Solution to Practice Questions

- covariance or $Cov(X,Y)$ or $\sigma_{X,Y} = E[(X-\mu_X)(Y-\mu_Y)]$
  ** Please be noted that the expression in your note is wrong. The term $E[(X-\mu_X)E(Y-\mu_Y)]$ should actually be $E[(X-\mu_X)(Y-\mu_Y)]$. Apologies for the typo.

$$\begin{aligned}
\sigma_{X,Y} &= E[(X-\mu_X)(Y-\mu_Y)] \\
&= E(XY - X\mu_y - \mu_X Y + \mu_x\mu_y) \\
&= E(XY) - \mu_y E(X) - \mu_x E(Y) + \mu_x\mu_y \\
&= E(XY) - \mu_X\mu_Y - \mu_X\mu_Y + \mu_X\mu_Y \\
&= E(XY) - \mu_X\mu_Y.
\end{aligned}$$

- Show that $MSE(W) = Var(W) + [Bias(W)]^2$

from

$$\begin{aligned}
MSE(W) &= E[(W-\theta)^2] \\
&= E(W^2 - 2W\theta + \theta^2) \\
&= E(W^2) - 2\theta E(W) + \theta^2 \qquad (1)
\end{aligned}$$

We know that $Var(W) = E(W^2) - [E(W)]^2$.
Thus, from (1), we have

$$\begin{aligned}
MSE(W) &= Var(W) + [E(W)]^2 - 2\theta E(W) + \theta^2 \\
&= Var(W) + [E(W) - \theta]^2 \\
&= Var(W) + [Bias(W)]^2.
\end{aligned}$$