left[1 - \frac{1}{1.1538}\right] + 0.8 \times \left[1 - \frac{1}{1.384632}\right] \\
&= 0.2489
\end{aligned}
$$

Expected utility gained from deposit contract is **higher than** expected utility gained from primary investment. Therefore the investor would prefer deposit contract to primary investment.

- The **economic reason** why investors prefer deposit contract which offers lower expected payoff but higher liquidity is that they are risk averse. They are willing to sacrifies some payoffs for risk reduction. "Each investor prefers the more liquid asset. A **risk-averse** investor prefers this smoother pattern of returns; holding the illiquid asset is risky because it delivers a low amount when liquidated early, on date 1. Note that **if investors were not risk averse** and had constant marginal utility of consumption, **they would not prefer this particular liquid asset**."
- Graphically,
  - Eu_deposits = $EU_{\text{deposit contract}} = 0.2489$.
  - Eu_primary investment = $EU_{\text{primary investment}} = 0.2444$.
  - d1 = 1.1538, d2= 1.3846.
  - *Expected payoff on primary investment = 1.352.
  - ** Expected payoff on deposit contract = 1.3384.
  - According to the assumption, utility is a concave function. $u'(c) = C^{-2} > 0$and $u''(c) = -2C^3 < 0; \forall C > 0$.
  - From the figure, expected utility from holding deposit contract is greater than expected utility from primary investment.



(d) Show numerically that the bank is able to offer the deposit contract (liquid asset), even though the bank invests in the primary investment (illiquid asset).

ANSWER. If the bank receives \$1 from each of the 100 investors, it receives \$100 in deposits on date T = 0. If the bank invests in the illiquid asset, it will need to liquidate some of the illiquid asset at T = 1 to pay 1.1538 to those who withdraw. At T = 1, the bank's entire portfolio isworth \$100. Suppose 20 depositors withdraw 1.1538 each, then 20(1.1538) = 23.076 assets must be liquidated: 23.076 percent of the portfolio must be liquidated. If 23.076 assets are liquidated, then 76.924 will remain until T = 2, when they will be worth R = 1.44 each. On date 2, there remain 80 depositors, each will receive

$$\frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} = \frac{76.924 \times 1.44}{80}$$

$$= 1.384632$$

Depositors prefer the more liquid asset to the illiquid asset. A bank can provide the more liquid deposit which has a smaller loss from early liquidation than is available from holding the illiquid assets directly. This liquidity transformation service is one of the most important functions of banks. If the bank offers the more liquid deposits and invests in the illiquid assets, it can create liquidity. It is an equilibrium (a Nash equilibrium) for 20 depositors to withdraw at T = 1, because if all depositors expect 20 to withdraw at T = 1, only type 1 depositors will withdraw because the 80 type 2 depositors prefer the 1.384632 available at T = 2 to the 1.1538 available at T = 1.

(e) Explain and show numerically what action investors will take in the following situation,

i. Suppose that all depositors forecast that 30 depositors will withdraw at date 1.

ii. Suppose that all depositors forecast that 65 depositors will withdraw at date 1.

In each situation, would a bank run occur and how many depositors actually withdraw given their forecast? Explain.

ANSWER

i. Suppose that all depositors forecast that 30 depositors will withdraw at date 1. Then, if 30 depositors withdraw 1.15 each, then 30(1.1532) = 34.596 assets must be liquidated: 34.596 percent of the portfolio must be liquidated. If 34.596 assets are liquidated, then 65.404 will remain until T = 2, when they will be worth R = 1.44 each. On date 2, there remain 70 depositors, each will receive

$$\frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} = \frac{65.404 \times 1.44}{70}$$

$$= 1.345454$$

If all depositors expect 20 to withdraw at T = 1, only type 1 depositors will withdraw at date 1 because the 80 type 2 depositors prefer the 1.347 available at T = 2 to the 1.1538 available at T = 1. Therefore, a bank run would not occur.

ii. Suppose that all depositors forecast that 65 depositors will withdraw at date 1. Then, if 65 depositors withdraw 1.15 each, then 65(1.1538) = 74.997 assets must be liquidated: 74.997 percent of the portfolio must be liquidated. If 74.997 assets are liquidated, then 25.003 will remain until T = 2, when they will be worth R = 1.44 each. On date 2, there remain 35 depositors, each will receive

$$\frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} = \frac{25.003 \times 1.44}{35}$$

$$= 1.028695$$

If all depositors expect 65 to withdraw at T = 1, all depositors will withdraw at date 1 because the 80 type 2 depositors prefer the 1.1538 available at T = 1 to the 1.028695 available at T = 2 to . Therefore, a bank run would occur at T = 1.

(f) Suppose that all depositors forecast that $\hat{f}$ fraction of depositors will withdraw. What is the tipping point (value of $\hat{f}$, for which type 2 depositors will be indifferent between withdraw at date 1 and withdraw at date 2)?

Consider how much is left to pay depositors whowait until date 2 to withdraw if a fraction $\hat{f}$ of initial depositors withdraw at date 1. If all depositors forecast that fraction $\hat{f}$ depositors will withdraw at date 1, they will be indifferent between to withdraw at date 1 and to witdraw at date 2. This means that they would receive the same amount regardless of timing of withdrawal.

Explain how to calculate. Suppose that all depositors forecast that fraction w depositors will withdraw at date 1. Then, if fraction w depositors withdraw 1.1538 each, then 1.1538(100$\hat{f}$) assets must be liquidated: 1.1538(100$\hat{f}$) percent of the portfolio must be liquidated. If fraction of assets are liquidated, then (100-1.1538(100$\hat{f}$)) percent of asset will remain until T = 2, when they will be worth R = 1.44 each. On date 2, there remain (1-$\hat{f}$100) depositors, each will receive

$$\frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} = \frac{(100 - 1.1538(100\hat{f}) \times 1.44}{100 - \hat{f}(100)}$$

$$= \begin{array}{c}\text{the amount a type 2 investor} \\ \text{expect to receive} \\ \text{if withdraw at date 2}\end{array}$$

A type 2 investor would be indifferent between to withdraw at date 1 and to withdraw at date 2, when the amount he/she receive if withdraw at date two is equal to that he/she would receive if withdraw at date one, which is equal to 1.1538. Hence,

$$\frac{\text{the value of the bank's entire portfolio at date 2}}{\text{the number of depositors remains at date 2}} = \frac{\left[100 - 1.1538(100\hat{f})\right] \times 1.44}{100 - \hat{f}(100)}$$

$$1.1538 = \frac{\left[100 - 1.1538(100\hat{f})\right] \times 1.44}{100 - \hat{f}(100)}$$

$$\hat{f} = 0.56375$$

A bank run would occur if all depositors forecast that more than 56.375 depositors withdraw at date 1. A bank run would not occur if all depositors forecast that less than 56.375 depositors withdraw at date 1.