

**EE 422 Mathematical Economics 2 (1/2016)**

**Assignment 5**

**The due date for this assignment is (Wednesday) 16<sup>th</sup> November 2016.**

1. Solve the following two difference-equation systems (10 points each):

(a)  $x_{t+1} + x_t + 2y_t = 24$

$y_{t+1} + 2x_t - 2y_t = 9$  (with  $x_0 = 10$  and  $y_0 = 9$ )

(b)  $x_{t+1} - x_t - \frac{1}{3}y_t = -1$

$x_{t+1} + y_{t+1} - \frac{1}{6}y_t = 8\frac{1}{2}$  (with  $x_0 = 5$  and  $y_0 = 4$ )

2. Solve the following two differential-equation systems (10 points each):

(a)  $x'(t) - x(t) - 12y(t) = -60$

$y'(t) + x(t) + 6y(t) = 36$  (with  $x(0) = 13$  and  $y(0) = 4$ )

(b)  $x'(t) - 2x(t) + 3y(t) = 10$

$y'(t) - x(t) + 2y(t) = 9$  (with  $x(0) = 8$  and  $y(0) = 5$ )

3. As special cases of the differential-equation system,  $x' = f(x, y)$  and  $y' = g(x, y)$ , assume that

(a)  $f_x = 0$                        $f_y > 0$                        $g_x > 0$                       and                       $g_y = 0$

(b)  $f_x = 0$                        $f_y < 0$                        $g_x < 0$                       and                       $g_y = 0$

For each case, construct an appropriate phase diagram, draw the streamlines, and determine the nature of equilibrium. (10 points each)

4. Analyze the local stability of each of the following nonlinear systems (10 points each):

(a)  $x' = e^x - 1$                        $y' = ye^x$

(b)  $x' = x + 2y$                        $y' = x^2 + y$

(c)  $x' = 1 - e^y$                        $y' = 5x - y$

(d)  $x' = x^3 + 3x^2y + y$                        $y' = x(1 + y^2)$

5. The following system possesses a zero-valued Jacobian. Construct a phase diagram, and deduce the locations of all the equilibria that exist: (10 points)

$x' = x + y$                        $y' = -x - y$