

Lecture 5

CAPM

Pym Manopimoke, PhD, Bank of Thailand

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Markowitz vs. the CAPM

- Using the Markowitz model, the number of parameters that must be estimated is huge for active portfolio management

↪ For a portfolio of N (100) securities we need:

σ_i 's	N	100
$E(r_i)$'s	N	100
$Cov(r_i, r_j)$'s	$\frac{1}{2}N(N-1)$	4950
<i>Total</i>	$\frac{1}{2}N(N+3)$	5150

↪ About how much data will we need when we have 500 securities?
1000 Securities?

- These parameters may be estimated with error
- Small errors in mean or covariance estimates often lead to unreasonable weights

Markowitz vs. the CAPM (cont.)

- A model of what returns should be will be useful
- The CAPM is an equilibrium model specifying a relation between expected rates of return and covariances for all assets
- Note that the Markowitz portfolio optimization problem is relevant for each investor, regardless of whether the equilibrium argument, and the CAPM, is correct or not

Equilibrium Concept

- If everyone in the economy holds an efficient portfolio, then how should securities be priced so that they are actually bought 100% in equilibrium?
 - For example, if based on the prices/expected returns our model comes up with, we found that no maximizing investor would like to buy IBM, then something is wrong.
 - IBM would be priced too high
 - The price of IBM would have to fall to the point where, in aggregate, investors would want to hold exactly the number of IBM shares outstanding
- So, what sort of prices (risk/return relationships) are feasible in equilibrium?

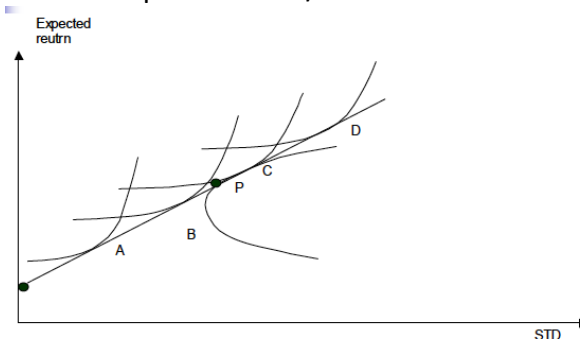
The CAPM World - Assumptions


- Perfect Competition
- No tax, transaction costs
- Rational, mean-variance optimizing agents
- There is a risk free asset (same borrowing and lending rate)
- Unlimited short sales and borrowing and lending
- Homogenous expectations
- All assets are marketable, i.e. can be bought and sold and are infinitely divisible

Note: Investors may differ in risk aversion and endowments

Conceptual Derivation of CAPM

- Investors solve the Markowitz's portfolio selection problem
- Investors have the same efficient frontier and optimal CAL
- Investors hold the same optimal risky portfolio
- The proportion of wealth each investor invests in the optimal risky portfolio depends on his/her risk aversion level




Optimal risky portfolio P

w1	w2	w3	w4
0.2	0.2	0.3	0.3

Investor's Wealth	Risk Aversion	Proportion of wealth in riskfree	Proportion of risky assets	Investor's \$ investment in each risky asset			
				w1	w2	w3	w4
A: 1000	highest	0.8 (\$800)	0.2 (\$200)	40	40	60	60
B: 10000	medium	0.04 (400)	0.96 (\$9600)	1920	1920	2880	2880
C: 10000	low	-0.1 (\$-1000)	1.1 (\$11000)	2200	2200	3300	3300
D: 1000	very low	-0.2 (\$-200)	1.2 (\$1200)	240	240	360	360
22000		0	22000	4400	4400	6600	6600

- Proportions of each security in the market portfolio = proportions of each asset in P

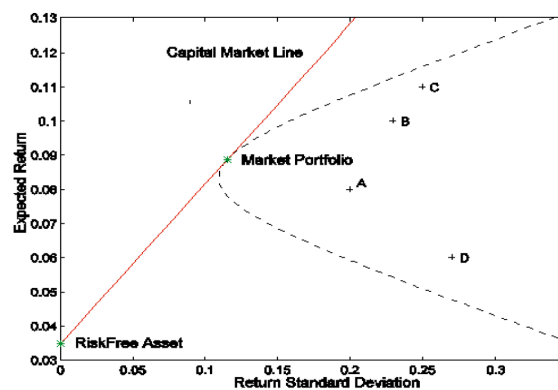
Market Portfolio

- What do we mean by the “market” portfolio?
 - Markowitz:
 - CAPM:

Definition: The “market” or total wealth portfolio is a portfolio of all risky securities held in proportion to their market value.

Implications of CAPM

- All investors hold the market portfolio
- The optimal CAL is the Capital Market Line (CML). This line gives us the set of efficient or optimal risk-return combinations



Implications of the CAPM (cont.)

- The risk premium of the market portfolio is proportional to the average risk aversion of investors in the market and the variance of the market portfolio

Implications of the CAPM (cont.)

- The expected return of individual securities depends only on its correlation with the market portfolio (beta risk)

Implications of the CAPM (cont.)

- The contribution of asset i to the variance of the market portfolio is
- The contribution of asset i to the risk premium of the market portfolio is

Implications of the CAPM (cont.)

- The reward to risk ratio for investment in asset i is
- The reward to risk ratio for investment in the market portfolio is

Arriving at CAPM

- Putting it all together we have the CAPM

Intuition

- Investors only want to hold a security in their portfolio if it provides a reasonable amount of extra reward (eg. expected return) in return for the risk (or variance) it adds to the portfolio
- For all securities, what a security adds to the portfolio will be just offset by what it adds in terms of expected return. The ratio of marginal return to marginal variance must be the same for all assets
 - What it adds in expected return is its expected excess return
 - What it adds in risk is proportional to its covariance with the portfolio
 - This is the intuition for the standard form of the CAPM which relates beta to expected return

A Formal Derivation of CAPM

- Let's do this more formally
 - Remember that, under our assumptions, all investors must hold the market portfolio
 - Based on our notion of equilibrium, every investor must be content with their portfolio holdings; if this were not the case then the prices of the securities would have to change
 - This is just a supply and demand argument; if some investors want to buy IBM, and noone wants to sell, prices will have to change (move up)
 - In equilibrium, everyone must be optimally invested
 - This must mean that, in equilibrium, noone can do anything to increase the Sharpe-ratio of their portfolio

- Suppose you currently hold the market portfolio and decide to invest in a small additional fraction of your wealth in GM, which you finance by borrowing at the risk free rate.

- Now what if we invest more in GM, and invest just enough less in the IBM so that our portfolio variance stays the same

- Remember that, we are holding the market portfolio, which is also the tangency portfolio
- This portfolio has the highest Sharpe Ratio of all portfolios
- Therefore, by definition, we cannot increase its expected return while keeping variance constant
- For this to be true, it must be that

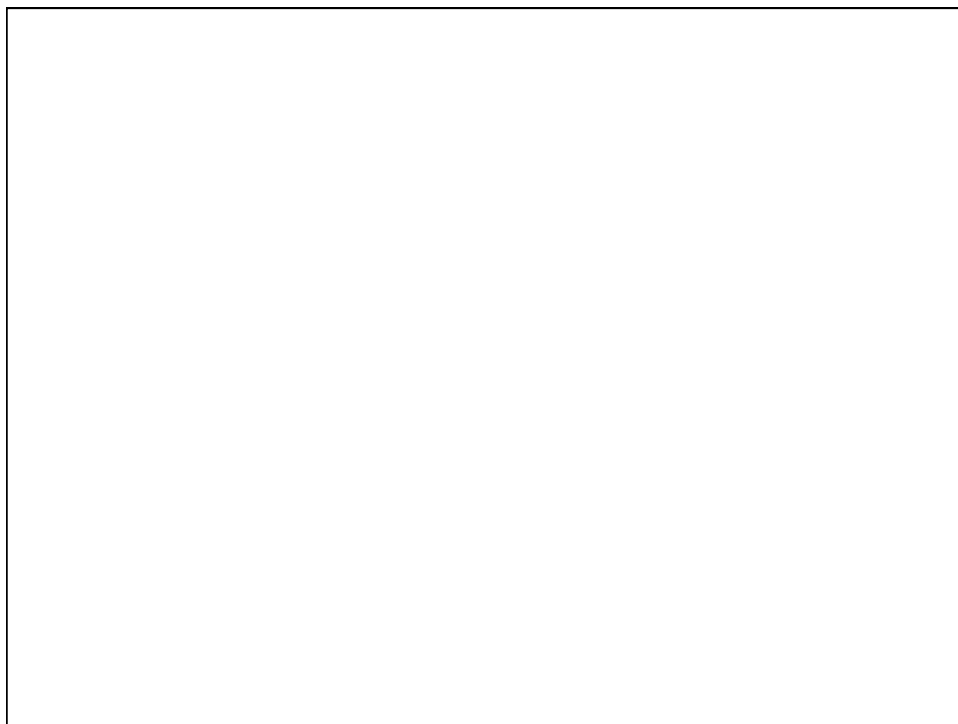
- Note that this also holds for the portfolio of assets as well
- We can use the market portfolio in place of IBM

Security Market Line

Systematic vs Idiosyncratic Risk

Interpret beta as a regression coefficient:

- Use historical returns and estimate the time-series regression



Beta Estimates

- Zero beta
- Negative beta
- Lower beta
- Higher beta

Example

Asset	$E(r)$	σ
A	8%	20%
B	10%	23%
C	11%	25%
D	6%	27%

Correlations				
Assets	A	B	C	D
A	1.0	0.0	0.3	-0.2
B	0.0	1.0	0.2	-0.2
C	0.3	0.2	1.0	-0.2
D	-0.2	-0.2	-0.2	1.0

- We also assume that $r_f = 3.5\%$.
- The resulting MVE portfolio has weights on the risky assets of:

$$w_{MVE} = \begin{bmatrix} 0.2515 \\ 0.3053 \\ 0.2270 \\ 0.2161 \end{bmatrix}$$

1. Calculate the β s for each of the four assets:

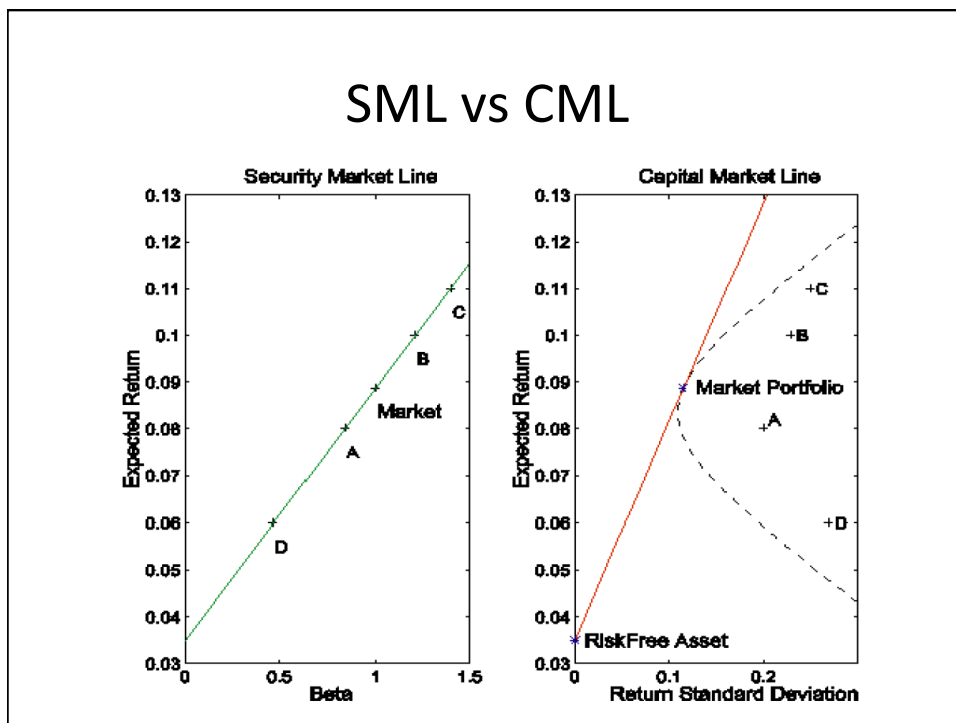
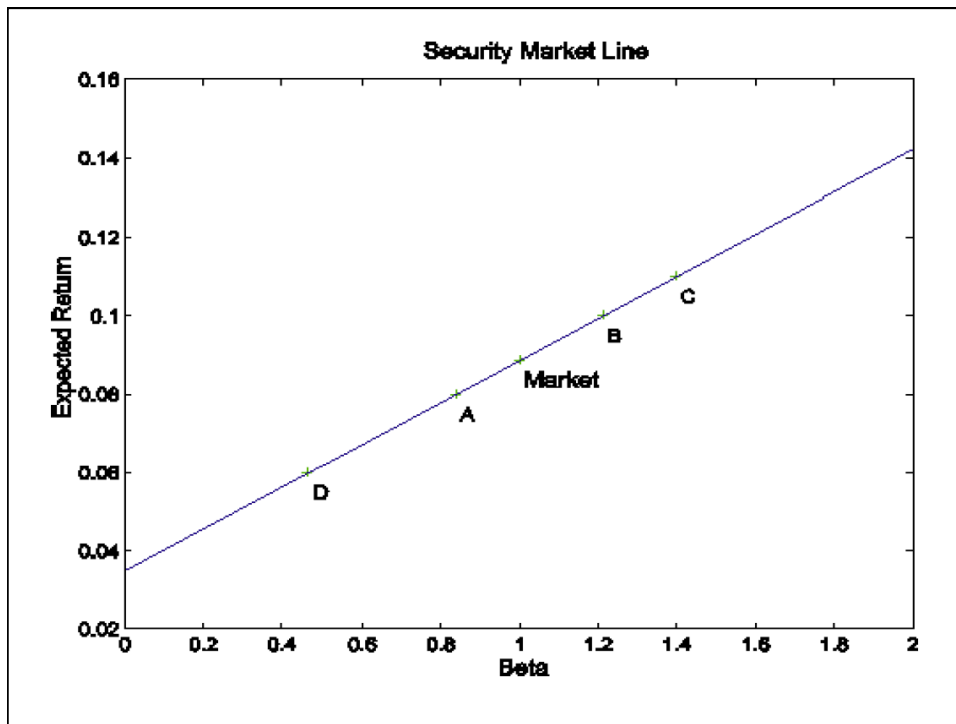
$$\beta_i = \frac{COV(r_i, r_m)}{\sigma_m^2}$$

- 1.1 use the equation for the covariance of two portfolios

$$\begin{aligned} COV(r_A, r_m) &= COV(r_A, w_A r_A + w_B r_B + w_C r_C + w_D r_D) \\ &= w_A COV(r_A, r_A) + w_B COV(r_A, r_B) \\ &\quad + w_C COV(r_A, r_C) + w_D COV(r_A, r_D) \end{aligned}$$

- 1.2 use the equation for the variance of a portfolio:

$$\begin{aligned} \sigma_m^2 &= \sum_{i=A}^D \sum_{j=A}^D w_i w_j COV(r_i, r_j) \\ &= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + w_D^2 \sigma_D^2 + \underbrace{2w_A w_B COV(r_A, r_B) + \dots}_{6 \text{ covariance terms}} \end{aligned}$$



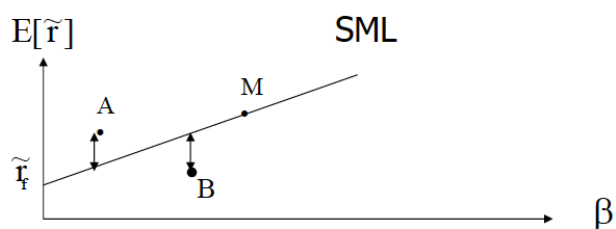
CAPM and Applications

- Whether the CAPM holds in the data is a hotly debated topic among academics
- Even if the CAPM is wrong empirically, there are several reasons we want to spend some time with it:
 - Portfolio performance measurement
 - Identify inefficient priced assets
 - Discount uncertain cash flow for pricing projects, IPO, non-traded assets

Performance Measurement

- Compare the performance of mutual funds and fund managers
 - Example: $E[\tilde{r}_m]=15\%$ and $r_f=5\%$. The average returns of fund A and B are 16% and 18%. Their betas are 1 and 1.4 Which fund manager has better information than the market?
 - Jensen's alpha measures the performance of a portfolio or security relative to the market

Identify inefficient securities



- All efficiently priced assets fall on the SML
- Which security should we buy/sell?

Compute NPV of Uncertain Cash Flows

- Calculating NPV for a project or IPO