

From the given data set (assig8-2.dta):

- Estimate the above models using MLE with Newton-Raphson algorithm.
- Perform hypothesis testing whether  $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$  using LR-test and Wald test.
- Estimate the above models using MLE with BHHH algorithm, make comparison of the estimated result with the result from (1), and give explanation why are they different?

a) Estimate model using MLE with Newton-Raphson algorithm:

$$\hat{z}_i = -1.4004 + 0.4835\hat{X}_1 + 1.454\hat{X}_2 + 2.1732\hat{X}_3 + 1.8555\hat{X}_4$$

b) LR test result

Prob > chi2 = 0.000, thus we can reject the null hypothesis.  
All explanatory variables are significant under the LR test.

Wald test result

Prob > chi2 = 0.001, thus we can reject the null hypothesis.  
All explanatory variables are significant under the Wald test.

c) Estimate model using MLE with BHHH algorithm:

$$\hat{z}_i = -1.4 + 0.4833\hat{X}_1 + 1.4533\hat{X}_2 + 2.1726\hat{X}_3 + 1.855\hat{X}_4$$

The estimated result differs from (a) a little bit — all the coefficient values when using BHHH are smaller than using Newton-Raphson.

This difference is caused by the usage of different algorithm.  
MLE is sensitive to this.

Let  $\Phi(\cdot)$  = Cumulation standard normal probability distribution function and

$$z_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} \quad (3)$$

d. Estimate the models using MLE with Newton-Ralphson algorithm.

$$\hat{z}_i = -0.7729 + 0.3592 \hat{x}_1 + 1.1396 \hat{x}_2$$

Assume that there exists heteroskedasticity in the model as:  $\sigma_i^2 = \exp(\gamma x_{4i})^2$ , then,

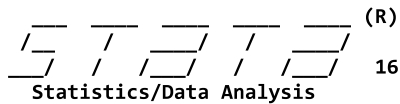
$\Phi(\cdot)$  = Cumulation standard normal probability distribution function  $\Phi z_i / \exp(\gamma x_{4i})$

e. Estimate the models with heteroskedasticity using MLE with Newton-Ralphson algorithm. Perform LR-test whether there exists significant heteroskedasticity.

$$\hat{z}_i = -0.6388 + 0.2921 \hat{x}_1 + 0.9417 \hat{x}_2$$

When perform the LR test,  $\text{Prob} > \chi^2 = 0.000$ .

Thus, we can reject  $H_0$  — meaning all explanatory variables are significant. Thus, there exists significant heteroskedasticity.



(R)

16.0

MP - Parallel Edition

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Notes:

1. Unicode is supported; see [help unicode advice](#).
2. More than 2 billion observations are allowed; see [help obs advice](#).
3. Maximum number of variables is set to 5000; see [help set maxvar](#).
4. New update available; type `-update all-`

```
1 . log using "C:\Users\user\Documents\BE TU\Year2\EE426\HW4\4.7.smcl"
```

---

```
name: <unnamed>  
log: C:\Users\user\Documents\BE TU\Year2\EE426\HW4\4.7.smcl  
log type: smcl  
opened on: 17 Feb 2021, 23:23:32
```

```
2 . do "C:\Users\user\AppData\Local\Temp\STD41d4_000000.tmp"
```

```
3 . program ml_logit  
1. args lnf theta  
2. quietly replace `lnf'=ln(1/(1+exp(-`theta')))) if $ML_y1==1  
3. quietly replace `lnf'=ln(1-(1/(1+exp(-`theta')))) if $ML_y1==0  
4. end
```

```
4 .  
end of do-file
```

```
5 . use "C:\Users\user\Documents\BE TU\Year2\EE426\HW4\assign4.dta"
```

```
6 . ml model lf ml_logit (y= x1 x2 x3 x4)
```

```
7 . ml maximize
```

```
initial:      log likelihood = -90.109133  
alternative:  log likelihood = -86.130008  
rescale:     log likelihood = -86.130008  
Iteration 0:  log likelihood = -86.130008  
Iteration 1:  log likelihood = -66.355929  
Iteration 2:  log likelihood = -63.355226  
Iteration 3:  log likelihood = -57.763041  
Iteration 4:  log likelihood = -55.063873  
Iteration 5:  log likelihood = -54.628535  
Iteration 6:  log likelihood = -54.627603  
Iteration 7:  log likelihood = -54.627603
```

```
Log likelihood = -54.627603  
Number of obs   =      130  
Wald chi2(4)    =      22.79  
Prob > chi2     =      0.0001
```

	y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
	x1	.4835128	.1686119	2.87	0.004	.1530395	.813986
	x2	1.454009	.5001373	2.91	0.004	.4737578	2.43426
	x3	2.173186	.7757021	2.80	0.005	.652838	3.693535
	x4	1.855464	.7138855	2.60	0.009	.4562739	3.254653
	_cons	-1.400447	.5531237	-2.53	0.011	-2.484549	-.316344

8 . test x1=x2=x3=x4=0

- ( 1) [eq1]x1 - [eq1]x2 = 0
- ( 2) [eq1]x1 - [eq1]x3 = 0
- ( 3) [eq1]x1 - [eq1]x4 = 0
- ( 4) [eq1]x1 = 0

chi2( 4) = 22.79  
 Prob > chi2 = 0.0001

9 . est store unres

10 . ml model lf ml\_logit (y= )

11 . ml maximize

initial: log likelihood = -90.109133  
 alternative: log likelihood = -86.130008  
 rescale: log likelihood = -86.130008  
 Iteration 0: log likelihood = -86.130008  
 Iteration 1: log likelihood = -86.129902  
 Iteration 2: log likelihood = -86.129902

Number of obs = 130  
 Wald chi2(0) = .  
 Prob > chi2 = .

Log likelihood = -86.129902

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
_cons	.5026289	.1809802	2.78	0.005	.1479141 .8573436

12 . est store res

13 . lrtest unres res

Likelihood-ratio test  
 (Assumption: res nested in unres)

LR chi2(4) = 63.00  
 Prob > chi2 = 0.0000

14 . ml model lf ml\_logit (y= x1 x2 x3 x4), tech (bhhh)

15 . ml maximize

initial: log likelihood = -90.109133  
 alternative: log likelihood = -86.130008  
 rescale: log likelihood = -86.130008  
 Iteration 0: log likelihood = -86.130008  
 Iteration 1: log likelihood = -64.805009  
 Iteration 2: log likelihood = -58.076419  
 Iteration 3: log likelihood = -55.104476  
 Iteration 4: log likelihood = -54.737891  
 Iteration 5: log likelihood = -54.673222  
 Iteration 6: log likelihood = -54.653417  
 Iteration 7: log likelihood = -54.641919  
 Iteration 8: log likelihood = -54.636424  
 Iteration 9: log likelihood = -54.632812  
 Iteration 10: log likelihood = -54.630848  
 Iteration 11: log likelihood = -54.629561  
 Iteration 12: log likelihood = -54.628821  
 Iteration 13: log likelihood = -54.628345  
 Iteration 14: log likelihood = -54.628063  
 Iteration 15: log likelihood = -54.627885  
 Iteration 16: log likelihood = -54.627777  
 Iteration 17: log likelihood = -54.62771  
 Iteration 18: log likelihood = -54.627669  
 Iteration 19: log likelihood = -54.627644  
 Iteration 20: log likelihood = -54.627628  
 Iteration 21: log likelihood = -54.627618  
 Iteration 22: log likelihood = -54.627612  
 Iteration 23: log likelihood = -54.627609  
 Iteration 24: log likelihood = -54.627607  
 Iteration 25: log likelihood = -54.627605

```

Log likelihood = -54.627605
Number of obs   =      130
Wald chi2(4)    =      16.34
Prob > chi2     =      0.0026

```

y	OPG		z	P> z	[95% Conf. Interval]	
	Coef.	Std. Err.				
x1	.4833318	.1542635	3.13	0.002	.180981	.7856827
x2	1.453275	.597925	2.43	0.015	.2813639	2.625187
x3	2.172641	.8589781	2.53	0.011	.489075	3.856207
x4	1.854961	.7142729	2.60	0.009	.4550117	3.25491
_cons	-1.400063	.5625155	-2.49	0.013	-2.502573	-.2975527

```
16 . do "C:\Users\user\AppData\Local\Temp\STD41d4_000000.tmp"
```

```

17 . program ml_probit
    1. args lnf theta
    2. tempvar z
    3. quietly g double `z'=`theta'
    4. quietly replace `lnf'=ln(normal(`z')) if $ML_y1==1
    5. quietly replace `lnf'=ln(1-normal(`z')) if $ML_y1==0
    6. end

```

```
18 .
end of do-file
```

```
19 . ml model lf ml_probit (y= x1 x2)
```

```
20 . ml maximize
```

```

initial:      log likelihood = -90.109133
alternative:  log likelihood = -87.504336
rescale:      log likelihood = -86.291737
Iteration 0:  log likelihood = -86.291737
Iteration 1:  log likelihood = -78.555778
Iteration 2:  log likelihood = -65.283096
Iteration 3:  log likelihood = -60.75495
Iteration 4:  log likelihood = -60.695531
Iteration 5:  log likelihood = -60.695503
Iteration 6:  log likelihood = -60.695503

```

```

Log likelihood = -60.695503
Number of obs   =      130
Wald chi2(2)    =      22.61
Prob > chi2     =      0.0000

```

y	OPG		z	P> z	[95% Conf. Interval]	
	Coef.	Std. Err.				
x1	.3592444	.0871351	4.12	0.000	.1884627	.5300262
x2	1.139607	.3224845	3.53	0.000	.5075491	1.771665
_cons	-.7729211	.2765064	-2.80	0.005	-1.314864	-.2309785

```
21 . do "C:\Users\user\AppData\Local\Temp\STD41d4_000000.tmp"
```

```

22 . program ml_probit_het
    1. args lnf theta sigma
    2. tempvar z s
    3. quietly g double `s'=exp(`sigma')
    4. quietly g double `z'=`theta'/'s'
    5. quietly replace `lnf'=ln(normal(`z')) if $ML_y1==1
    6. quietly replace `lnf'=ln(1-normal(`z')) if $ML_y1==0
    7. end

```

```

23 .
    end of do-file

24 . ml model lf ml_probit_het (y=x1 x2) (x4, noconstant)

25 . ml maximize

```

```

initial:      log likelihood = -90.109133
alternative:  log likelihood = -83.932039
rescale:     log likelihood = -83.932039
rescale eq:  log likelihood = -70.42663
Iteration 0: log likelihood = -70.42663
Iteration 1: log likelihood = -69.416378
Iteration 2: log likelihood = -67.899365
Iteration 3: log likelihood = -64.594892
Iteration 4: log likelihood = -62.956299
Iteration 5: log likelihood = -61.359985
Iteration 6: log likelihood = -59.838297
Iteration 7: log likelihood = -59.474018
Iteration 8: log likelihood = -59.404563
Iteration 9: log likelihood = -59.404451
Iteration 10: log likelihood = -59.404451

```

```

Number of obs   =      130
Wald chi2(2)    =      16.42
Prob > chi2     =      0.0003

```

Log likelihood = -59.404451

	y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
<b>eq1</b>							
	x1	.2921402	.0773776	3.78	0.000	.140483	.4437975
	x2	.9417319	.280057	3.36	0.001	.3928302	1.490634
	_cons	-.6388431	.2144267	-2.98	0.003	-1.059112	-.2185746
<b>eq2</b>							
	x4	1.183929	.5851991	2.02	0.043	.0369599	2.330898

```

26 . est store unres1

27 . ml model lf ml_probit_het (y= ) (x4, noconstant)

28 . ml maximize

```

```

initial:      log likelihood = -90.109133
alternative:  log likelihood = -83.932039
rescale:     log likelihood = -83.932039
rescale eq:  log likelihood = -70.42663
Iteration 0: log likelihood = -70.42663
Iteration 1: log likelihood = -69.976275
Iteration 2: log likelihood = -69.973883
Iteration 3: log likelihood = -69.973882

```

```

Number of obs   =      130
Wald chi2(0)    =          .
Prob > chi2     =          .

```

Log likelihood = -69.973882

	y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
<b>eq1</b>							
	_cons	.6158099	.2005698	3.07	0.002	.2227003	1.008919
<b>eq2</b>							
	x4	-8.17863	3.212406	-2.55	0.011	-14.47483	-1.882429

29 . est store res1

30 . lrtest unres1 res1

Likelihood-ratio test  
(Assumption: res1 nested in unres1)

LR chi2(2) = **21.14**  
Prob > chi2 = **0.0000**

31 . log close

name: <unnamed>  
log: C:\Users\user\Documents\BE TU\Year2\EE426\HW4\4.7.smcl  
log type: smcl  
closed on: 17 Feb 2021, 23:31:29

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32 .