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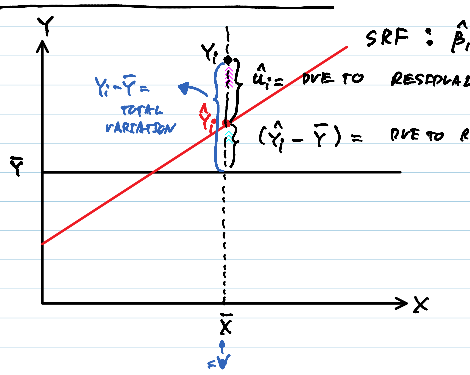
ANALYSIS OF VARIANCE



SRF : $\hat{\beta}_1 + \hat{\beta}_2 X_i$

$$\boxed{TSS = ESS + RSS}$$

TSS = TOTAL SUM OF SQUARES

$= \Sigma (Y_i - \bar{Y})^2$

ESS = EXPLAINED SUM OF SQUARES

$= \Sigma (\hat{Y}_i - \bar{Y})^2 = \hat{\beta}_2^2 \Sigma x_i^2$

RSS = RESIDUAL SUM OF SQUARES

$= \Sigma (Y_i - \hat{Y}_i)^2$

OR $= \Sigma \hat{u}_i^2$.

$$\underbrace{\Sigma y_i^2}_{TSS} = \underbrace{\Sigma \hat{y}_i^2}_{ESS} + \underbrace{\Sigma \hat{u}_i^2}_{RSS}$$

$$\boxed{\Sigma y_i^2 = \hat{\beta}_2^2 \Sigma x_i^2 + \Sigma \hat{u}_i^2}$$

ANOVA TABLE

ANALYSIS OF VARIANCE

SUM OF SQUARES

MEAN SUM OF SQUARES $= \frac{SS}{df}$

SOURCE OF VARIATION	SS	df	MSS
DUE TO REGRESSION (ESS)	$\hat{\beta}_2^2 \Sigma x_i^2$	1	$\hat{\beta}_2^2 \Sigma x_i^2 / 1$ — (1)
DUE TO RESIDUALS (RSS)	$\Sigma \hat{u}_i^2$	$n-2$	$\Sigma \hat{u}_i^2 / n-2 = \hat{\sigma}^2$ — (2) (OUR OLD KNOWLEDGE)
TSS	Σy_i^2	$n-1$	$\Sigma y_i^2 / n-1$ — (3)

THEREFORE,

$$F = \frac{MSS \text{ OF } ESS \; (1)}{MSS \text{ OF } RSS \; (2)}$$

$$= \frac{\hat{\beta}_2^2 \Sigma x_i^2}{\Sigma \hat{u}_i^2 / (n-2)}$$

$$= \frac{\hat{\beta}_2^2 \Sigma x_i^2}{\hat{\sigma}^2}$$

$\sim$ F - DISTRIBUTION WITH 1 df in THE NUMERATOR AND $(n-2)$ df IN THE DENOMINATOR.

DECISION RULE : IF $F^* >$ CRITICAL F VALUE $(F_{\alpha, 1, n-2})$,

THEN REJECT H_0. (i.e, X COULD BE USED TO EXPLAIN Y)

: IF $F^* <$ CRITICAL F VALUE $(F_{\alpha, 1, n-2})$

THEN, FAIL TO REJECT H_0.

PRATICE

$\alpha = 0.05$	$\alpha = 0.01$
FIND $F_{\alpha,1,5} = 6.61$	FIND $F_{\alpha,1,5} = 16.3$
$F_{\alpha,1,8} =$ ----	$F_{\alpha,1,8} =$ ---
$F_{\alpha,2,5} =$ ---	$F_{\alpha,2,5} =$ ____

$F_{\alpha,2,5} = $ ___ $F_{\alpha,2,5} = $ ___

$F_{\alpha,3,5} = $ ___ $F_{\alpha,3,5} = $ ___

BACK TO OUR WAGE - EDU EXAMPLE:

$H_0 : \beta_2 = 0$ (EDU CANNOT BE USED TO EXPLAIN WAGE)

$H_1 : \beta_2 \neq 0$ (EDU CAN BE USED TO EXPLAIN WAGE)

$n = 13$

CALCULATION

SOURCE OF VARIATION	SS	df	MSS	
ESS	95.4255	1	95.4255	①
RSS	9.6928	n-2 = 11	.8811	②
TSS	105.1183	n-1 = 12		

$$F^* = \frac{\text{MSS OF ESS}}{\text{MSS OF RSS}} = \frac{95.4255}{0.8811}$$

$$= 108.3026$$

CRITICAL F VALUE ($F_{0.05,1,11}$) = 4.84.

SINCE F^* > CRITICAL F, THEN WE REJECT H_0.

IT IMPLIES THAT <u>EDU STATISTICALLY INFLUENCE WAGE</u>.

EXTENSIONS OF THE TWO-VARIABLE LINEAR REGRESSION MODEL.

(CHAPTER 6)

AGENDA: I REGRESSION THROUGH THE ORIGIN (R-T-O) OR INTERCEPTLESS MODEL:

$$Y_i = \beta_2 X_i + u_i .$$

II SCALING AND UNITS OF MEASUREMENT

III FUNCTIONAL FORMS OF REGRESSION MODELS

I REGRESSION THROUGH THE ORIGIN (R-T-O)

$$Y_i = \beta_2 X_i + u_i \quad \text{(PRF)}$$

EX: CAPM MODEL

CAPITAL ASSET PRICING MODEL

↓ VIA OLS

WE OBTAIN $\hat{Y}_i = \hat{\beta}_2 X_i$.

OLS ESTIMATOR FOR β_2

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\hat{u}_i = Y_i - \hat{\beta}_2 X_i$$

$$\sum \hat{u}_i^2 = \sum (Y_i - \hat{\beta}_2 X_i)^2 \rightarrow \text{SUM OF SQUARED ERROR.}$$

$$\sum \hat{u}_i^2 = \sum (Y_i - \hat{\beta}_2 X_i)^2$$ → SUM OF SQUARED ERROR.

$$\boxed{\min_{\hat{\beta}_2} \sum \hat{u}_i^2 = \sum (Y_i - \hat{\beta}_2 X_i)^2}$$

"OUR OBJECTIVE FUNCTION"

FONC : $\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_2} = 2 \sum \underbrace{(Y_i - \hat{\beta}_2 X_i)}_{=\hat{u}_i}(-X_i) = 0$

FIRST-ORDERED NECESSARY CONDITION

$$\boxed{\sum \hat{u}_i X_i = 0}$$

FROM $2 \sum (Y_i - \hat{\beta}_2 X_i)(-X_i) = 0$

$$\sum (Y_i - \hat{\beta}_2 X_i)(-X_i) = 0$$

$$\sum X_i Y_i - \hat{\beta}_2 \sum X_i^2 = 0$$

$$\boxed{\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}}$$

*** R-T-O MODEL

$$\boxed{\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}}$$

$x_i = (X_i - \bar{X})$, $y_i = (Y_i - \bar{Y})$

MODEL w/ INTERCEPT

Q: IS $\hat{\beta}_2$ UNBIASED ?

A: CHECK IT OUT...

$$\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2}$$

$$\hat{\beta}_2 = \frac{\sum X_i (\beta_2 X_i + u_i)}{\sum X_i^2}$$

$$\hat{\beta}_2 = \frac{\cancel{\sum X_i} \beta_2 \cancel{X_i}}{\cancel{\sum X_i^2}} + \frac{\sum X_i u_i}{\sum X_i^2}$$

$$\hat{\beta}_2 = \beta_2 + \frac{\sum X_i u_i}{\sum X_i^2}$$

$$\boxed{E(\hat{\beta}_2) = \beta_2}$$

$\because E\left[\frac{\sum X_i u_i}{\sum X_i^2}\right] = 0$