EE211
PRINCIPLES OF MICROECONOMICS

Topic 7.2:

Production Function in the Long Run

Topics

- Isoquant
- Isocost
- Production equilibrium and Expansion Path
- Relationship between Expansion Path and LRTC
- Long-run Costs of Production: LRTC, LRAC, LRMC
- Relationship between Long-run and Short-run Costs
- The Meaning of Returns to Scale
- Economies and Diseconomies of Scale

Production in the Long Run

- In the long run, all inputs are variable. Hence, there is no fixed cost.
- To determine the optimal amount of inputs (say, L & K) for a given price (w & r), the firm face one of the two problems:

1. Output maximization
   - To maximize the output (Q) under the constraint of a given cost (C_0).
2. Cost Minimization
   - To minimize the cost (C) under the constraint of a given output (Q_0).

Long-Run Production Function

- Suppose there are two inputs: L and K.
- Long-run production function can be written as:

$$Q = f(L, K)$$

Graph

Isoquant

- **Isoquant** illustrates all combinations of inputs that yield the same level of output (Q).

- Slope of isoquant is the Marginal Rate of Technical Substitution (MRTS):

$$\frac{\Delta K}{\Delta L} = MRTS$$

Graph

Properties of Isoquants

- Higher isoquants mean higher levels of output.
- Isoquants cannot cross nor be tangent to each other.
- Isoquants always have negative slope.

$$MRTS = \frac{\Delta K}{\Delta L} = -\frac{MP_L}{MP_K}$$

- MRTS is assumed to be diminishing.

Isocost

- Isocost represents all combinations of inputs that can be purchased for the same cost, given that the input prices are fixed.
- Equation: $wL + rK = C_0$
- Slope of isocost = $-\frac{w}{r}$

Graph

Changes in Isocost

- When C_0 increases.

- When r increases.

Production Equilibrium: Output Maximization

- Firm's objective:

$$\max_{L,K} \boldsymbol{Q = f(L,K)} \qquad \boldsymbol{subject\ to} \qquad \boldsymbol{wL + rK = C_0}$$

- Equilibrium condition

E = (L*,K*) is the equilibrium when:

1. $wL^* + rK^* = C_0$
2. $MRTS = -\frac{w}{r}$ (i.e. $-\frac{MP_L}{MP_K} = -\frac{w}{r}$)

➔ $\frac{MP_L}{w} = \frac{MP_K}{r}$

Graph: Output Maximization

Production Equilibrium: Cost Minimization

- Firm's objective:

$$\min_{L,K} C = wL + rK \qquad subject\ to\ \ f(L,K) = Q_0$$

- Equilibrium conditions

E = (L*,K*) is the equilibrium when:

1. $f(L^*, K^*) = Q_0$
2. $\frac{MP_L}{MP_K} = \frac{w}{r}$

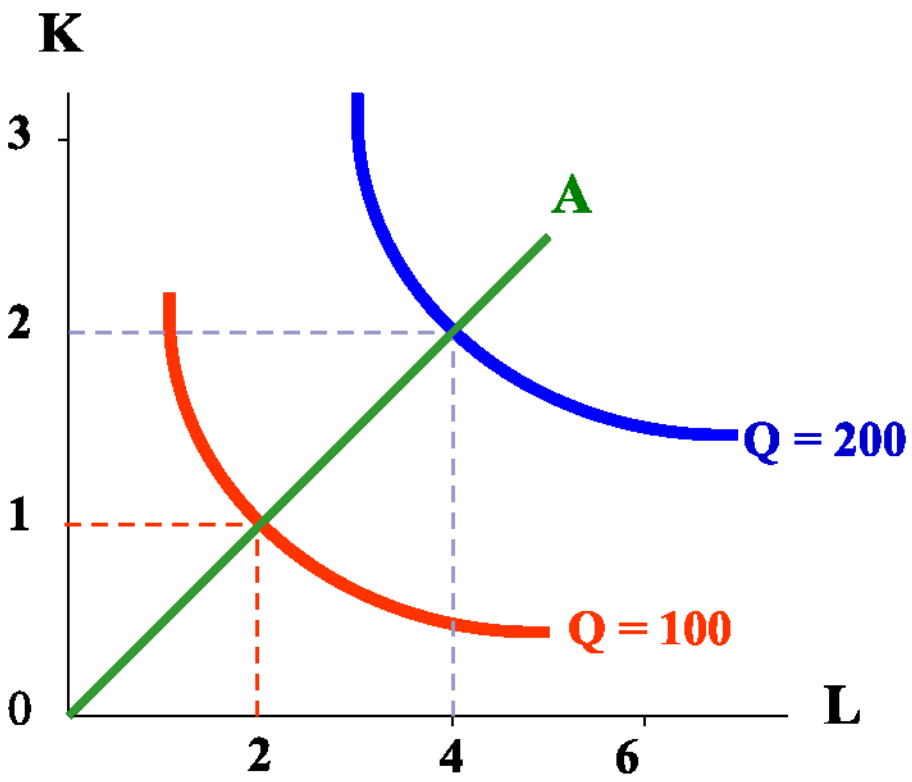
Graph: Cost Minimization

Returns to Scale

- Returns to scale is a property of production function that tells us what happens to output when all inputs are increased by exactly the same proportions.
  - Constant return to scale
    - $f(aL, aK) = af(L, K)$
  - Increasing return to scale
    - $f(aL, aK) > af(L, K)$
  - Decreasing return to scale
    - $f(aL, aK) < af(L, K)$

Graph: Returns to Scale (1)

- Constant returns to scale



Example:

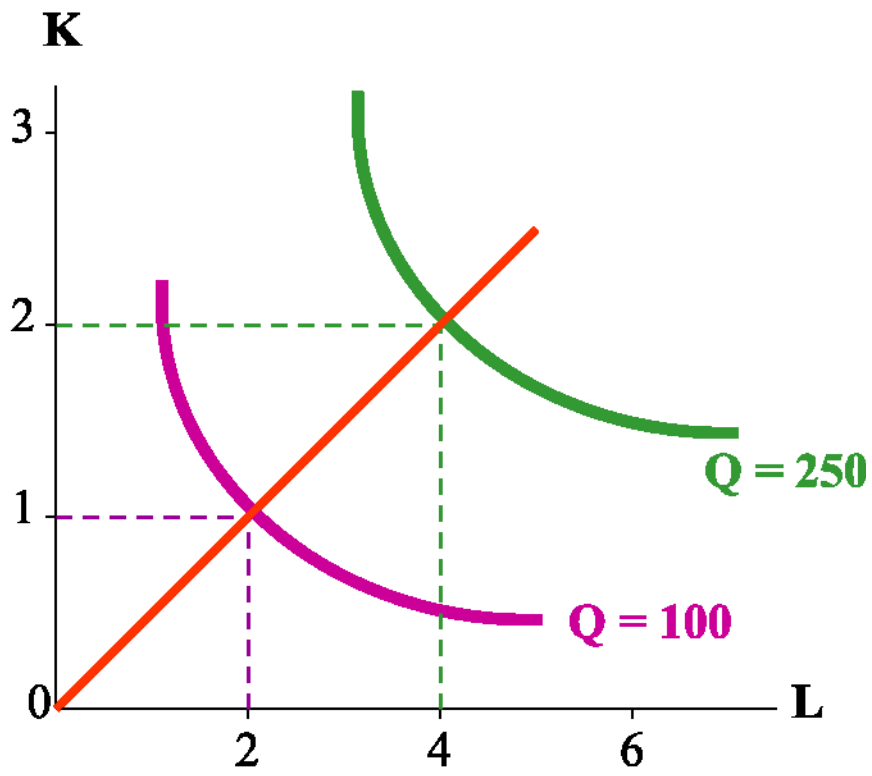
et $Q = 5L^{1/2}K^{1/2}$.

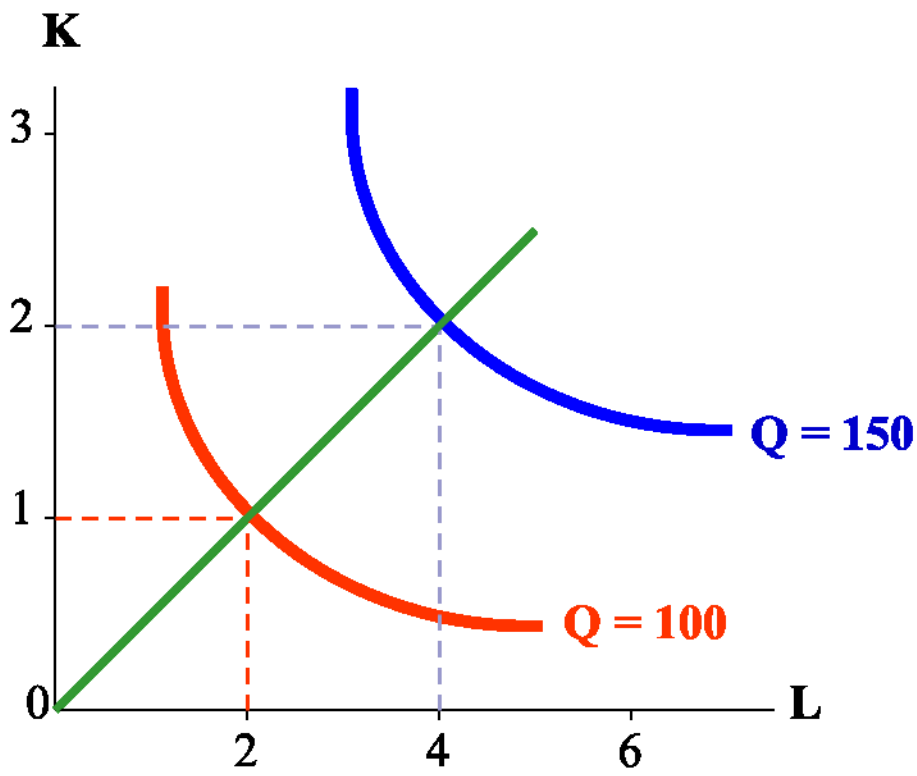
uppose both L and K
ouble. What happens to Q?

Graph credit: Ajarn Nantavut Phipatseritham

Graph: Returns to Scale (2)

- Increasing returns to scale
- Decreasing returns to scale





Graph credit: Ajarn Nantavut Phipatseritham

Least-Cost Combinations & Expansion Path

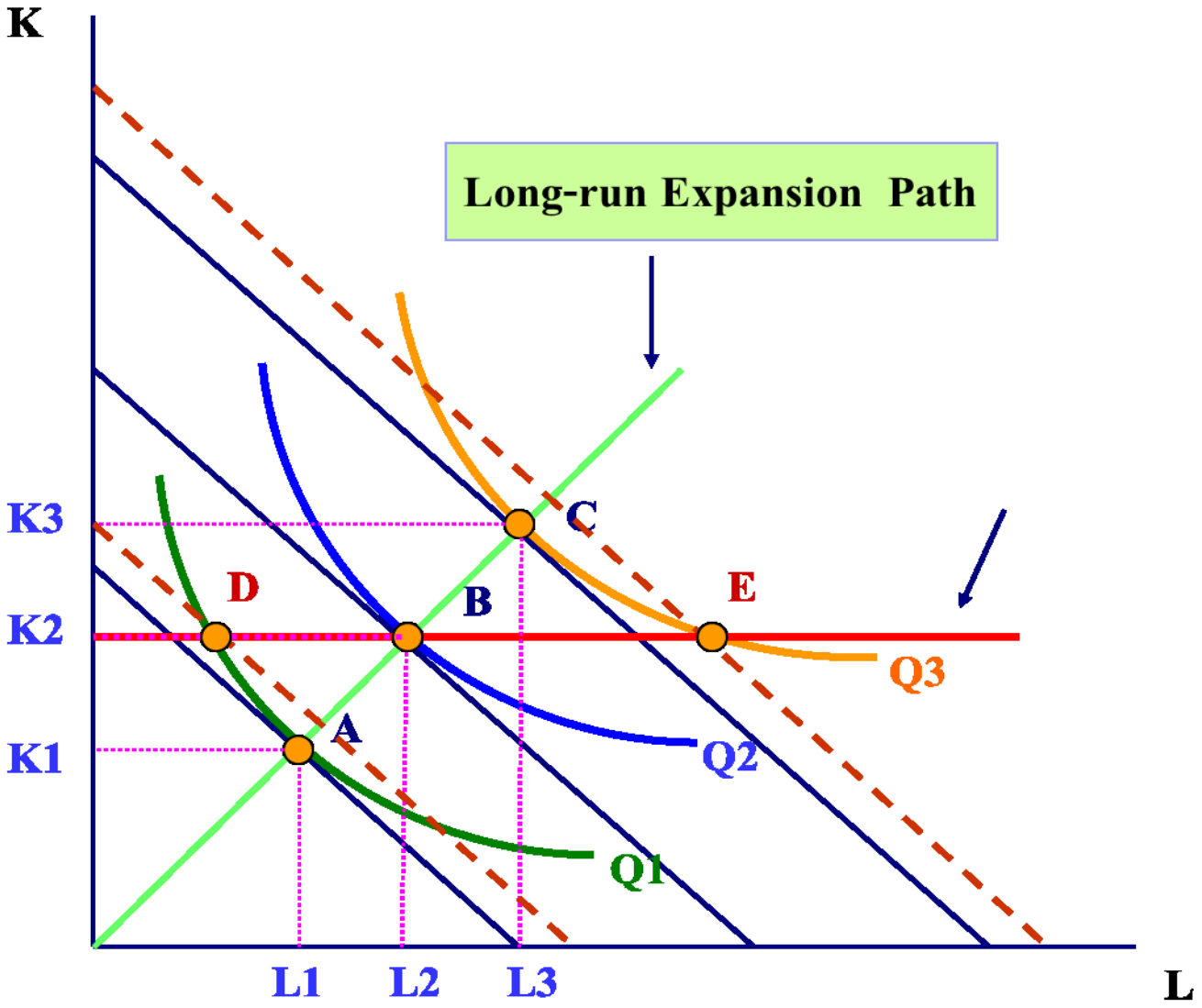
- **Expansion path** is a line that connects all the tangent points between the isocost lines and isoquants, for given w and r.

  - It indicates the optimal L and K that minimizes the total cost for each given quantity level.

    (i.e. It includes all least-cost combinations of inputs).

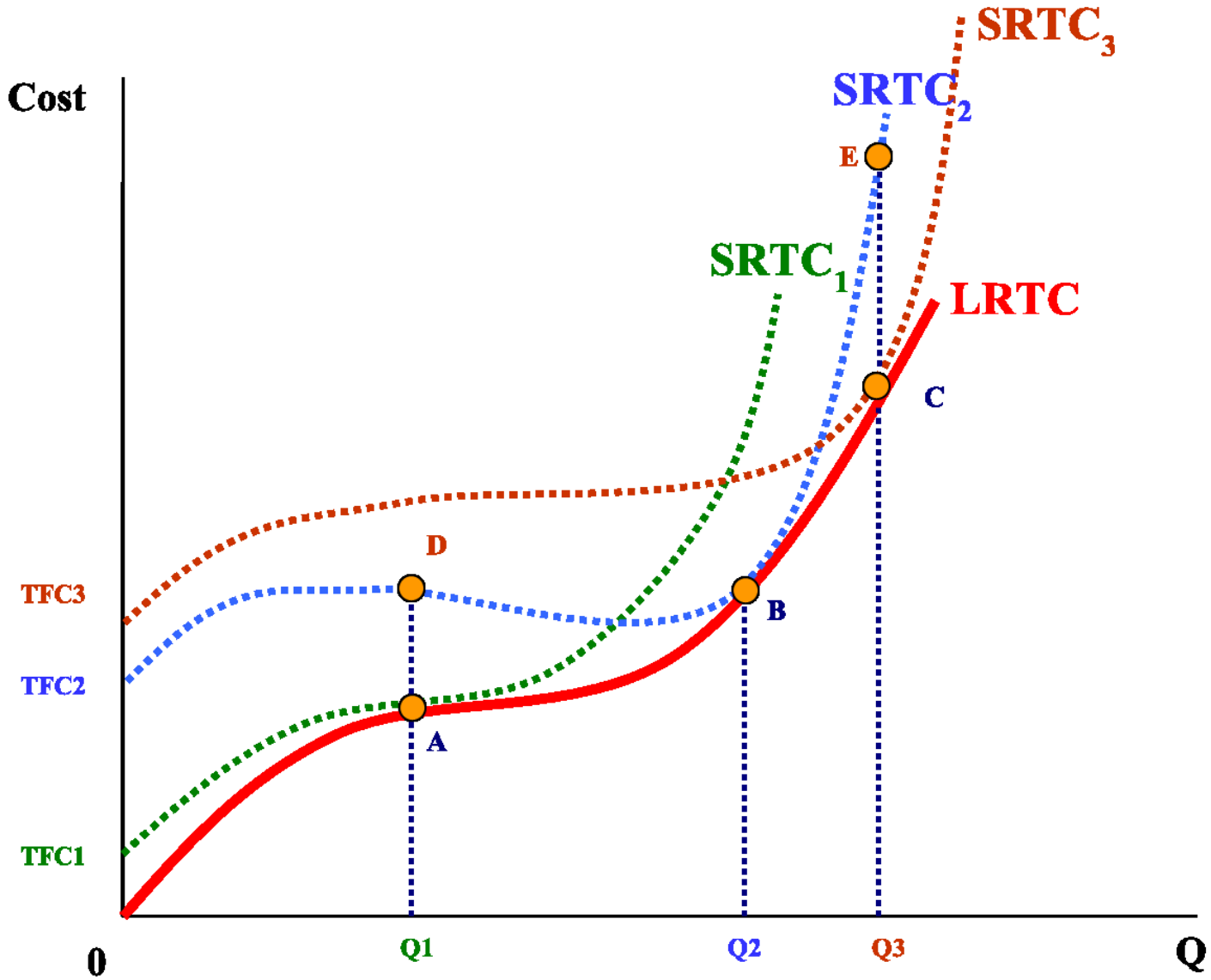
  - It indicates the optimal L and K that maximizes the output Q for each given cost level.

Expansion Path & LRTC (1)



Graph credit: Ajarn Nantavut Phipatseritham

Expansion Path & LRTC (2)



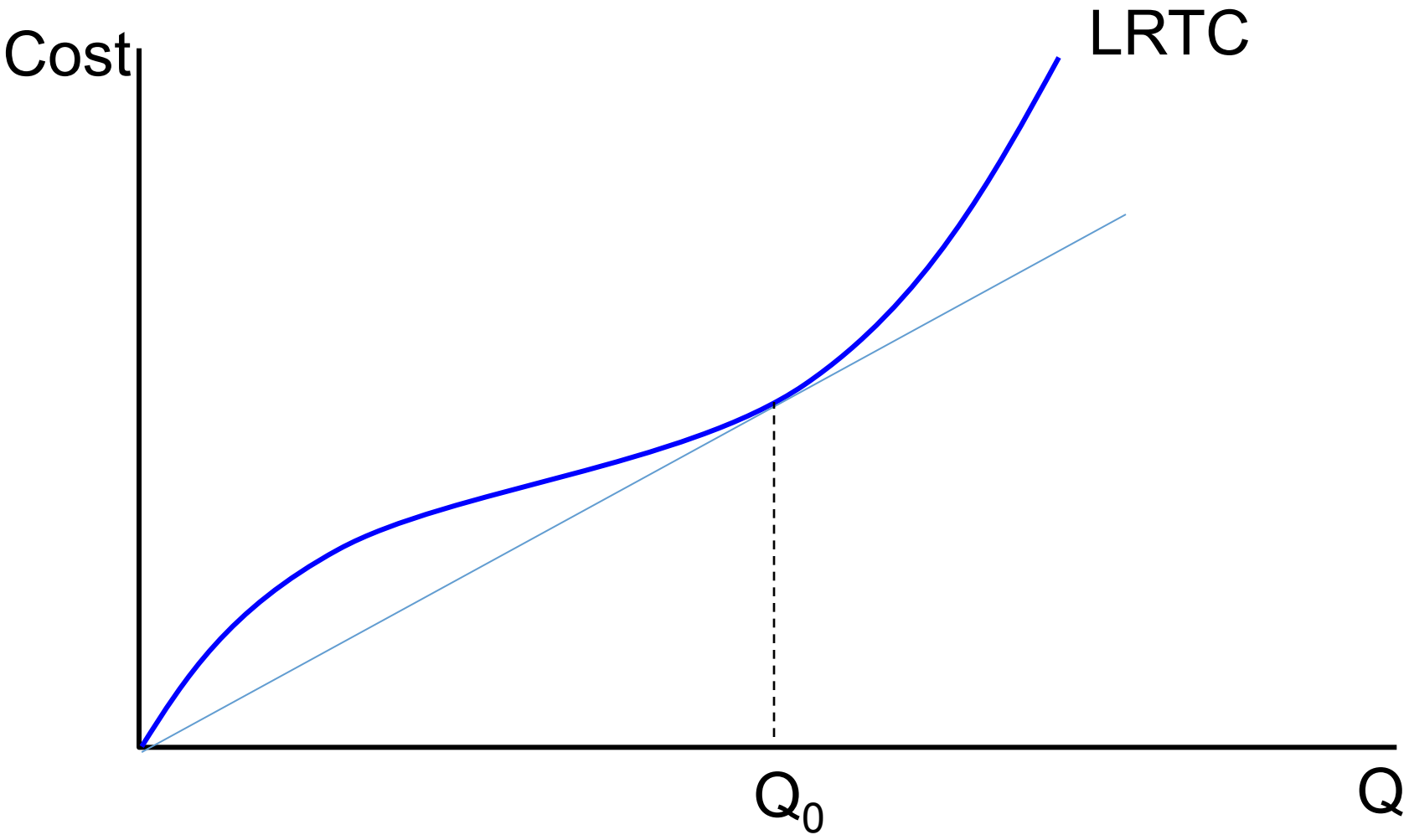
Graph credit: Ajarn Nantavut Phipatseritham

Exercise

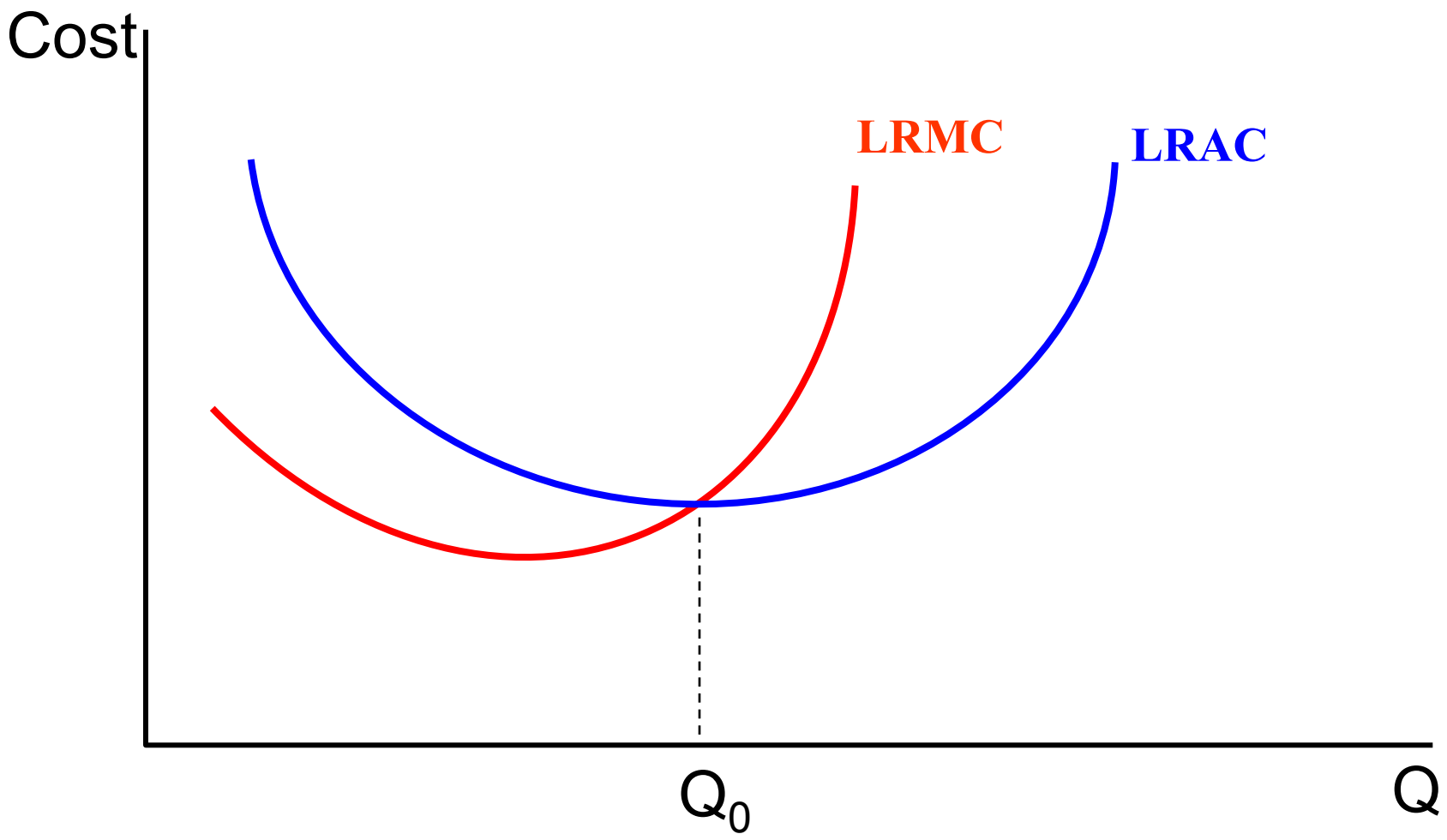
- Fill out the missing numbers in the following table, and use this information to draw the LRTC and SRTC, given w =2 and r =10. (Use the attached handout.)

Point	L	K	Q	wL	rK	LRTC	SRTC when K=4
A	10	2	50			40	n/a
B	15	3	100	30			n/a
C	20	4	200		40		n/a
D	25	5	350				n/a
E	10	4	50		40	n/a	60
F	35	4	350			n/a	

Long-Run Total Cost Curve

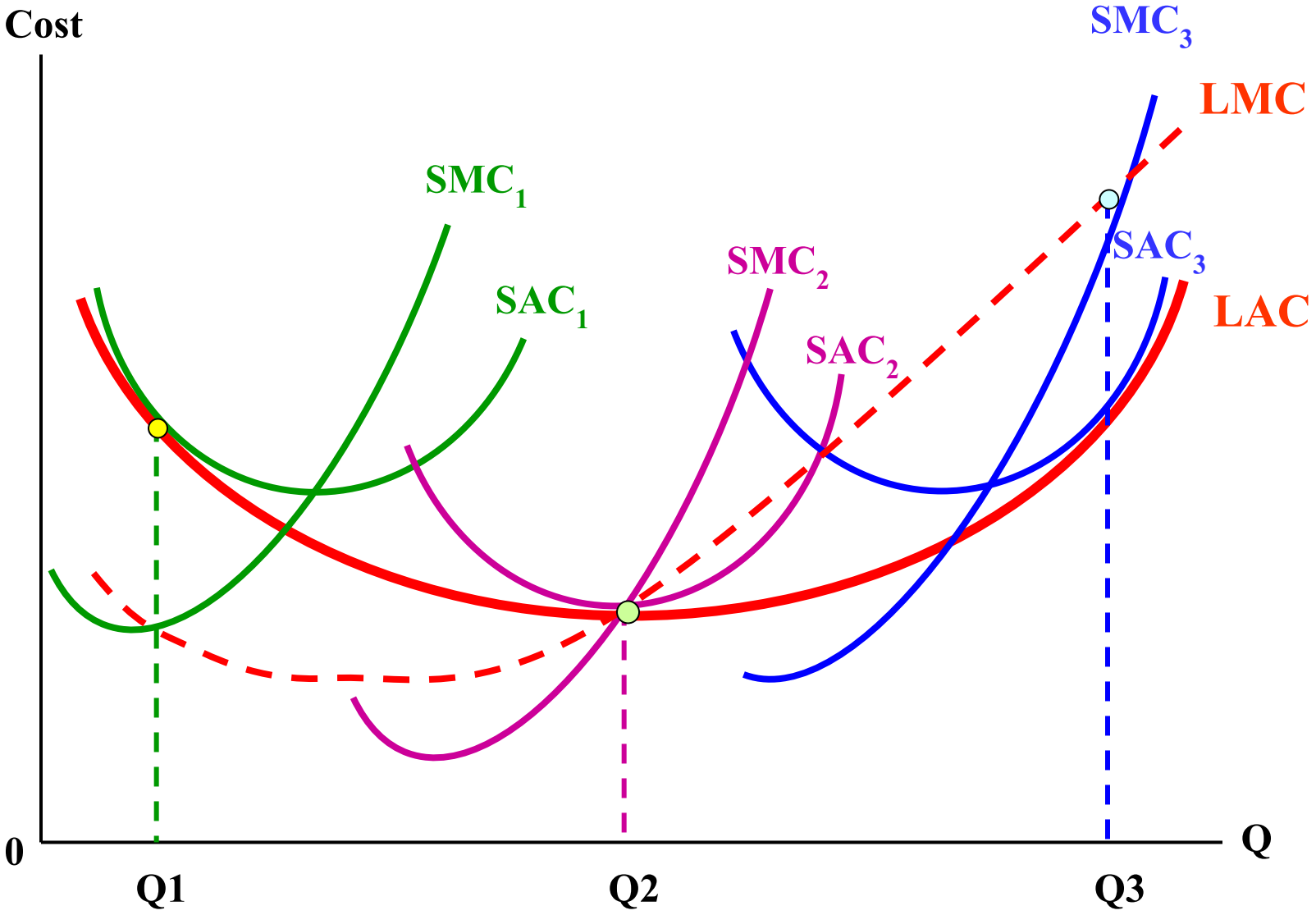


LRTC, LRAC, LRMC



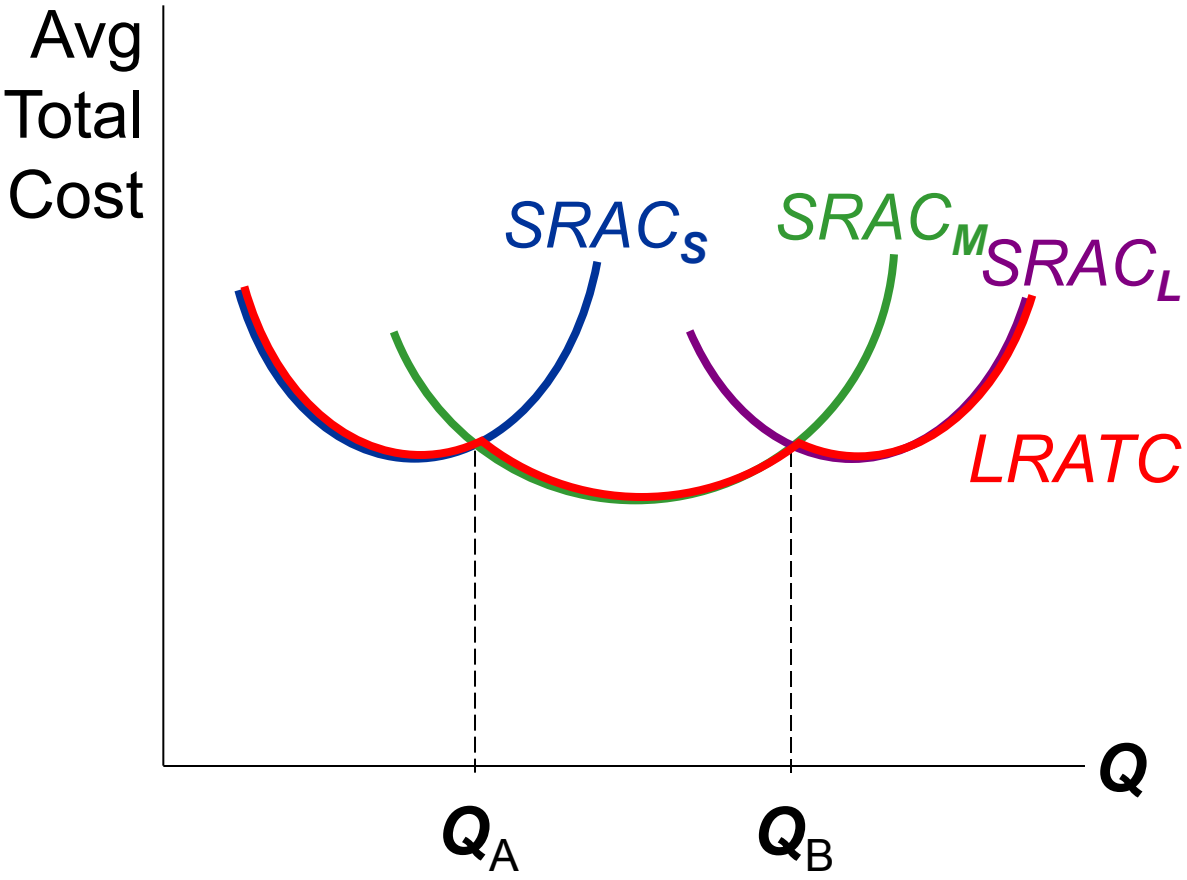
Graph credit: Ajarn Nantavut Phipatseritham

Relationship between Long-run and Short-run Costs

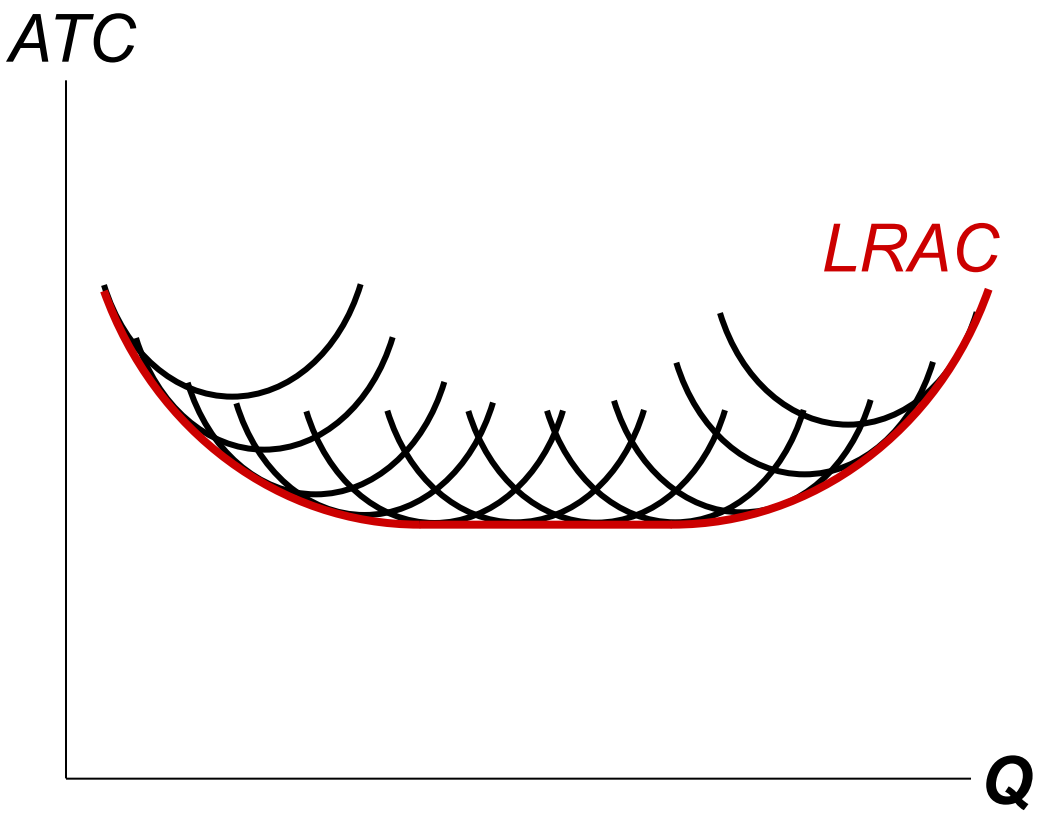


Graph credit: Ajarn Nantavut Phipatseritham

Example: Firm with 3 factory sizes



LRAC & SRAC



Economies of Scale

- Economies of scale:
  - *ATC* falls as *Q* increases.
- Constant returns to scale:
  - *ATC* stays the same as *Q* increases.
- Diseconomies of scale:
  - *ATC* rises as *Q* increases.

Graph: Economies of Scale